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Amplification of electromagnetic waves in dusty nonstationary plasmas

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The propagation of electromagnetic radiation in multicomponent unmagnetized plasmas with dust particulates is investigated. It is demonstrated that the effect of capture of electrons by the dust particles can give rise to an amplification of the electromagnetic waves. The results could be important for the interpretation of high-frequency waves in space and astrophysical dusty plasmas.

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Physics of dusty plasmas is recently studied rather intensively because of its importance for a number of applications in space plasmas, the earth's environment, as well as in the laboratory (e.g., HF plasma etching) [1–6]. The dust particles are highly charged ($Z_d \sim 10^3$) and are of size a much less than the Debye length $r_{\rm De}$. The characteristic feature of the dust particles is that their charges are not fixed. Thus, dusty plasmas, i.e., plasmas with embedded macroscopic particles with great and not constant charges, should be considered as open systems with an external sink (and/or source) of electrons and ions [7]. In open systems, the so-called plasma-maser effect can be important.

The plasma-maser instability is a relatively new nonlinear process in plasma turbulence [8,9]. This effect arises due to a coupling between two types of waves. The first type involves resonant modes satisfying the condition of Cherenkov resonance with plasma particles, whereas the second one encompasses the nonresonant modes for which both the Cherenkov and the scattering conditions are not fulfilled. The effect is basically an energy up- (or down-) conversion where the energy flows from the resonant oscillations to the nonresonant waves in the process of their nonlinear interaction with plasma particles. One of the attractive features of the plasma-maser interaction is that the wave amplification takes place without inverse particle population. The maser instability is considered to play an important role in interpretation of numerous anomalous radiation phenomena in laboratory and astrophysical plasmas.

The plasma-maser interaction is mostly effective in open systems, where the direct mode coupling cannot be balanced by processes of reversed absorption (the latter is due to the quasilinear evolution of the resonant waves), because of the energy source and/or sink outside the system [10].

Recently, the plasma-maser instability of Langmuir and electromagnetic waves in the presence of dust-acoustic turbulence has been considered [11]. In this paper, the interaction due to the direct nonlinear coupling of the waves was investigated. Thus the effects of the system's nonstationarity have been neglected. This situation takes place if some external source provides constant particle distribution.

Here, we demonstrate that because of variable charge of the dust particles, the plasma-maser instability can be sufficiently effective even in the presence of quasilinear evolution of resonant turbulence. It is interesting to note that because of compensation of the direct nonlinear coupling and the reversed absorption terms in closed plasma systems, the total effect in an open system (e.g., with variable number of particles) can be described using only parameters of the external source (in our case this is the process of capture of plasma electrons by dust particles; see for details [10]). The analogous problem of "parametric" amplification of electromagnetic waves in systems with electron-ion recombination has been considered in [12].

The current on a dust particle can be described by [7]

$$I(q) = \sum_\alpha \int e_\alpha f_\alpha \sigma_\alpha(v,q) v d\mathbf{v}, \qquad (1)$$

where the subscript $\alpha = e, i$ describes electrons or ions, f_α is the distribution function, $v \equiv |\mathbf{v}|$ is the absolute value of particle speed $\mathbf{v}$, q is the charge of the dust particle, σ_α is the charging cross section,

$$\sigma_\alpha = \pi a^2 \left(1 - \frac{2e_\alpha q}{a m_\alpha v^2}\right) \quad \text{if} \quad \frac{2e_\alpha q}{a m_\alpha v^2} < 1,$$
$$\sigma_\alpha = 0 \quad \text{if} \quad \frac{2e_\alpha q}{a m_\alpha v^2} \geq 1, \qquad (2)$$

and, finally, m_α is the mass of electrons and ions. The last inequality in (2) gives restriction on particle charging speeds only for electrons ($e_e = -e$; we point out that the dust particles are negatively charged, i.e., $q < 0$). Therefore, only sufficiently fast electrons can charge the dust particles.

For equilibrium distribution functions, we have

$$I_{\rm eq}(-Z_d e) = 0. \qquad (3)$$

If the equilibrium distributions are thermal, then the equilibrium charge is given by

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$$\frac{\omega_{pe}^2}{v_{T_e}}\exp\left(-\frac{Z_d e^2}{aT_e}\right)=\frac{\omega_{pi}^2}{v_{T_i}}\left(\frac{T_i}{T_e}+\frac{Z_d e^2}{aT_e}\right), \quad (4)$$

where $\omega_{p\alpha}=(4\pi n_\alpha e_\alpha^2/m_\alpha)^{1/2}$ is the plasma frequency (electron or ion), $v_{T_\alpha}=(T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity, and T_α is the corresponding temperature.

Furthermore, it is convenient to introduce the dimensionless variables (see [7])

$$\tau\equiv\frac{T_i}{T_e}, \qquad z\equiv\frac{Z_d e^2}{aT_e}. \quad (5)$$

For hydrogen plasmas, Eq. (4) gives $z\simeq 2.5$ if $\tau=1$ and $z\simeq 1.9$ if $\tau=0.1$. Usually, Z_d is of order 10^3–10^4, but the total negative charge on the dust particles is close to (and does not exceed as a rule) the total charge of electrons. Thus the following dimensionless parameter is of order unity [7]:

$$\mu\equiv\frac{Z_d n_d}{n_e}\sim 1, \quad (6)$$

where n_d is the dust density. However, more precise calculation (which was obtained for a dust cloud in a thermal plasma [13]) gives that the following parameter is of order unity:

$$P\equiv\frac{n_d}{n_e}\frac{aT_e}{e^2}=\frac{\mu}{z}. \quad (7)$$

When $z\sim 1$, these two conditions coincide.

The charging dissipative process is characterized by the charging frequency ν_d [7]

$$\nu_d\equiv-\left.\frac{\partial I(q)}{\partial q}\right|_{q=-Z_d e}=\frac{1}{\sqrt{2\pi}}\left[\frac{\omega_{pi}^2 a}{v_{T_i}}+\frac{\omega_{pe}^2 a}{v_{T_e}}\mathrm{e}^{-z}\right]$$
$$=\frac{1}{\sqrt{2\pi}}\frac{\omega_{pi}^2 a}{v_{T_i}}(1+\tau+z). \quad (8)$$

Note that the charging frequency reflects the process of charging of the dust particles, thus it contains contributions from both electrons and ions. To obtain the last expression in the right-hand side of (8), Eq. (4) has been used. Comparison of the rate (8) with the ion-ion collision frquency [7] demonstrates that this process can be the most important dissipative process for dusty plasmas. However, for our purposes the frequency characterizing the rate of capture of electrons by dust particles ν_{ed} is more interesting,

$$\nu_{ed}=\frac{n_d}{n_e}\int f_e\sigma_e v d\mathbf{v}=4\sqrt{\frac{\pi}{2}}a^2 v_{T_e} n_d \mathrm{e}^{-z}=\nu_d P\frac{\tau+z}{1+\tau+z}=2\omega_{p0}\sqrt{\frac{Pn_d a^3}{2}}\sqrt{\frac{n_e}{n_0}}\exp\left(-\frac{\mu_0}{P}\frac{n_0}{n_e}\right), \quad (9)$$

where $\omega_{p0}=(4\pi n_0 e^2/m_e)^{1/2}$, $\mu_0=Z_d n_d/n_0$, and n_0 is the electron density at the initial moment.

To derive an expression for the growth rate Γ which is the result of the maser instability, the methods of [10] can be used. The growth rate describes the change of a number of quanta N_k of electromagnetic waves, $dN_k/dt=2\Gamma N_k$. For the case of an electromagnetic wave with frequency ω propagating in a nonstationary plasma, the rate Γ obtained in [10] is given by

$$\Gamma=-\frac{\omega}{4}\frac{\partial^2}{\partial\omega\partial t}\varepsilon_{\omega\mathbf{k}}^{(\mathrm{tr})}(t), \quad (10)$$

where the superscript (tr) denotes the transversal character of the dielectric function. Here, we note that the charging process of dust particles (as well as its perturbation) also influences high-frequency dielectric function of electromagnetic waves leading to a contribution which is additional to the usual term with electron plasma frequency in $\varepsilon_{\omega\mathbf{k}}^{(\mathrm{tr})}(t)$. However, for the frequencies $\omega\gg\nu_d,\nu_{ed}$ this contribution is small (see the analogous problem for longitudinal waves [7]). Thus, below we take into account only the electron contribution to $\varepsilon_{\omega\mathbf{k}}^{(\mathrm{tr})}(t)$. We have

$$\Gamma=-\frac{2\pi e^2}{m_e\omega^2}\frac{dn_e}{dt}=\frac{1}{2}\frac{\omega_{pe}^2}{\omega^2}\nu_{ed}=\frac{4\pi\omega_{p0}n_e e^2}{m_e\omega^2}\sqrt{\frac{Pn_d a^3}{2}}\sqrt{\frac{n_e}{n_0}}\exp\left(-\frac{\mu_0}{P}\frac{n_0}{n_e}\right). \quad (11)$$

Now, we can find functions $n_e(t)$, $\varepsilon(t)$, and $\omega(t)$ whose temporal dependence is due to the process (9) of capture of electrons by dust particles. The law describing change of the quanta number with time is given by

$$N_k(t)=N_0\exp\left[2\int_0^t dt_1\Gamma(t_1)\right]=N_0\left(1+\frac{\omega_{p0}^2}{k^2c^2}\right)\left[1+\frac{\omega_{p0}^2}{k^2c^2}\frac{n_e(t)}{n_0}\right]^{-1}, \quad (12)$$

where the dispersion relation for electromagnetic waves $\omega^2=k^2c^2+\omega_{p0}^2 n_e(t)/n_0$ has been used. For the function $n_e(t)$, two different cases can be considered. The first case corresponds to such an evolution of the dust-plasma system, when the parameter P (7) remains constant. The latter takes place for slow development of the instability, i.e., when the rate Γ (11) is much less than the charging frequency. This can be realized for sufficiently high frequency electromagnetic waves, when $\omega\gg\omega_{pe}$. In this case, the function $n_e(t)$ is determined by the following relation:

$$\sqrt{2\mu_0 n_d a^3}\,\omega_{p0}t=8\int_{\sqrt{\mu_0/P}}^{\sqrt{\mu_0 n_0/Pn_e}}dy\exp\left(y^2\right). \quad (13)$$

For the second case, the rate (9) of capture of plasma electrons by dust particles is constant. This can take place for long-wavelength electromagnetic waves when $\omega\sim\omega_{pe}$. We have

$$n_e(t)=n_0\exp(-\nu_{ed}t). \quad (14)$$

Equation (12) together with (13) and (14) gives us the corresponding temporal dependences of the quanta number of electromagnetic waves. We note that the rate of change of the wave energy $dW(t)/dt$ does not coincide with the rate $dN(t)/dt$ due to adiabatic change of wave frequency $\omega(t)$ because of the system's nonstationarity (see [10]). It is easy to see that the temporal dependence of the wave frequency obeys the equation $d\omega(t)/dt = -\Gamma\omega(t)$. Therefore the wave energy density $W(t)$ changes in time twice as slowly as the number of quanta $N(t)$.

When $n_e \to 0$ we formally obtain from (12)

$$\frac{N_\infty}{N_0} = 1 + \frac{\omega_{p0}^2}{k^2c^2} = \frac{\omega_0^2}{\omega_0^2 - \omega_{p0}^2}, \tag{15}$$

where ω_0 is the electromagnetic wave frequency at the initial moment. Note that the actual dependence $n_e(t)$ is absent in the result (15). However, we should mention that the described process is effective when the total electric charge is much greater than that of the dust particles (i.e., $n_0 \gg Z_d n_d$) and/or electron temperature significantly exceeds $Z_d e^2/a$. Note that parameter P can be of order unity from the beginning of the process (in contrast to parameter μ).

The expression (15) becomes infinite when $k \to 0$. However, in this limit we can use (12) to find

$$\frac{N_{\mathbf{k}\to 0}(t)}{N_0} = \frac{n_0}{n_e(t)}. \tag{16}$$

To conclude, we have studied the effect of amplification of electromagnetic radiation due to the plasma-maser instability in open dusty plasma systems. It is found that the capture of plasma electrons by dust particles leads to an increasing of the number of electromagnetic quanta. The obtained results should be useful for understanding the enhanced emission of electromagnetic waves in multicomponent dusty plasmas such as those in planetary dust rings, interstellar clouds, and the earth's mesopause.

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[1] U. de Angelis, V. Farmisano, and M. Giordano, J. Plasma Phys. **40**, 399 (1988).
[2] C.K. Goertz, Rev. Geophys. **27**, 271 (1989).
[3] V. Tsytovich, U. de Angelis, and R. Bingham, J. Plasma Phys. **42**, 429 (1989).
[4] U. de Angelis, R. Bingham, and V. Tsytovich, J. Plasma Phys. **42**, 445 (1989).
[5] O. Havnes, U. de Angelis, R. Bingham, C. Goertz, G. Morfill, and V. Tsytovich, J. Atmos. Terr. Phys. **52**, 637 (1990).
[6] R. Bingham, U. de Angelis, V.N. Tsytovich, and O. Havnes, Phys. Fluids B **3**, 811 (1991).
[7] V.N. Tsytovich and O. Havnes, Commun. Plasma Phys. Contr. Fusion **15**, 267 (1993).
[8] V.N. Tsytovich, L. Stenflo, and H. Wilhelmsson, Phys. Scr. **11**, 251 (1975).
[9] M. Nambu, Phys. Rev. Lett. **34**, 387 (1975).
[10] V.S. Krivitsky, V.N. Tsytovich, and S.V. Vladimirov, Phys. Rep. **218**, 141 (1992).
[11] M. Nambu, P.K. Shukla, and S.V. Vladimirov, Phys. Lett. A **180**, 441 (1993).
[12] V.S. Krivitsky and S.V. Vladimirov, Phys. Scr. **47**, 90 (1993).
[13] O. Havnes, G.E. Morfill, and C.K. Goertz, J. Geophys. Res. **89**, 10 999 (1984).