Ion-acoustic solitons in electron–positron–ion plasmas

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(Received 20 September 1994; accepted 1 December 1994)

The ion-acoustic solitons are investigated in three-component plasmas, whose constituents are electrons, positrons, and singly charged ions. It is found that the presence of the positron component in such a multispecies plasma can result in reduction of the ion-acoustic soliton amplitudes. © *1995 American Institute of Physics.*

I. INTRODUCTION

It is widely thought that electron–positron plasmas have presumably appeared in the early universe,[1,2] and are frequently encountered in active galactic nuclei[3] and in pulsar magnetospheres.[4,5] An electron–positron plasma is usually characterized as a fully ionized gas consisting of electrons and positrons, the masses of which are equal. Recently, there has been a great deal of interest in studying linear as well as nonlinear wave motions in such plasmas.[6–12] The nonlinear studies have been focused on the nonlinear self-consistent structures,[6,7] such as envelope solitons, vortices, etc.

However, most of the astrophysical plasmas usually contains ions, in addition to the electrons and positrons. Clearly, the properties of wave motions in an electron–positron–ion plasma should be different from those in two-component electron–positron plasmas. For example, Rizzato[13] and Berezhiani *et al.*[14] have investigated envelope solitons of electromagnetic waves in three-component electron–positron–ion plasmas.

In this paper, we present an investigation of the nonlinear ion-acoustic waves in the presence of cold ions and hot electrons and positrons. In our model, the ion dynamics is governed by the hydrodynamic equations, whereas the electron and positron fluids follow the Boltzmann distribution. Accordingly, the phase velocity of the oscillations is assumed to be smaller (larger) than the thermal velocity of the electrons and positrons (ions). It is found that the maximum amplitude of ion-acoustic solitons can be considerably reduced when a substantial fraction of hot positrons is present in the system. The results of our investigation should be useful in understanding the nonlinear properties of low phase velocity sound perturbations in an electron–positron–ion plasma.

The paper is organized in the following fashion. In Sec. II, we present the governing equations for nonlinear ion-acoustic waves in an electron–positron–ion plasma. Here, we investigate the influence of a positron component on finite-amplitude ion-acoustic solitons and discuss the small-amplitude limit. Section III contains a brief summary.

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II. FORMULATION

We consider a three-component plasma consisting of electrons, positrons, and singly charged positive ions. The nonlinear propagation of low phase velocity (in comparison with the electron and positron thermal velocities) ion-acoustic waves is governed by the ion continuity equation,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_i)=0; \tag{1}$$

the ion momentum equation,

$$\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial x}=-\frac{e}{m_i}\frac{\partial \varphi}{\partial x}; \tag{2}$$

as well as the Boltzmann distribution for the electrons and positrons, which read, respectively, as

$$n_e=n_{e0}\exp\left(\frac{e\varphi}{T_e}\right), \tag{3}$$

$$n_p=n_{p0}\exp\left(-\frac{e\varphi}{T_p}\right). \tag{4}$$

Equations (1)–(4) are closed with the help of Poisson's equation,

$$\frac{\partial^2 \varphi}{\partial x^2}=4\pi e(n_e-n_i-n_p). \tag{5}$$

In (1)–(5), n_α is the number density of the particle species α (where α equals e for the electrons, i for the ions, and p for the positrons), v_i is the ion fluid velocity, e is the magnitude of the electron charge, m_i is the ion mass, T_e (T_p) is the electron (positron) temperature, and φ is the electrostatic potential. The unperturbed electron and positron densities are denoted by n_{e0} and n_{p0}, respectively. At equilibrium, we have $n_{i0}+n_{p0}=n_{e0}$, where n_{i0} is the unperturbed ion number density.

In the linear limit, (1)–(5) yield the dispersion relation of the ion-acoustic waves in an electron–positron–ion plasma. We have

$$\omega^2=\frac{k^2c_s^2}{1+k^2\lambda_D^2}\frac{1-p}{1+pT_e/T_p}, \tag{6}$$

where ω and k are the frequency and the wave number, $c_s\equiv(T_e/m_i)^{1/2}$ is the ion-sound velocity in electron–ion plasmas without positrons, $\lambda_D^2\equiv(r_{De}^{-2}+r_{Dp}^{-2})^{-1}$, $r_{De(p)}\equiv(T_{e(p)}/4\pi n_{e(p)0}e^2)^{1/2}$ is the electron (positron) Debye

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length, and $p=n_{p0}/n_{e0}$ is the ratio between the unperturbed positron and electron number densities. We note that $p<1$ and the phase velocity of long wavelength (in comparison with λ_D) ion-acoustic waves is reduced when a fraction of the positron component is present.

In the following, we seek stationary solutions of the nonlinear equations (1)–(5). Accordingly, we introduce a new variable $\xi=x-Vt$ and assume steady state in the moving frame, where V is the constant velocity of the nonlinear structure. Thus, all the physical variables depend on ξ and V. Furthermore, we assume the perturbations vanish at $\xi\to\pm\infty$.

Introducing dimensionless quantities, viz.

$$\Phi=\frac{e\varphi}{T_e},\quad M=\frac{V}{c_s},\quad \xi=\frac{\xi}{r_{De}},\quad x=\frac{x}{r_{De}}, \tag{7}$$

we obtain from (1) and (2) the ion number density,[15,16]

$$n_i=\frac{|M|}{\sqrt{M^2-2\Phi}}\,n_{i0}, \tag{8}$$

which can then be substituted, along with (3) and (4), in (5), so that Poisson's equation can be expressed as

$$\frac{\partial^2\Phi}{\partial\xi^2}=-\frac{d}{d\Phi}V(\Phi), \tag{9}$$

where the effective Sagdeev potential[15] for our purposes reads as

$$V(\Phi)=1+p\,\frac{T_p}{T_e}-\exp(\Phi)-p\,\frac{T_p}{T_e}\exp\left(-\frac{T_e}{T_p}\,\Phi\right)+(1-p)|M|(|M|-\sqrt{M^2-2\Phi}). \tag{10}$$

In (10), the arbitrary constant $V(0)$ is chosen to be zero.

Multiplying (9) by $\partial\Phi/\partial\xi$ and recognizing that for localized perturbations (solitons), Φ as well as $\partial\Phi/\partial\xi$ vanish at $\xi\to\pm\infty$, we obtain "the energy law,"

$$\frac{1}{2}\left(\frac{\partial\Phi}{\partial\xi}\right)^2+V(\Phi)=0. \tag{11}$$

In order for the ion-acoustic soliton to exist, the effective potential $V(\Phi)$ must have a local maximum in the point $\Phi=0$, and the equation $V(\Phi)=0$ should have at least one real solution $\Phi_0\neq 0$, which determines the amplitude Φ_0 of the soliton (as a function of M). The local maximum of the effective potential $V(\Phi)=0$ at the point $\Phi=0$ exists if

$$M^2>\frac{1-p}{1+pT_e/T_p}. \tag{12}$$

We note that the condition (12) is a consequence of the inequality $[d^2V(\Phi)/d\Phi^2]|_{\Phi=0}<0$. Furthermore, it follows from (12) that only supersonic ion-acoustic solitons can exist in electron–positron–ion plasmas.

The equation $V(\Phi)=0$ [for M obeying the inequality (12)] can have only one real nonzero solution; this solution Φ_0 being positive. Figure 1 displays the dependence of the effective potential $V(\Phi)$ on the normalized electrostatic potential Φ for $M=1.4$, $p=0.1$, and $T_e=T_p$. It emerges that in electron–positron–ion plasmas, the ion-acoustic solitons would have an electrostatic potential hump.

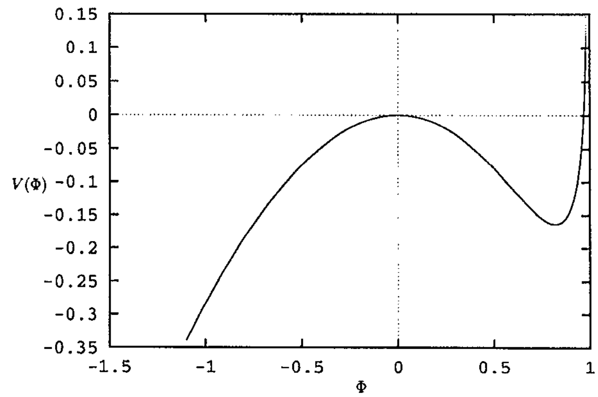

FIG. 1. The variation of $V(\Phi)$ against Φ for $M=1.4$, $p=0.1$, and $T_e=T_p$.

The nonzero solution of the equation $V(\Phi)=0$ can exist only if

$$V(\Phi_{0\max})\geqslant 0, \tag{13}$$

where $\Phi_{0\max}=(\frac{1}{2})M^2$ [see (10)], in addition to (12). The condition (13) can be written as

$$\exp\left(\frac{M^2}{2}\right)\leqslant 1+p\,\frac{T_p}{T_e}-p\,\frac{T_p}{T_e}\exp\left(-\frac{T_e}{T_p}\,\frac{M^2}{2}\right)+(1-p)M^2. \tag{14}$$

We see that the condition (14) restricts significantly the region of the parameter M, for which the existence of ion-acoustic solitons is possible. For example, for $p=0.1$ and $T_p=T_e$, the condition (14) results in the limitation $M\leqslant 1.51$, for which the maximum possible value of the soliton amplitude is $\Phi_{0\max}\approx 1.14$. We note that the value $\Phi_{0\max}\approx 1.14$ is less than the maximum possible amplitude of the ion-acoustic soliton in two-component plasmas (without positrons), $\Phi_{0\max}\approx 1.26$ (corresponding to $M\approx 1.59$). The numerical analysis shows that such a reduction of the amplitude of the ion-acoustic solitary waves in electron–positron–ion plasmas (in comparison with the case of a plasma without the positrons) is inherent, not only in the case $T_e\sim T_p$, but also for cases $T_e\gg T_p$ and $T_e\ll T_p$. The maximum possible Mach number M and, correspondingly, the maximum possible amplitude of the soliton depends on the parameter p significantly. For example, for $p=0.5$ and $T_p=T_e$, the maximum value of M is approximately equal to 1.10 and associated with $\Phi_{0\max}\approx 0.61$. The profiles of the ion-acoustic solitons of maximum possible amplitude for two cases, namely $p=0.1$, $T_e=T_p$ and $p=0.5$, $T_e=T_p$, are presented in Fig. 2 (the wave of the higher amplitude corresponds to the former case). It is seen that the amplitudes of the solitons are drastically reduced in the presence of significant fraction of the positrons. Figure 3 displays the dependence of the Mach number on the fractional number p for a given wave amplitude $\Phi_0\approx 0.61$ and $T_e=T_p$. We see that the existence of the ion-acoustic solitons of such an amplitude is possible for $M\geqslant M_0$, where $M_0\approx 1.10$ (that corresponds to the inequality $M^2\geqslant 2\Phi_0$). Furthermore, for the case $T_e=T_p$, the solitons with the amplitude $\Phi_0\approx 0.61$ can exist only if

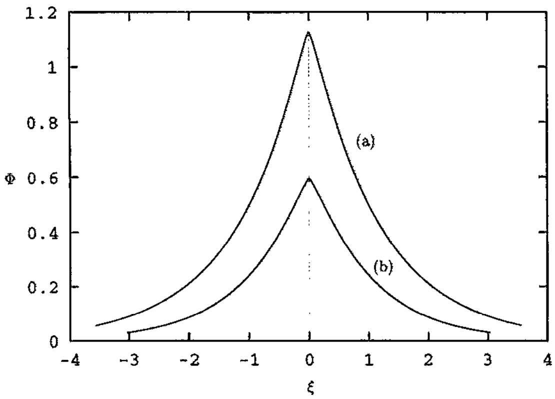

FIG. 2. The ion-acoustic soliton potentials $\Phi(\xi)$ for two cases. (a) $\Phi_0 \approx 1.14$, $M \approx 1.51$, $p=0.1$, and $T_e=T_p$; and (b): $\Phi_0 \approx 0.61$, $M \approx 1.10$, $p=0.5$, and $T_e=T_p$.

$p \leqslant 0.5$. As we have seen, the amplitude $\Phi_0 \approx 0.61$ is the maximum possible one in a plasma with $p=0.5$ and $T_e=T_p$. The speed of the solitons of a given amplitude decreases as the fractional number p increases.

A complete analytical investigation of the ion-acoustic solitons in the electron–positron–ion plasma is possible for small-amplitude waves ($\Phi \ll 1, T_e/T_p, M^2/2$). Here, the specific results can be obtained by expanding $V(\Phi)$ in powers of Φ up to the third-order terms $\sim \Phi^3$. Accordingly, (9) takes the form

$$\frac{\partial^2 \Phi}{\partial \xi^2} + \left(-1 - p\,\frac{T_e}{T_p} + \frac{1-p}{M^2} \right) \Phi + \left[-1 + p\left(\frac{T_e}{T_p}\right)^2 + \frac{3(1-p)}{M^4} \right] \frac{\Phi^2}{2} = 0. \tag{15}$$

It is easy to verify that (15) admits the soliton solution

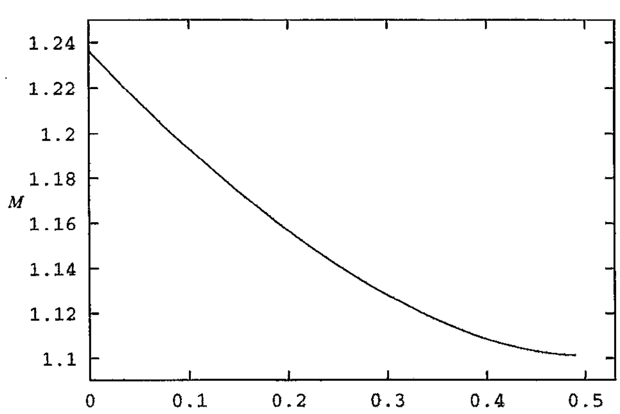

FIG. 3. The dependence of the Mach number M of the ion-acoustic soliton on the fractional number p for a given soliton amplitude $\Phi_0 \approx 0.61$ and $T_e=T_p$.

$$\Phi = \frac{3[1+p(T_e/T_p)-(1-p)/M^2]}{-1+p(T_e/T_p)^2+3(1-p)/M^4} \times \cosh^{-2}\left[\frac{\xi}{2} \left(1 + p\,\frac{T_e}{T_p} - \frac{1-p}{M^2} \right)^{1/2} \right]. \tag{16}$$

The dynamical equation for small-amplitude, long-wavelength (in comparison with λ_D ion-acoustic waves in electron–positron–ion plasmas can be derived from (1)–(5) following the standard procedure.[17,18] It turns out that the wave evolution is governed by a Boussinesq equation,

$$\frac{r_{De}^2}{c_s^2} \frac{1+pT_e/T_p}{1-p} \frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{1+pT_e/T_p} \frac{\partial^4 \Phi}{\partial x^4} + \frac{1}{2} \frac{1}{1+pT_e/T_p} \left[1 - p\left(\frac{T_e}{T_p}\right)^2 - \frac{3}{1-p} \left(1 + p\,\frac{T_e}{T_p} \right)^2 \right] \frac{\partial^2 (\Phi)^2}{\partial x^2} = 0, \tag{17}$$

where the space variable is in the unit of r_{De}. For the unidirectional propagation, (17) reduces to the Korteweg–de Vries equation,

$$\frac{r_{De}}{c_s} \left(\frac{1+pT_e/T_p}{1-p} \right)^{1/2} \frac{\partial \Phi}{\partial t} + \frac{\partial \Phi}{\partial x} + \frac{1}{2} \frac{1}{1+pT_e/T_p} \frac{\partial^3 \Phi}{\partial x^3} - \frac{1}{2} \frac{1}{1+pT_e/T_p} \left[1 - p\left(\frac{T_e}{T_p}\right)^2 - \frac{3}{1-p} \left(1 + p\,\frac{T_e}{T_p} \right)^2 \right] \Phi \frac{\partial \Phi}{\partial x} = 0. \tag{18}$$

The stationary solution of (18) is obviously given by (16), provided that the Mach number M is related to the soliton amplitude Φ_0 by

$$M^2 = \frac{1-p}{1+pT_e/T_p} \left(1 + \frac{\Phi_0}{3} \times \frac{\{-1+p(T_e/T_p)^2+[3/(1-p)][1+p(T_e/T_p)]^2\}}{(1+pT_e/T_p)} \right). \tag{19}$$

It follows that the Mach number is related with the maximum soliton amplitude and the other plasma parameters in a complex manner. However, in the limit of $p=0$, the well-known result ($M^2=1+2\Phi_0/3$) is recovered.[17]

III. SUMMARY

In summary, we have considered the nonlinear propagation of ion-acoustic waves in a three-component plasma, whose constituents are electrons, ions, and positrons. By employing the hydrodynamic model for the ion fluid and the kinetic description for hot electron and positron fluids, the existence of finite-amplitude ion-acoustic solitons of positive electrostatic potential has been established. The nonlinear ion-acoustic waves are accompanied by compressional ion and electron number density perturbations ($\delta n_i \equiv n_i - n_{i0} > 0$, $\delta n_e \equiv n_e - n_{e0} > 0$), as well as a rarefractive perturbation of the positron number density. It is found that the amplitudes of intense ion-acoustic solitons are reduced by a factor of one-half when a significant fraction of the positrons is present in the plasma. The positron component also hinders the propagation speed of the solitons. Furthermore, the nonlinear evolution of small-amplitude ion-acoustic solitary pulses is governed by a Boussinesq equation. The latter reduces to the Korteweg–de Vries equation for the unidirectional propagation. We stress that the results of our investigation should be useful in understanding the properties of low phase velocity localized electrostatic perturbations that may appear in astrophysical plasmas, such as those in the early universe and active galactic nuclei.

In closing, we mention that for the physical parameters of our interest, we have been unable to find double layer solutions in an electron–positron–ion plasma. However, it is suggested that more numerical analyses have to be carried out in order to establish the existence of double layers in multispecies plasmas, whose constituents are cold inertial ions and Boltzmann distributed hot electrons and positrons. The study of double layers in such plasmas is under consideration and the results shall be reported elsewhere.

ACKNOWLEDGMENTS

Two of the authors (S. I. P. and S. V. V.) would like to thank the Alexander von Humboldt Foundation for financial support, and K. Elsässer and M. Y. Yu for hospitality.

[1] W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), p. 763.
[2] M. J. Rees, in *The Very Early Universe*, edited by G. W. Gibbons, S. W. Hawking, and S. Siklas (Cambridge University Press, Cambridge, 1983).
[3] H. R. Miller and P. J. Witta, *Active Galactic Nuclei* (Springer-Verlag, Berlin, 1987), p. 202.
[4] P. Goldreich and W. H. Julian, Astrophys. J. **157**, 869 (1969).
[5] F. C. Michel, Rev. Mod. Phys. **54**, 1 (1982).
[6] P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **135**, 1 (1986).
[7] M. Y. Yu, P. K. Shukla and L. Stenflo, Astrophys. J. **309**, L63 (1986).
[8] F. C. Michel, *Theory of Neutron Star Magnetospheres* (Chicago University Press, Chicago, 1991).
[9] T. Tajima and T. Taniuti, Phys. Rev. A **42**, 3587 (1990).
[10] P. K. Shukla, N. L. Tsintsadze, and L. N. Tsintsadze, Phys. Fluids B **5**, 811 (1993).
[11] P. K. Shukla, Europhys. Lett. **22**, 695 (1993).
[12] P. K. Shukla and L. Stenflo, Astrophys. Space Sci. **209**, 323 (1993).
[13] F. B. Rizzato, Plasma Phys. **40**, 289 (1988).
[14] V. I. Berezhiani, M. Y. El-Ashry, and U. A. Mofiz, Phys. Rev. E **50**, 448 (1994).
[15] R. Z. Sagdeev, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, p. 23.
[16] S. I. Popel and K. Elsässer, Phys. Lett. A **190**, 460 (1994).
[17] R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972), p. 19.
[18] L. Stenflo, N. L. Tsintsadze, and T. D. Buadze, Phys. Lett. A **135**, 37 (1989).