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On electric forces in a time-dependent medium

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Abstract

Forces acting on a non-stationary medium in the presence of a high-frequency electrostatic field include the ponderomotive force, the dissipative force (taking into account the slow non-stationarity of the system), and the force due to the full change of the field momentum. For the fields of normal modes (propagating waves), only the ponderomotive force acts on the medium.

Electromagnetic properties of a non-stationary medium are interesting for many applications to plasmas as well as optics and condensed matter physics. Forces acting on the medium due to the presence of electric and magnetic fields are closely connected with energy and momentum densities of the fields [1–4]. Although the corresponding expressions in a stationary dispersive medium are well known [1,2], the problem for a time-dependent medium is less trivial. The main point is that in a (weakly) time-dependent transparent medium an imaginary part of the dielectric function appears [1,5] which depends on the slow time scale. In a closed system, waves propagate adiabatically, conserving the wave action (number of quanta) [3,5]. The conservation of the quanta number is connected with the slow time evolution of the system thus determining its scale [6]. In contrast, the slow time scale in open systems is in general indefinite and depends on the nature of the energy exchange with external sources and sinks. For forces acting on the medium in the presence of high-frequency fields, we can expect a similar situation.

To find the ponderomotive force, the macroscopic [1,5,7–9] as well as microscopic (based on concrete microscopic e.g. plasma models, see Refs. [9–14]) consideration can be used. In this Letter, we employ the macroscopic approach based on general theory of electrodynamics of a continuous fluid-like medium with linear dependence of its susceptibility on density. We demonstrate that, generally, a high-frequency electrostatic field in a non-stationary medium creates three types of forces (see also Ref. [11]): the ponderomotive force, the dissipative force (which can include terms because of the medium nonstationarity), and, finally, the force due to the change of the field momentum. We find that for normal modes, the general macroscopic consideration is sufficient to establish that only the ponderomotive force acts on the medium. Moreover, although for arbitrary fields acting in open systems the general approach lacks a detalization necessary to establish the scale of the slow time evolution, we demonstrate that even in this case the full force can be written in a form similar to that for propagating waves.

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We invoke the geometrical optics approximation when the electrostatic field amplitude is written as

$$\begin{aligned} E(t,r) &= \tfrac{1}{2}\int \frac{\mathrm{d}k}{(2\pi)^3} E_k(t)\exp(\mathrm{i}k\cdot r) + \text{c.c.} \\ &= \tfrac{1}{2}\int \frac{\mathrm{d}k}{(2\pi)^3}\frac{k}{|k|} E_k(t)\exp(\mathrm{i}k\cdot r) + \text{c.c.} \\ &= \tfrac{1}{2}\int \frac{\mathrm{d}k}{(2\pi)^3}\frac{k}{|k|} E_k^{(\mathrm{a})}(t) \\ &\quad\times \exp\Bigl(-\mathrm{i}\int^{t} \omega(t')\,\mathrm{d}t' + \mathrm{i}k\cdot r\Bigr) + \text{c.c.} \end{aligned} \tag{1}$$

In the linear approximation, the general relationship (allowing for causality) between the spatial Fourier components of the charge density $\rho_k(t)$, polarization density $P_k(t)$, and electric field $E_k(t)$, is given by

$$\begin{aligned} \rho_k(t) &= \hat{P}_k(t)\,\mathrm{i}k\cdot E_k(t) \\ &= \int_{-\infty}^{t} \frac{\mathrm{d}t'}{2\pi}\mathrm{i}|k| P_k(t,t')\,E_k(t'). \end{aligned} \tag{2}$$

Here, we note that the conjugate charge density ρ^* contains the polarization operator $\hat{P}^+$ adjoint to $\hat{P}$; as was demonstrated in Ref. [11], in a Vlasov plasma, the adjoint operator P^+ corresponds to reversion of time in the Vlasov equation.

The averaged over the high frequency $\omega(t)$ force acting on the medium is defined by a variation of the free energy [1],

$$f = -\frac{\delta\mathcal{F}}{\delta r} = -\frac{\delta\mathcal{F}_0}{\delta r} - \frac{\delta\mathcal{F}_E}{\delta r} = f_0 + f_E, \tag{3}$$

where $\delta\mathcal{F}_0$ is the change of the free energy of the medium in the absence of electric fields and $\delta\mathcal{F}_E$ is the change of the free energy because of the fields. The spatial Fourier component of the force f_E (below we omit the subscript E since only this part of (3) will be considered) for a fluid-like homogeneous medium (i.e. we allow for a weak inhomogeneity of the field amplitude only) with linear dependence of its susceptibility on density can be written as

$$f_k(t) = \langle\rho E\rangle_k(t) - \nabla\langle E\cdot\hat{P}E\rangle_k(t). \tag{4}$$

For the concrete model of plasma collisionless hydrodynamics, this expression coincides with Eq. (4) of Ref. [14] written for electrostatic waves. Note that the last term on the right-hand side of Eq. (4) was not taken into account in Ref. [11].

In a stationary system, the medium polarization P_k depends only on $t-t'$. If the parameters of the system change sufficiently slow comparing with the characteristic period of oscillations, P_k is a rapidly changing function of $\tau = t - t'$ and a slowly changing function of another argument which depends linearly on τ and can be written in the most general form as $t - \alpha\tau$, where α is a constant. Thus we have

$$P_k(t,t') \approx P_k(\tau,t) - \alpha\tau\frac{\partial P_k(\tau,t)}{\partial t}. \tag{5}$$

Since the work [5], it is customary to assume $\alpha = 1/2$. However, on this stage the choice of the second argument in the form of $\tau/2$ is in fact arbitrary. In this situation, the authors of Ref. [15] concluded that the magnitude of the imaginary part of the dielectric function due to the system's nonstationarity depends on the model used to describe the system. However, it is possible to demonstrate [6] that in general the "effective" slow time scale of a closed system always (i.e. independently of internal processes in the system) corresponds to $\alpha = 1/2$, thus ensuring the conservation of the total energy and wave action (number of quanta). For an open system, the effective time scale is determined by the energy exchange with external sources and sinks, and therefore no general answer can be provided. Below, we demonstrate that the slow time scale is irrelevant in calculating the full force acting on the time-dependent medium, i.e. the final result can be written in a similar form without time derivatives of the slowly changing quantities such as amplitudes and (high) frequencies of the electric fields and the medium characteristics.

We note, however, that the above irrelevance does not mean that the slow time evolution does not affect the physical result. In the case of propagating waves, the slow time dependence is contained in the wave amplitude which satisfies the dispersion equation. The dispersion relation connects the (slow) time evolution of the field amplitude with the change of the wave eigenfrequency as well as with the change of the medium parameters. In the case of non-wave fields when there is no dispersion relation, the resulting full force still can be written in the similar form using Poisson equation with the (external) charges generating the field. In this case, the time evolution of the amplitude and frequency of the electric field as well as the time scale of the medium parameters also depend on the external sources.

In Eq. (2), we expand the electric field amplitude and phase in τ. We have

$$E_{\boldsymbol{k}}^{(\mathrm{a})}(t-\tau) \simeq E_{\boldsymbol{k}}^{(\mathrm{a})}(t) - \tau \frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t)}{\partial t}, \tag{6}$$

and

$$\exp\left(-\mathrm{i}\int_t^{t-\tau} \omega(t')\,\mathrm{d}t'\right) \simeq \exp\left[\mathrm{i}\omega(t)\tau\right]\left(1 - \frac{\mathrm{i}\tau^2}{2}\frac{\partial\omega(t)}{\partial t}\right). \tag{7}$$

Using Eq. (5), we thus find

$$\begin{aligned}
\rho_{\boldsymbol{k}}(t,\tau) \approx{}& \mathrm{i}|\boldsymbol{k}| \int_0^\infty \frac{\mathrm{d}\tau}{4\pi}\Bigg(P_{\boldsymbol{k}}^{(0)}(t)\, E_{\boldsymbol{k}}^{(\mathrm{a})}(t) \\
&- \alpha\tau \frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial t} E_{\boldsymbol{k}}^{(\mathrm{a})}(t) - \tau P_{\boldsymbol{k}}^{(0)}(t) \frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t)}{\partial t} \\
&- \frac{\mathrm{i}\tau^2}{2} P_{\boldsymbol{k}}^{(0)}(t) \frac{\partial \omega(t)}{\partial t} E_{\boldsymbol{k}}^{(\mathrm{a})}(t)\Bigg) \\
&\times \exp\left(\mathrm{i}\omega(t)\,\tau - \mathrm{i}\int^t \omega(t')\,\mathrm{d}t'\right) + \text{c.c.} \\
\approx{}& \frac{\mathrm{i}|\boldsymbol{k}|}{2}\Bigg(P_{\omega\boldsymbol{k}}^{(0)}(t)\, E_{\boldsymbol{k}}^{(\mathrm{a})}(t) + \mathrm{i}\alpha \frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega\partial t} E_{\boldsymbol{k}}^{(\mathrm{a})}(t) \\
&+ \mathrm{i}\frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial \omega}\frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t)}{\partial t} \\
&+ \frac{\mathrm{i}}{2}\frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega^2}\frac{\partial\omega(t)}{\partial t} E_{\boldsymbol{k}}^{(\mathrm{a})}(t)\Bigg) \\
&\times \exp\left(-\mathrm{i}\int^t \omega(t')\,\mathrm{d}t'\right) + \text{c.c.},
\end{aligned} \tag{8}$$

where

$$P_{\omega\boldsymbol{k}}^{(0)}(t) = \int_0^\infty \frac{\mathrm{d}\tau}{2\pi} P_{\boldsymbol{k}}^{(0)}(t,\tau) \exp\left[\mathrm{i}\omega(t)\,\tau\right]. \tag{9}$$

In Eq. (8), the partial time derivative applies only to the time dependence of the polarization function (i.e. with the fixed frequency ω). Thus the full time derivative of the polarization is given by

$$\frac{\mathrm{d}P_{\omega\boldsymbol{k}}^{(0)}(t)}{\mathrm{d}t} = \left.\frac{\partial P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial t}\right|_{\omega=\omega(t)} + \frac{\partial P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega}\frac{\partial\omega(t)}{\partial t}. \tag{10}$$

Assuming in Eq. (8) slow spatial inhomogeneity of the electric field amplitude (i.e. putting $\boldsymbol{k} \to \boldsymbol{k} - \mathrm{i}\boldsymbol{\nabla}$, where $|\boldsymbol{\nabla}| \ll |\boldsymbol{k}|$, and spatial inhomogeneity is of the order of the time nonstationarity), we can write

$$\begin{aligned}
\langle\rho \boldsymbol{E}\rangle_{\boldsymbol{k}}(t,\boldsymbol{r}) \approx{}& \tfrac{1}{4}\Bigg(\mathrm{i}kP_{\omega\boldsymbol{k}}^{(0)}(t)\,|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&+ P_{\omega\boldsymbol{k}}^{(0)}(t)\,\boldsymbol{\nabla}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&- \boldsymbol{k}\alpha\frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega\partial t}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&- \boldsymbol{k}\frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial\omega}[E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})]^*\frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})}{\partial t} \\
&- \frac{\boldsymbol{k}}{2}\frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega^2}\frac{\partial\omega(t)}{\partial t}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&+ \boldsymbol{k}\frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial\boldsymbol{k}}\cdot\boldsymbol{\nabla}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2\Bigg) + \text{c.c.}
\end{aligned} \tag{11}$$

Together with the second term on the right-hand side of Eq. (4) we obtain

$$\begin{aligned}
\boldsymbol{f}_{\boldsymbol{k}}(t,\boldsymbol{r}) \approx{}& \tfrac{1}{4}\Bigg(-P_{\boldsymbol{k}}^{(0)}(t)\,\boldsymbol{\nabla}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&+ \mathrm{i}\boldsymbol{k}P_{\omega\boldsymbol{k}}^{(0)}(t)\,|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&- \boldsymbol{k}\alpha\frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega\partial t}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&- \boldsymbol{k}\frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial\omega}[E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})]^*\frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})}{\partial t} \\
&- \frac{\boldsymbol{k}}{2}\frac{\partial^2 P_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega^2}\frac{\partial\omega(t)}{\partial t}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2 \\
&+ \boldsymbol{k}\frac{\partial P_{\boldsymbol{k}}^{(0)}(t)}{\partial\boldsymbol{k}}\cdot\boldsymbol{\nabla}|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2\Bigg) + \text{c.c.}
\end{aligned} \tag{12}$$

Next, we introduce the medium dielectric permittivity according to

$$P_{\omega\boldsymbol{k}}^{(0)}(t)\, E_{\boldsymbol{k}}(t,\boldsymbol{r}) = -\frac{1}{4\pi}[\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t) - 1]E_{\boldsymbol{k}}(t,\boldsymbol{r}), \tag{13}$$

and suppose that the imaginary part of the dielectric function $\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t)$ is of the same (small) order as the corrections due to the medium non-stationarity. Therefore (12) can be rewritten as

$$f_k(t,r) = \frac{1}{16\pi}\Bigg[[\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t) - 1]\nabla|E^{(a)}_k(t,r)|^2 + 2k\,\mathrm{Im}\,\varepsilon^{(0)}_{\omega k}(t)|E^{(a)}_k(t,r)|^2 + (2\alpha-1)k\frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega\partial t}|E^{(a)}_k(t,r)|^2 + \frac{\mathrm{d}}{\mathrm{d}t}\left(k\frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega}|E^{(a)}_k(t,r)|^2\right)\Bigg]. \tag{14}$$

We see that the full expression (14) includes the ponderomotive force (the first term on the right-hand side), the dissipative force (the second and third terms), as well as the rate of change of the momentum of the field (the last term) which also accounts for the momentum flow in space, i.e. the full time derivative there is (in contrast to (10))

$$\frac{\mathrm{d}N_k(t,r)}{\mathrm{d}t} \equiv \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega}|E^{(a)}_k(t,r)|^2\right) = \frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega\partial t}|E^{(a)}_k(t,r)|^2 + \frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega}\frac{\partial|E^{(a)}_k(t,r)|^2}{\partial t} + \frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega^2}\frac{\partial\omega(t)}{\partial t}|E^{(a)}_k(t,r)|^2 - \frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial k}\cdot\nabla|E^{(a)}_k(t,r)|^2. \tag{15}$$

The terms with partial time derivatives of Eq. (14) for $\alpha = 1$ coincide with those found on the basis of collisionless plasma hydrodynamics by the authors of Ref. [14] (if we neglect the external magnetic field, consider all electric fields as longitudinal and allow for the slow time variation of their frequency ω).

If the electric field corresponds to a normal mode of the medium (i.e. $\omega(t) = \omega_k(t)$ is the solution of a dispersion equation), Eq. (14) can be significantly simplified. Indeed, the dispersion equation of the (longitudinal) waves in a slowly nonstationary medium is

$$D(t,r) \simeq \int\frac{\mathrm{d}R}{(2\pi)^3}\int_0^\infty\frac{\mathrm{d}\tau}{2\pi}\left(\varepsilon(\tau,t;R) - \alpha\tau\frac{\partial\varepsilon(\tau,t;R)}{\partial t}\right)\int\mathrm{d}k\,E^{(a)}_k(t-\tau,r-R) \times\exp\left(-\mathrm{i}\int_t^{t-\tau}\omega_k(t')\,\mathrm{d}t' + \mathrm{i}k\cdot(r-R)\right) = 0. \tag{16}$$

Using expansions (6) and (7), we obtain

$$D_k(t,r) \approx \Bigg[\Bigg(\mathrm{Re}\,\varepsilon^{(0}_{\omega k}(t) + \mathrm{i}\,\mathrm{Im}\,\varepsilon_{\omega k}(t) + \mathrm{i}\alpha\frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega\partial t} + \frac{\mathrm{i}}{2}\frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega^2}\frac{\partial\omega_k(t)}{\partial t}\Bigg) + \mathrm{i}\frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega}\frac{\partial E^{(a)}_k(t,r)}{\partial t}\Bigg]\Bigg|_{\omega=\omega_k(t)}E^{(a)}_k(t,r) = 0. \tag{17}$$

In the first approximation, we have

$$\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)\Big|_{\omega=\omega_k(t)} = 0. \tag{18}$$

The approximation next to (18) gives us

$$\frac{1}{E^{(a)}_k(t,r)}\frac{\partial E^{(a)}_k(t,r)}{\partial t} = -\left(\frac{\partial\,\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega}\right)^{-1} \times\left(\mathrm{Im}\,\varepsilon_{\omega k}(t) + \alpha\frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega\partial t} + \frac{1}{2}\frac{\partial^2\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t)}{\partial\omega^2}\frac{\mathrm{d}\omega_k(t)}{\mathrm{d}t}\right)\Bigg|_{\omega=\omega_k(t)}. \tag{19}$$

Therefore we see that Eq. (11) can be written as

$$\langle\rho E\rangle_k(t) = -\frac{1}{16\pi}[\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t) - 1]\nabla|E^{(a)}_k(t,r)|^2 = \frac{1}{16\pi}\nabla|E^{(a)}_k(t,r)|^2. \tag{20}$$

Thus in the case of propagating waves only the ponderomotive force

$$f^{(p)}_{i,k}(t,r) = \frac{1}{16\pi}[\mathrm{Re}\,\varepsilon^{(0)}_{\omega k}(t) - 1]\nabla|E^{(a)}_k(t,r)|^2 = -\frac{1}{16\pi}\nabla|E^{(a)}_k(t,r)|^2 \tag{21}$$

really contributes to expression (14) for the full force.

If the electrostatic high-frequency field does not correspond to any propagating wave, instead of dispersion Eq. (17) we have

$$\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{D}_{\boldsymbol{k}}(t,\boldsymbol{r}) = 4\pi\rho_{\boldsymbol{k}}^{\mathrm{ext}}(t,\boldsymbol{r}), \tag{22}$$

where $\rho_{\boldsymbol{k}}^{\mathrm{ext}}(t,\boldsymbol{r})$ is the external charge density amplitude. Thus instead of (17) we have

$$\begin{aligned}&\left[E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})\left(\mathrm{Im}\,\varepsilon_{\omega\boldsymbol{k}}(t) + \alpha\frac{\partial^2\,\mathrm{Re}\,\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega\partial t}\right.\right.\\&\quad\left.+\frac{1}{2}\frac{\partial^2\,\mathrm{Re}\,\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega^2}\frac{\partial\omega(t)}{\partial t}\right)\\&\quad\left.\left.+\frac{\partial\,\mathrm{Re}\,\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t)}{\partial\omega}\frac{\partial E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})}{\partial t}\right]\right|_{\omega=\omega(t)}\\&\quad= -\frac{4\pi}{|\boldsymbol{k}|}\rho_{\boldsymbol{k}}^{\mathrm{ext}}(t,\boldsymbol{r}) + \mathrm{i}\,\mathrm{Re}\,\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t)\,E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r}).\end{aligned} \tag{23}$$

Moreover, $\langle\rho\boldsymbol{E}\rangle$ in Eq. (4) should be changed to

$$\langle\rho\boldsymbol{E}\rangle \to \langle(\rho+\rho^{\mathrm{ext}})\boldsymbol{E}\rangle. \tag{24}$$

This corresponds to accounting for the work of external sources (see Ref. [1], §15). Thus we finally find that the full force acting on the system again can be written as

$$\boldsymbol{f}_{i,\boldsymbol{k}}^{(\mathrm{p})}(t,\boldsymbol{r}) = \frac{1}{16\pi}[\mathrm{Re}\,\varepsilon_{\omega\boldsymbol{k}}^{(0)}(t) - 1]\nabla|E_{\boldsymbol{k}}^{(\mathrm{a})}(t,\boldsymbol{r})|^2. \tag{25}$$

In summary, we demonstrated that the full force of the high-frequency electrostatic field acting on a time-dependent medium includes, in addition to the ponderomotive force, the dissipative force and force due to change of the momentum of the wave. Because of the dispersion relation, for fields of normal modes (propagating waves) only the ponderomotive force (21) containing the slow changing wave amplitudes actually contributes to the full force. For non-wave fields, the corresponding equation (25) contains also the slowly changing dielectric permittivity of the nonstationary medium.

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