On plasma crystal formation

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It is shown that charged dust grains in a sheath region with plasma ion flow can attract each other in the wake potential cone of an upstream dust particulate. Because of the periodic nature of the potential, periodic structures of the dust grains can be formed in a base plane of the cone. © *1996 American Institute of Physics.* [S1070-664X(96)04902-0]

Recently, the possibility of formation of Coulomb quasilattices[1] involving the micrometer sized highly charged dust particulates has been demonstrated experimentally.[2–6] In the experiments, the dust-crystal structure is observed in the sheath region where there is balance between the gravitational and electrostatic forces. The distance between the dust grains is of the order of a Debye length λ_D.

It is well known that in the sheath region strong ion flow is established; according to the Bohm criterion the speed of the flow exceeds the ion acoustic velocity.[7,8] The presence of dust affects plasma–wall interactions; some effects have been investigated in Refs. 9–11.

The drag force on the test particle in a plasma with finite flows necessarily includes collective effects.[12] Recently, it has been demonstrated that if the speed of the flow exceeds the velocity of ion oscillations in the flow, an oscillating stationary wake is formed "behind" the static test particle.[13] The effect is similar to the Cooper pairing[14] of electrons in superconductors, and has been studied for two-component electron–ion plasmas[15] as well as for moving dust particles.[16] We note that the generation of the wake acoustic fields in the case of the stationary dust particle is analogous to the generation of the electromagnetic fields for the charge at rest placed in a medium moving faster than light.[17]

However, the oscillating potential has been calculated in Ref. 13 just on the line "behind" the test particle down the flow. At the same time, from experiments there is clear evidence of formation of a crystal-like structure in the direction perpendicular to the flow. The possibility of the oscillating potential in this direction was not discussed. In this Letter, we show how the collective mechanism can be responsible for the oscillatory potential in the direction perpendicular to the flow, and the attraction due to the wake potential can overcome the static Coulomb repulsion. We also discuss how the regular polygon structure can result from the wake potential. The characteristic spacing in this case is of the order of a Debye length λ_D, in agreement with the experiments.

We consider the cylindrical geometry (ρ,φ,z); the plasma ion flow is in the $-z$ direction with velocity v_{i0}. The test dust particle of the charge Q is placed on the position (0,0). Furthermore, we calculate the potential "behind" the test particle down the flow within the wake cone: $|z| > \rho\sqrt{M^2-1}$, where $M=v_{i0}/v_s$ is the Mach number, $v_s=(T_e n_i/m_i n_e)^{1/2}$ is the sound velocity, T_e is the electron temperature, $n_{i(e)}$ is the ion (electron) density, and m_i is the ion mass. Note that because we are interested in static dust particulates placed in the ion flow, the Čerenkov angle depends only on the speed of the flow. The static Coulomb repulsion between the dust particles is strong if the distance between them is less than the Debye length. Here, we note that the main contribution to the effective Debye length λ_D in typical discharge conditions[4] is due to plasma electrons: $\lambda_D = \lambda_{De} \equiv (T_e/4\pi n_e e^2)^{1/2}$, where $e=-|e|$ is the electron charge. Below, we are interested in the case where dust particles are subject to the Debye screening potential as well as the wake potential which arises when the ion flow velocity exceeds the critical speed.

We write the electrostatic potential of the static dust particle as

$$\Phi(\mathbf{r})=Q\int \frac{d\mathbf{k}}{2\pi^2k^2}\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{\varepsilon(\mathbf{k},-k_z v_{i0})}, \tag{1}$$

where $\mathbf{k}=(\mathbf{k}_\perp,k_z)$. The dielectric response function of the plasma in the presence of the finite ion flow with the speed v_{i0} [$v_s<v_{i0}\ll v_{Te}$, where $v_{Te}=(T_e/m_e)^{1/2}$ is the electron thermal velocity and m_e is the electron mass] is calculated under condition

$$kv_{Ti}\ll|k_z v_{i0}|\ll kv_{Te}, \tag{2}$$

where $v_{Ti}=(T_i/m_i)^{1/2}$ is the ion thermal velocity, T_i is the ion temperature, and, as we stated already, the flow is in $-z$ direction.

For the plasma dielectric response function, we have

$$\varepsilon(\mathbf{k},-k_z v_{i0})=1+\frac{1}{k^2\lambda_D^2}-\frac{\omega_{pi}^2}{(-k_z v_{i0}+i0)^2}, \tag{3}$$

where $\omega_{pi}=(4\pi n_i e^2/m_i)^{1/2}$ is the ion plasma frequency. Note the imaginary part $+i0$ (appearing due to causality) which is important when integrating over k_z.

Furthermore, following Ref. 13, we separate in Eq. (1) [using the inverse of function (3)] the static Debye and oscillating wake potentials

$$\Phi(\mathbf{r})=\Phi_D(\mathbf{r})+\Phi_W(\mathbf{r}), \tag{4}$$

where

$$\Phi_D(\mathbf{r})=(Q/|\mathbf{r}|)\exp(-|\mathbf{r}|/\lambda_D), \tag{5}$$

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and $\Phi_W(\mathbf{r})$ involves the collective effects caused by the oscillations in the ion flow:

$$\Phi_W(\mathbf{r})=Q\int \frac{d\mathbf{k}}{2\pi^2 k^2}\frac{k^2\lambda_D^2\omega_{\mathbf{k}}^2 \exp(i\mathbf{k}\cdot\mathbf{r})}{(1+k^2\lambda_D^2)[(-k_z v_{i0}+i0)^2-\omega_{\mathbf{k}}^2]}. \tag{6}$$

Here, $\omega_{\mathbf{k}}=kv_s/(1+k^2\lambda_D^2)^{1/2}$ is the characteristic frequency of the oscillations in the flow; it naturally appears when we equal to zero the plasma response function (3). The potential (6) describes the strong resonant interaction between the oscillations in the ion flow and the test particulate when $|k_z v_{i0}|$ is close to $\omega_{\mathbf{k}}$.

In dimensionless units $\mathbf{k}$ normalized by the inverse of the Debye length, we can write Eq. (6) as

$$\Phi_W(\mathbf{r})=\frac{Q}{\lambda_D M^2}\int \frac{dk_z\, d\mathbf{k}_\perp}{2\pi^2}\frac{k^2}{1+k^2} \times\frac{\exp(i\mathbf{k}\cdot\mathbf{r}/\lambda_D)}{[(k_z-i0)^2+k_0^2][(k_z-i0)^2-k_1^2]}, \tag{7}$$

where $k^2=k_z^2+k_\perp^2$ and $k_{0,1}^2=\pm(1-M^{-2}+k_\perp^2)/2+[k_\perp^2 M^{-2}+(1-M^{-2}+k_\perp^2)^2/4]^{1/2}$. Note that the contribution from the poles at $k_z=\pm ik_0$ provides the nonoscillating part which modifies the static Debye shielding scale (5) in plasmas with finite ion flows.[10]

After integration over k_z in (7) we find

$$\Phi_W(\mathbf{r})=-\frac{Q}{\lambda_D M^2}\int \frac{d\mathbf{k}_\perp}{2\pi|k_1|}\frac{k_\perp^2+k_1^2}{k_0^2+k_1^2} \times\frac{\exp(ik_\perp\rho\cos\varphi/\lambda_D)}{1+k_\perp^2+k_1^2}\sin\left(\frac{|k_1 z|}{\lambda_D}\right). \tag{8}$$

Taking into account the cylindrical symmetry of the wake cone, we obtain

$$\Phi_W(\rho,z)=-\frac{Q}{\lambda_D M^2}\int_0^\infty \frac{k_\perp\, dk_\perp}{|k_1|}\frac{k_\perp^2+k_1^2}{k_0^2+k_1^2}\frac{J_0(k_\perp\rho/\lambda_D)}{1+k_\perp^2+k_1^2} \times\sin\left(\frac{|k_1 z|}{\lambda_D}\right), \tag{9}$$

where J_0 is the Bessel function of zeroth order. It is difficult to find the mathematically exact expression for the potential (9). Thus below we present an estimation to evaluate the asymptotic behavior of the function. The main contribution to the integral (9) comes from $k_\perp\sim 1$. Then for distances $\rho>\lambda_D$ and $|z|>\lambda_D\sqrt{M^2-1}$ we find the following approximate expression for the wake potential:

$$\Phi_W(\rho,z)\simeq\frac{Q}{1-M^{-2}}\sqrt{\frac{\lambda_D}{2\pi\rho}} \times\left(\frac{\cos[(\pi/4)+(z_-/\lambda_D\sqrt{M^2-1})]}{z_-} +\frac{\cos[(\pi/4)-(z_+/\lambda_D\sqrt{M^2-1})]}{z_+}\right), \tag{10}$$

where $z_\pm\equiv|z|\pm\rho\sqrt{M^2-1}>0$ (we remind that the oscillating potential exists only in the wake of the test particle, i.e., for $z<0$ and $|z|>\rho\sqrt{M^2-1}$). Obviously, this function oscillates as we change ρ or z. Because oscillating potential (10) is proportional to the same dust particle charge Q as the static Debye potential (5), and contains no screening exponential, there are regions in space which correspond to the change of the effective potential sign and, hence, to the most probable positions of the particulates. We stress that these regions are not only on the line $\rho=0$ [by setting $\rho=0$ in Eq. (9), we can recover the results of Ref. 13; note that the latter cannot be found from (10) because of different asymptotics used], but also on the line $z=-z_0=$const, where $z_0>0$. The effective periodic spacing in the plane perpendicular to the flow is of the order of a Debye length; note that this can be seen not only from approximate expression (10), but also from the more exact formula (9).

To conclude, we have demonstrated that collective effects can provide the oscillating potential not only on the line (as in Ref. 13) but also in the plane perpendicular to the direction of the ion flow downstream from the dust particle. Since in the experiments the most probable effective spacing is of the order of a Debye length (the estimation is $0.2\sim 1.7\lambda_D$, see, e.g., Ref. 4), we see that the polygon structure can appear in plasma crystal formation. Indeed, because the wake potential cannot change the sign of the effective potential at distances less than the λ_D, the dust particulates are not expected to be arranged with distances less than the Debye length. At the same time, the characteristic spacing of the polar radius vector in the plane perpendicular to the flow is also of order λ_D. Therefore, we can expect the particulates on the equal distances λ_D on the periphery of the circle of the radius of order λ_D. This may correspond to a polygon of an order not higher than a hexagon. Note that hexagonal as well as square structures were observed in the experiments.[2–6]

Here, we would like to mention a hydrodynamical analogy. In fluid dynamic study, it has been known as a Stokes solution that a cluster of spheres, falling in a viscous fluid under gravity, can form regular structures with the spheres at the vertices of a regular horizontal polygon.[18–20] Plasma dust particles in a potential wake produced by a dust particle in a plasma ion flow can attract each other forming a periodic structure in a way somewhat similar to that in fluid dynamics. We stress that the proposed mechanism can provide a qualitative explanation of the arrangement of particles downstream of the test dust particulate. For a quantitative picture, it is necessary to calculate the potential of an ensemble of dust particulates as well as the contribution of other factors such as gravitation, plasma inhomogeneity, etc. Moreover, the regular arrangement of the particles in the first (with regard to the ion flow) layer of the plasma crystal needs more investigations; we mention only that if the regular quasicrystal structure is established in downstream layers, this may also affect the arrangement of particles in the first layer.

Note added in proof. We note that a recent numerical simulation by Melandsø and Goree[21] demonstrated formation of downstream wake string around charged bodies in a supersonic plasma flow which is in agreement with analytical solution in Ref. 13 and in the present Letter.

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[1]H. Ikezi, Phys. Fluids **29**, 1764 (1986).
[2]J. H. Chu and L. I, Phys. Rev. Lett. **72**, 4009 (1994).
[3]J. H. Chu, J. B. Du, and L. I, J. Phys. D: Appl. Phys. **27**, 296 (1994).
[4]H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Möhlmann, Phys. Rev. Lett. **73**, 652 (1994).
[5]Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys. **33**, L804 (1994).
[6]A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[7]F. F. Chen, *Introduction to Plasma Physics* (Plenum, New York, 1974), Chap. 8.
[8]O. Ishihara, I. Alexeff, H. J. Doucet, and W. D. Jones, Phys. Fluids **21**, 2211 (1978).
[9]M. Y. Yu, H. Saleem, and H. Luo, Phys. Fluids B **4**, 3427 (1992).
[10]S. Hamaguchi and R. T. Farouki, Phys. Plasmas **1**, 2110 (1994).
[11]J.-X. Ma and M. Y. Yu, Phys. Plasmas **2**, 1343 (1995).
[12]N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).
[13]S. V. Vladimirov and M. Nambu, Phys. Rev. E **52**, 2172 (1995).
[14]P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), p. 102.
[15]M. Nambu and H. Akama, Phys. Fluids **28**, 2300 (1985).
[16]M. Nambu, S. V. Vladimirov, and P. K. Shukla, Phys. Lett. A **203**, 40 (1995).
[17]B. M. Bolotovskii and S. N. Stolyarov, Sov. Phys. Usp. **35**, 143 (1992) [Usp. Fiz. Nauk **162**, 177 (1992)].
[18]K. O. L. F. Jayaweera, B. J. Mason, and G. W. Slack, J. Fluid Mech. **20**, 121 (1964).
[19]L. M. Hocking, J. Fluid Mech. **20**, 129 (1964).
[20]F. P. Bretherton, J. Fluid Mech. **20**, 401 (1964).
[21]F. Melandsø and J. Goree, Phys. Rev. E **52**, 5312 (1995).