Spontaneous emission effects in nonlinear interactions of nonresonant waves with resonant plasma fluctuations

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Spontaneous emission effects on propagation of nonresonant waves in plasmas in the presence of resonant fluctuations are studied. It is demonstrated that in closed plasma systems the number of nonresonant quanta is conserved as an adiabatic invariant. The conservation is due to the vanishing polarizational contribution that resulted from the symmetry of the system as well as to the balance of the direct nonlinear coupling and reverse absorption by particle collisions. Energy of the nonresonant waves as well as their amplitudes may vary with time even when the resonant field fluctuations are at the thermal level. [S1070-664X(96)02402-5]

I. INTRODUCTION

The problem of conversion of wave energy with a frequency variation is important for many laboratory and space plasmas. Recently, there has been much interest in the study of a relatively new type of nonlinear wave–wave and wave–particle coupling, which can provide amplification of high-frequency waves in the presence of low-frequency oscillations.[1–11] In the process, the low-frequency oscillations $(\omega,\mathbf{k})$ can be in Čerenkov resonance with plasma particles

$$\omega-\mathbf{k}\cdot\mathbf{v}=0, \tag{1}$$

while the high-frequency nonresonant waves $(\Omega,\mathbf{K})$ are neither in the linear resonance.

$$\Omega-\mathbf{K}\cdot\mathbf{v}\neq 0, \tag{2}$$

nor in the scattering resonance (off the resonant waves),

$$\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}\neq 0, \tag{3}$$

with plasma particles. There are many examples of such systems[8] including the case of unmagnetized plasma, where low-frequency ion-acoustic waves are resonant and high-frequency Langmuir waves are nonresonant.

In the literature, the effect of coupling of the nonresonant waves, the resonant oscillations, and plasma particles has been referred to as "plasma turbulent bremsstrahlung"[1] or "nonlinear plasma maser."[5,8] A similar mechanism can be also responsible for down-frequency conversion of wave energy,[5,12] particle heating,[13] and stochastic acceleration,[14] as well as evolution of resonant waves.[15,16]

It has been demonstrated that the plasma maser is mostly effective in open systems with external energy sources/sinks[8,17] or in the presence of external fields.[9] In closed plasma systems without external fields, conservation of the adiabatic invariant (number of nonresonant quanta) is observed.[8,9,11,18] The conservation of the nonresonant quanta in this case is closely connected with symmetry properties of the system leading to the zero polarizational contribution,[6,8,9] as well as with cancellation of the direct nonlinear coupling by the reverse absorption effect due to the quasilinear evolution of the system.[8,9]

Most of the recent studies considered low-frequency resonant turbulence with a sufficiently enhanced level, thus neglecting the effects of spontaneous emission in the process. In some earlier studies,[8,13] the problem was pointed out, and the contribution of some terms due to spontaneous effects was calculated. However, detailed study of the effects (like that of bremsstrahlung of plasma particles[19]) has not been done yet. At the same time, interest in the spontaneous emission in turbulent bremsstrahlung increased recently when a possible change of the nonresonant wave energy because of spontaneous fluctuations of particle distributions was pointed out.[20,21]

In this paper, we present an investigation of all possible contributions due to spontaneous emission in the evolution of the nonresonant wave amplitude. We demonstrate that spontaneous fluctuations of plasma particle distributions, and the field fluctuations associated with them, produce polarizational "virtual" fields which, in turn, affect the interaction of the nonresonant and resonant waves. In closed plasma systems, when there is no energy exchange with external sources and/or sinks, all the contributions of the spontaneous emission terms naturally lead to conservation of the number of the nonresonant quanta. We note that a similar conclusion was previously made for resonant waves with amplitudes sufficiently higher than the thermal level where spontaneous effects are negligible (see, e.g., Ref. 8). As in the latter case, the conservation is connected with symmetry of the system, which results in cancellation of the polarizational contribution. However, in the situation with low-frequency thermal fluctuations, the plasma particle collisions (and the accompanied slow evolution of the averaged particle distribution) play the key role in cancellation of the direct coupling term;

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unlike the case of the high level of resonant wave turbulence where the quasilinear evolution of the system plays the key role.

In closed systems, while the number of the nonresonant quanta is constant, change of the energy and amplitude of the resonant waves should be observed, even in the case of a thermal level of resonant fluctuations. In open plasma systems, spontaneous particle and field fluctuations can lead to change of the nonresonant quanta.

In our analysis of the nonstationarity problem, we allow the temporal variation of the nonresonant wave frequency together with the nonresonant field energy. We note that the frequency variation was also found to be critical in the study of nonlinear evolution of Buneman instability,[22,23] while the nonstationary nature of the field energy plays an important role in the quasilinear evolution, e.g., of the ion acoustic turbulence.[24,25]

The paper is arranged as follows: In Sec. II, we formulate a perturbation theory for random test nonresonant waves in the presence of electrostatic resonant fluctuations. In Sec. III we develop a general nonlinear equation for fields of the random nonresonant waves. Effects of nonstationarity of the system are introduced in Sec. IV. In Sec. V we describe a dispersion relation for the nonresonant wave in the presence of resonant fluctuations in the nonstationary system. Temporal change of the nonresonant wave amplitude, number of quanta, and energy is discussed in Sec. VI. In Sec. VII we comment on the situation where the nonresonant test wave is nonrandom or regular. Concluding remarks are stated in Sec. VIII.

II. PERTURBATION THEORY

For simplicity, we consider propagation of longitudinal nonresonant waves (with random phases) in the presence of electrostatic resonant fluctuations in unmagnetized plasma. The energy of the resonant field fluctuations is supposed to be close to the thermal level, thus we neglect induced effects.

Electron distribution is described by the function of their momenta $f_{\mathbf{p}}$ normalized as follows:

$$n_e = \int \frac{d\mathbf{p}}{(2\pi)^3} f_{\mathbf{p}}; \tag{4}$$

its time evolution is defined by the Vlasov equation,

$$\frac{\partial f_{\mathbf{p}}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\mathbf{p}}}{\partial \mathbf{r}} + e\mathbf{E}\cdot\frac{\partial f_{\mathbf{p}}}{\partial \mathbf{p}} = \mathbf{0}, \tag{5}$$

where $e = -|e|$ is the electron charge, and all fields contributing to the total electric field $\mathbf{E}$ are supposed to be longitudinal.

Next, we divide the distribution function into the regular and fluctuating parts,

$$f_{\mathbf{p}} = \Phi_{\mathbf{p}} + \delta f_{\mathbf{p}}, \quad \Phi_{\mathbf{p}} = \langle f_{\mathbf{p}} \rangle, \tag{6}$$

where the angular bracket means averaging over the statistical ensemble. The fluctuating distribution $\delta f_{\mathbf{p}}$ can be represented as a sum of spontaneous emission term $\delta f_{\mathbf{p}}^{(0)}$ and a group of terms proportional to the corresponding order of the electric field,

$$\delta f_{\mathbf{p}} = \delta f_{\mathbf{p}}^{(0)} + \sum_{j\geqslant 1} \delta f_{\mathbf{p}}^{(j)}, \quad \delta f_{\mathbf{p}}^{(j)} \propto E^j. \tag{7}$$

Here, we assume that in the first approximation regular fields are absent, i.e.,

$$\langle \mathbf{E} \rangle = 0. \tag{8}$$

The zeroth-order spontaneous term satisfies the equation

$$\frac{\partial \delta f_{\mathbf{p}}^{(0)}}{\partial t} + \mathbf{v}\cdot\frac{\partial \delta f_{\mathbf{p}}^{(0)}}{\partial \mathbf{r}} = 0. \tag{9}$$

Furthermore, following Ref. 19, we divide the pertubations of the distribution function $\delta f_{\mathbf{p}}^{(j)}$ into the part $\delta f_{\mathbf{p}}^{R(j)}$, which is proportional to the zeroth-order regular distribution function $\Phi_{\mathbf{p}}$, and part $\delta f_{\mathbf{p}}^{S(j)}$, which is proportional to the zeroth-order spontaneous fluctuations $\delta f_{\mathbf{p}}^{(0)}$ of particle distribution. Therefore, for Fourier-components,

$$A_k = (2\pi)^{-4} \int A(t,\mathbf{r}) \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})\, dt\, d\mathbf{r}, \tag{10}$$

we can easily find the corresponding equations of the perturbation theory,

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\,\delta f_{\mathbf{p},k}^{R(1)} = -e\mathbf{E}\cdot\frac{\partial \Phi_{\mathbf{p}}}{\partial \mathbf{p}}, \tag{11}$$

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\,\delta f_{\mathbf{p},k}^{R(j+1)} = -e\int d^{(2)}\left[\mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{R(j)} - \left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{R(j)} \right\rangle\right], \tag{12}$$

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\,\delta f_{\mathbf{p},k}^{S(1)} = -e\int d^{(2)}\left[\mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{(0)} - \left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{(0)} \right\rangle\right], \tag{13}$$

$$-i(\omega - \mathbf{k}\cdot\mathbf{v})\,\delta f_{\mathbf{p},k}^{S(j+1)} = -e\int d^{(2)}\left[\mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{S(j)} - \left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial \mathbf{p}}\delta f_{\mathbf{p},k_2}^{S(j)} \right\rangle\right], \tag{14}$$

where $d^{(2)} = dk_1 dk_2 \delta(k - k_1 - k_2)$, $k = (\mathbf{k},\omega)$, $\delta(k) \equiv \delta(\mathbf{k}) \times \delta(\omega)$, and $dk = d\mathbf{k}\, d\omega$.

Ensemble average of the squared zeroth-order spontaneous fluctuations is given by[19,21,26]

$$\langle \delta f_{\mathbf{p},k}^{(0)} \delta f_{\mathbf{p}',k'}^{(0)} \rangle = \Phi_p \delta(\mathbf{p} - \mathbf{p}')\, \delta(k + k')\, \delta(\omega - \mathbf{k}\cdot\mathbf{v}); \tag{15}$$

this equation gives the relation between squared fluctuations of a number of particles in a given volume and an averaged number of particles in the volume.

For electric fields, we take into account the test nonresonant field $\mathbf{E}_K^N$ on frequency,

$$\Omega = \Omega_{\mathbf{K}} \tag{16}$$

(which will be considered linearly in what follows), the fluctuation resonant field $\mathbf{E}_k^{(0)}$ with frequency $\omega \neq \omega_{\mathbf{k}}$ (we take into account effects up to second order in this field), and the polarization "virtual" field $\mathbf{E}^V$ on beat frequency $\Omega - \omega$; we assume that the latter field corresponds to forced oscillations only, i.e., there is no dispersion relation between $\Omega - \omega$ and $\mathbf{K} - \mathbf{k}$. Therefore, the total electric field (we remind the reader that the total field is supposed to contain no regular component; the case of regular nonresonant wave is considered separately below) can be written as

$$\mathbf{E}=\mathbf{E}^{N}+\mathbf{E}^{(0)}+\mathbf{E}^{V}. \tag{17}$$

The fluctuation field $\mathbf{E}^{(0)}$ is connected with spontaneous particle fluctuation $\delta f_{\mathbf{p}}^{(0)}$ via Poisson's equation

$$\mathbf{E}_{k}^{(0)}=-\frac{4\pi ie\mathbf{k}}{|\mathbf{k}|^{2}\varepsilon_{k}}\int\frac{d\mathbf{p}}{(2\pi)^{3}}\,\delta f_{\mathbf{p},k}^{(0)}, \tag{18}$$

where the linear dielectric permittivity is given by the standard formula

$$\varepsilon_{k}=1+\frac{4\pi e^{2}}{|\mathbf{k}|^{2}}\int\frac{d\mathbf{p}}{(2\pi)^{3}}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right). \tag{19}$$

And, finally, the ensemble average of oscillating fields defines their spectra as

$$\langle E_{i,k}E_{j,k'}\rangle=\frac{k_{i}k_{j}}{|\mathbf{k}|^{2}}|E|_{k}^{2}\delta(k+k'). \tag{20}$$

From (18) and (20), it is easy to find that

$$\langle E_{i,k}^{(0)}E_{j,k'}^{(0)}\rangle=\frac{k_{i}k_{j}}{|\mathbf{k}|^{2}}\,\frac{2e^{2}}{\pi|\mathbf{k}|^{2}|\varepsilon_{k}|^{2}}\times\int\frac{d\mathbf{p}}{(2\pi)^{3}}\,\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v})\,\delta(k+k'). \tag{21}$$

Thus, we have defined all the equations necessary to derive the nonlinear charge density perturbations which, in turn, contribute to the effective dielectric permittivity of the nonresonant waves. Note that for our purposes, it is sufficient to consider effects up to $\delta f_{\mathbf{p}}^{R(3)}$ which is proportional to the third order of the electric field, and up to $\delta f_{\mathbf{p}}^{S(2)}$ which is proportional to the second order of electric field.

III. GENERAL EQUATION FOR THE TEST NONRESONANT FIELDS

Taking into account perturbations of the distribution function $\delta f_{\mathbf{p}}^{R(2)}$ and $\delta f_{\mathbf{p}}^{R(3)}$ which connected only with the zeroth-order regular part $\Phi_{\mathbf{p}}$ of the particle distribution, we can write the nonlinear equation containing nonlinearities up to third order in the electric field,[19,27]

$$\begin{aligned}\varepsilon_{k}E_{k}&=-\frac{4\pi ie}{|\mathbf{k}|}(\rho^{R(2)}+\rho^{R(3)})\\&=\int d^{(2)}S_{1,2}(E_{1}E_{2}-\langle E_{1}E_{2}\rangle)\\&\quad+\int d^{(3)}\Sigma_{1,2,3}(E_{1}E_{2}E_{3}-E_{1}\langle E_{2}E_{3}\rangle-\langle E_{1}E_{2}E_{3}\rangle),\end{aligned} \tag{22}$$

where $E_{k}=\mathbf{k}\cdot\mathbf{E}_{k}/|\mathbf{k}|$, $E_{j}=E_{k_{j}}$, $j=1,2,3$, $d^{(3)}=dk_{1}dk_{2}dk_{3}\delta(k-k_{1}-k_{2}-k_{3})$, and second- and third-order nonlinear plasma responses (which are symmetrized over the last two indices) are given by[27–29]

$$\begin{aligned}S_{1,2}=S_{k,k_{1},k_{2}}&=\frac{2\pi ie^{3}}{|\mathbf{k}||\mathbf{k}_{1}||\mathbf{k}_{2}|}\int\frac{d\mathbf{p}}{(2\pi)^{3}}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\\&\quad\times\left[\left(\mathbf{k}_{1}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\omega_{2}-\mathbf{k}_{2}\cdot\mathbf{v}+i0}\left(\mathbf{k}_{2}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right.\\&\quad\left.+\left(\mathbf{k}_{2}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\omega_{1}-\mathbf{k}_{1}\cdot\mathbf{v}+i0}\left(\mathbf{k}_{1}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right]\Phi_{\mathbf{p}},\end{aligned} \tag{23}$$

and

$$\begin{aligned}\Sigma_{1,2,3}&=\Sigma_{k,k_{1},k_{2},k_{3}}\\&=\frac{2\pi e^{4}}{|\mathbf{k}||\mathbf{k}_{1}||\mathbf{k}_{2}||\mathbf{k}_{3}|}\int\frac{d\mathbf{p}}{(2\pi)^{3}}\,\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}_{1}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\\&\quad\times\frac{1}{\omega-\omega_{1}-(\mathbf{k}-\mathbf{k}_{1})\cdot\mathbf{v}+i0}\left[\left(\mathbf{k}_{2}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right.\\&\quad\times\frac{1}{\omega_{3}-\mathbf{k}_{3}\cdot\mathbf{v}+i0}\left(\mathbf{k}_{3}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\\&\quad\left.+\left(\mathbf{k}_{3}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\omega_{2}-\mathbf{k}_{2}\cdot\mathbf{v}+i0}\left(\mathbf{k}_{2}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right]\Phi_{\mathbf{p}}.\end{aligned} \tag{24}$$

Since we are interested in effects which are linear in the test nonresonant field $\mathbf{E}^{N}$, we have to linearize Eq. (22) with respect to the field. Furthermore, we invoke the standard logic of the weak turbulence theory.[19] In the second-order term involving the response S, only the combination $E^{(0)}E^{V}$ can appear since the virtual fields $\mathbf{E}^{V}$ are proportional to the nonresonant field. In the third-order term which contains the response Σ, only the field E_{2} can be the test wave field E^{N}, while all other terms will disappear after multiplying by E^{N} and ensemble averaging of Eq. (22); the two other fields in this term are the fields of zeroth-order fluctuations $E^{(0)}$. Terms containing virtual fields provide polarizational contribution to the nonlinear dielectric function because of the second order in field term in Eq. (22). Indeed, we can write

$$\begin{aligned}\varepsilon_{k}E_{k}^{RV}&=2\int d^{(2)}S_{1,2}(E_{1}^{N}E_{2}^{(0)}-\langle E_{1}^{N}E_{2}^{(0)}\rangle)\\&=2\int d^{(2)}S_{1,2}E_{1}^{N}E_{2}^{(0)}.\end{aligned} \tag{25}$$

Here, we assumed there are no correlations between the nonresonant fields and resonant fluctuations. After substitution of this expression into the second-order term of (22) (which contains $E^{(0)}E^{V}$), we obtain the same combination of fields $E^{(0)}E^{N}E^{(0)}$.

On the other hand, Eq. (22), which is similar to the basic equation in the standard theory of the evolution of nonresonant waves in the presence of resonant turbulence, does not account for all the contribution due to spontaneous plasma fluctuations (in fact, it accounts for part of them via the field fluctuations $E^{(0)}$; we stress, however, that in its derivation we did not use any assumption about the level of the field fluctuations, which means that the equation can be used for the study of a sufficiently enhanced level of plasma turbulence; in the latter case we just have turbulent resonant spectrum

$|E|_k^2$. In Eq. (22), we included only perturbations of the distribution function which are due to the regular part $\Phi_{\mathbf{p}}$. Now, we write contributions from the zeroth-order fluctuations $\delta f_{\mathbf{p}}^{(0)}$.

The corresponding charge densities are given by

$$\rho_k^{S(1)}=\int \frac{d\mathbf{p}dk_1}{(2\pi)^3}\frac{e}{i|\mathbf{k}_1|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times(E_{k_1}\delta f_{\mathbf{p},k-k_1}^{(0)}-\langle E_{k_1}\delta f_{\mathbf{p},k-k_1}^{(0)}\rangle) \tag{26}$$

and

$$\rho_k^{S(2)}=-\int \frac{d\mathbf{p}dk_1dk_2}{(2\pi)^3}\frac{e^2}{|\mathbf{k}_1||\mathbf{k}_2|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times\frac{1}{\omega_2-\mathbf{k}_2\cdot\mathbf{v}+i0}\left(\mathbf{k}_2\cdot\frac{\partial}{\partial\mathbf{p}}\right)(E_{k_1}E_{k_2}\delta f_{\mathbf{p},k-k_1-k_2}^{(0)}-E_{k_1}\langle E_{k_2}\delta f_{\mathbf{p},k-k_1-k_2}^{(0)}\rangle-\langle E_{k_1}E_{k_2}\delta f_{\mathbf{p},k-k_1-k_2}^{(0)}\rangle). \tag{27}$$

Charge density perturbation (26), after linearization on the nonresonant test field, is responsible for the generation of an additional virtual field which we denote as E^{SV}. We have

$$\varepsilon_k E_k^{SV}=-\int \frac{d\mathbf{p}dK}{(2\pi)^3}\frac{4\pi e^2}{|\mathbf{k}||\mathbf{K}|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times(E_K^N\delta f_{\mathbf{p},k-K}^{(0)}-\langle E_K^N\delta f_{\mathbf{p},k-K}^{(0)}\rangle)=-\int \frac{d\mathbf{p}dK}{(2\pi)^3}\frac{4\pi e^2}{|\mathbf{k}||\mathbf{K}|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\times\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)E_K^N\delta f_{\mathbf{p},k-K}^{(0)}. \tag{28}$$

The virtual fields E^{RV} and E^{SV}, together with charge density perturbation (27), contribute to the nonlinear dielectric function of the test nonresonant wave.

Thus, multiplying Eq. (22) by $E_{K'}^N$, integrating it over K', and averaging over a statistical ensemble, we obtain the equation where all contributions due to the spontaneous emission effects are consistently taken into account:

$$\int dK'\langle E_{K'}^N E_K^N\varepsilon_K\rangle=2\int dK'd^{(3)}\Sigma_{1,2,3}\langle E_{K'}^N E_1^{(0)}E_2^N E_3^{(0)}\rangle+2\int dK'd^{(2)}S_{1,2}\langle E_{K'}^N(E_{k_1}^{RV}+E_{k_1}^{SV})E_{k_2}^{(0)}\rangle-\int\frac{d\mathbf{p}dk_1dK'}{(2\pi)^3}\frac{4\pi e^2}{|\mathbf{K}||\mathbf{k}_1|}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\langle E_{K'}^N(E_{k_1}^{RV}+E_{k_1}^{SV})\delta f_{\mathbf{p},K-k_1}^{(0)}\rangle+\int\frac{d\mathbf{p}dkdK_1dK'}{(2\pi)^3}\frac{4\pi ie^3}{|\mathbf{K}||\mathbf{K}_1||\mathbf{k}|}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}+i0}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\Omega_1-\mathbf{K}_1\cdot\mathbf{v}+i0}\left(\mathbf{K}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times\langle E_{K'}^N E_k^{(0)}E_{K_1}^N\delta f_{\mathbf{p},K-k-K_1}^{(0)}\rangle. \tag{29}$$

Note that in this equation we did not assume any specific resonant conditions. Thus Eq. (29) is quite general and contains [taking into account expressions (25) and (28)] all effects of the corresponding order, in particular bremsstrahlung and scattering, see Ref. 19.

IV. EFFECTS OF NONSTATIONARITY IN THE SYSTEM

Here, we stress one important point. In our consideration of nonlinear terms (which contain $|E^{(0)}|^2$ and are small corrections to the linear dielectric permittivity of the nonresonant waves ε_K), it is sufficient to use Eq. (20) in calculating the correlation function of the nonresonant wave field $\langle E_{K'}^N E_K^N\rangle$. However, in a linear term we have to take into account the nonstationarity of the system which is connected with slow time evolution of the zeroth-order regular part of the distribution function $\Phi_{\mathbf{p}}$. Changing of $\Phi_{\mathbf{p}}$ in time can provide effects of the same order $|E^{(0)}|^2$ as the nonlinear terms.

Indeed, averaging Eq. (5) over a statistical ensemble, we obtain

$$\frac{\partial\Phi_{\mathbf{p}}}{\partial t}+\mathbf{v}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}=-\left\langle e\mathbf{E}\cdot\frac{\partial f_{\mathbf{p}}}{\partial\mathbf{p}}\right\rangle. \tag{30}$$

In the order we are interested in, it is sufficient to substitute in the right hand side of this equation the fluctuating electric field $E^{(0)}$, the zeroth-order fluctuations $\delta f_{\mathbf{p}}^{(0)}$, as well as the first-order perturbation $\delta f_{\mathbf{p}}^{R(1)}$ (the latter should contain only the electric field $E^{(0)}$). Thus, we find that the right hand side of Eq. (30) contains nothing but a Balescu–Lenard collision integral[30]:

$$\frac{\partial\Phi_{\mathbf{p}}}{\partial t}+\mathbf{v}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{r}}=-\left\langle e\mathbf{E}^{(0)}\cdot\frac{\partial}{\partial\mathbf{p}}(\delta f_{\mathbf{p}}^{(0)}+\delta f_{\mathbf{p}}^{R(1)})\right\rangle=ie^2\int\frac{dkdk'd\mathbf{p}'}{2\pi^2|\mathbf{k}|^2\varepsilon_k}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\left[\langle\delta f_{\mathbf{p}',k}^{(0)}f_{\mathbf{p},k'}^{(0)}\rangle-\frac{e^2}{|\mathbf{k}'|^2\varepsilon_{k'}}\int\frac{d\mathbf{p}''}{2\pi^2}\langle\delta f_{\mathbf{p}',k}^{(0)}f_{\mathbf{p}'',k'}^{(0)}\rangle\times\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right)\right]=2e^4\int\frac{d\mathbf{p}'d\mathbf{k}}{(2\pi)^3|\mathbf{k}|^4}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{\delta(\mathbf{k}\cdot\mathbf{v}-\mathbf{k}\cdot\mathbf{v}')}{|\varepsilon_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}}|^2}\times\left[\Phi_{\mathbf{p}'}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right)-\Phi_{\mathbf{p}}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}\right)\right], \tag{31}$$

where we took into account resonance (1) and used

$$\mathrm{Im}\,\frac{1}{\varepsilon_k}=-\frac{\mathrm{Im}\,\varepsilon_k}{|\varepsilon_k|^2}=\frac{4\pi^2e^2}{|\mathbf{k}|^2|\varepsilon_k|^2}\times\int\frac{d\mathbf{p}'}{(2\pi)^3}\,\delta(\omega-\mathbf{k}\cdot\mathbf{v}')\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}\right). \tag{32}$$

We note that the derivation of the Balescu–Lenard collision integral is not new, see, e.g., Ref. 31.

Slow time evolution of the function $\Phi_\mathbf{p}$ leads to a slow change in time of the nonresonant linear dielectric permittivity, $\varepsilon_K=\varepsilon_K(t)$, and, therefore, to a slow change in time of not only the amplitude but also the eigenfrequency $\Omega_\mathbf{K}=\Omega_\mathbf{K}(t)$ of the nonresonant waves. For weakly nonstationary systems we use the geometrical optics approximation and write[8,11,22,23]

$$E^N_\mathbf{K}(t)=E^{(a)}_\mathbf{K}(t)\exp\left[-i\int^t\Omega_\mathbf{K}(t')\,dt'\right], \tag{33}$$

where $E^{(a)}_\mathbf{K}(t)$ is the random amplitude of the nonresonant wave; the correlation function $\langle E^{(0)}_\mathbf{K}(t)E^{(0)}_{\mathbf{K}'}(t)\rangle$ is slowly changing with time. For the linear part (i.e., left hand side) of dispersion equation (29) we then obtain

$$\begin{aligned}\int dK'\langle E^N_{K'}E^N_K\rangle\varepsilon_K
&=\int dK'\int\frac{dt'\,dt\,d\tau}{(2\pi)^3}\langle E^{(a)}_{\mathbf{K}'}(t')E^{(a)}_\mathbf{K}(t-\tau)\rangle\varepsilon_\mathbf{K}(\tau,t-\tau)\exp\left[i\Omega't'+i\Omega t-i\int^{t'}\Omega_{\mathbf{K}'}(s)\,ds-i\int^{t-\tau}\Omega_\mathbf{K}(s)\,ds\right]\\
&\approx-\int dK'\,\delta(\mathbf{K}+\mathbf{K}')\int\frac{dt\,d\tau}{(2\pi)^2}\,\exp[i(\Omega+\Omega')t+i\Omega_\mathbf{K}(t)\tau]\\
&\quad\times\left[|E^N|^2_\mathbf{K}(t)-\frac{d|E^N|^2_\mathbf{K}(t)}{dt}\frac{\tau}{2}-|E^N|^2_\mathbf{K}(t)\frac{d\Omega_\mathbf{K}(t)}{dt}\frac{i\tau^2}{2}\right]\left[\varepsilon_\mathbf{K}(\tau,t)-\tau\frac{\partial\varepsilon_\mathbf{K}(\tau,t)}{\partial t}\right]\\
&=-|E^N|^2_\mathbf{K}(t)\left[\varepsilon_K(t)+i\gamma_\mathbf{K}(t)\frac{\partial\varepsilon_K(t)}{\partial\Omega}+i\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t}+\frac{i}{2}\frac{d\Omega_\mathbf{K}(t)}{dt}\frac{\partial^2\varepsilon_K(t)}{\partial\Omega^2}\right]_{\Omega=\Omega_\mathbf{K}(t)},\end{aligned} \tag{34}$$

where we used $\Omega_{-\mathbf{K}}=-\Omega_\mathbf{K}$,

$$\varepsilon_\mathbf{K}(\tau,t)=2\pi\delta(\tau)+\frac{4\pi e^2}{|\mathbf{K}|^2}\int\frac{d\mathbf{p}\,d\Omega}{(2\pi)^3}\times\exp(-i\Omega\tau)\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\Phi_\mathbf{p}(t), \tag{35}$$

$$\varepsilon_K(t)=\int\frac{d\tau}{2\pi}\,\exp(i\Omega\tau)\varepsilon_\mathbf{K}(\tau,t), \tag{36}$$

and

$$\langle E^{(a)}_{\mathbf{K}'}(t')E^{(a)}_\mathbf{K}(t)\rangle=-2\pi|E^N|^2_\mathbf{K}(t)\,\delta(\mathbf{K}+\mathbf{K}')\,\delta(t-t'). \tag{37}$$

Furthermore, in Eq. (34) we have (by definition)

$$2\gamma_\mathbf{K}(t)\equiv\frac{1}{|E^N|^2_\mathbf{K}(t)}\frac{\partial|E^N|^2_\mathbf{K}(t)}{\partial t}. \tag{38}$$

Equation (34) contains all the corrections due to the system's slow nonstationarity: terms of spectrum nonstationarity,[11,32,33] slow time evolution of the particle distribution function,[8,14] and the slow time change in the nonresonant eigenfrequency.[8,11] For simplicity, no spatial inhomogeneities are assumed in the system. Equation (31) should be used to calculate the time derivative of the linear dielectric permittivity in the case of the resonant fluctuations.

V. DISPERSION EQUATION FOR THE NONRESONANT WAVE

By substituting (25) and (28) in (29), and using (15), (18), (20), and (34), we find all the contributions (of order $|E^{(0)}|^2$) to the dispersion equation for the nonresonant waves.

The term proportional to the third-order plasma response Σ gives the direct nonlinear contribution to the dielectric permittivity of the test nonresonant wave,[8,9,11,20,21]

$$\varepsilon^D_K=2\int dk\Sigma_{k,K,-k}|E^{(0)}|^2_k. \tag{39}$$

After substitution of the field (25) into Eq. (22) and averaging over the statistical ensemble, we find a part of the polarizational contribution,[8,9,11]

$$\varepsilon^{RP}_K=4\int dk\frac{S_{k,K-k}S_{K,-k}}{\varepsilon_{K-k}}|E^{(0)}|^2_k. \tag{40}$$

Note that Eqs. (39) and (40) are similar to those for the above-thermal level of the resonant oscillations: see, e.g., Ref. 8.

The second term in the right hand side of (29) includes coupling of the virtual field E^{SV} with the field of zeroth-order fluctuations $E^{(0)}$ due to the second-order plasma nonlinearity, and its polarization contribution to the dielectric permittivity is given by

$$\varepsilon_K^{SP(1)} = -\int \frac{d\mathbf{p}dk}{(2\pi)^3} \frac{2ie^3}{\pi|\mathbf{K}||\mathbf{k}|} \frac{S_{k,K-k}}{|\mathbf{K}-\mathbf{k}|\varepsilon_{K-k}\varepsilon_k} \times \frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}} \left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}). \quad (41)$$

The third term in the right hand side of (29) includes

$$\varepsilon_K^{SP(2)} = -\int \frac{d\mathbf{p}dk}{(2\pi)^3} \frac{2ie^3}{\pi|\mathbf{K}||\mathbf{k}|} \frac{S_{K,-k}}{|\mathbf{K}-\mathbf{k}|\varepsilon_{K-k}\varepsilon_{-k}} \times \frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left[(\mathbf{K}-\mathbf{k})\frac{\partial}{\partial\mathbf{p}}\right]\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}), \quad (42)$$

which corresponds to coupling of E^{RV} with $\delta f_{\mathbf{p}}^{(0)}$ in $\rho^{S(1)}$, and

$$\varepsilon_K^{SP(3)} = -\int \frac{d\mathbf{p}dk}{(2\pi)^3} \frac{2e^4}{\pi m_e^2} \frac{((\mathbf{K}-\mathbf{k})\cdot\mathbf{K})^2}{|\mathbf{K}-\mathbf{k}|^2|\mathbf{K}|^2\varepsilon_{K-k}} \times \frac{1}{(\Omega-\mathbf{K}\cdot\mathbf{v})^4}\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}), \quad (43)$$

due to coupling of E^{SV} with $\delta f_{\mathbf{p}}^{(0)}$ in $\rho^{S(1)}$; here, m_e is the electron mass. Finally, the last contribution comes from the last term in the right hand side of (29) with coupling of E^N, $E^{(0)}$, and $\delta f_{\mathbf{p}}^{(0)}$ in $\rho^{S(2)}$. We have

$$\varepsilon_K^{SP(4)} = -\int \frac{d\mathbf{p}dk}{(2\pi)^3} \frac{2e^4}{\pi} \frac{1}{|\mathbf{K}|^2|\mathbf{k}|^2\varepsilon_k} \frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right) \times \frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}). \quad (44)$$

Thus the resulting dispersion equation, which follows from Eq. (29), is

$$\left[\varepsilon_K(t)+\varepsilon_K^D+\varepsilon_K^{RP}+\sum_{j=1}^{4}\varepsilon_K^{SP(j)}+i\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t} + \frac{i}{2}\frac{\partial^2\varepsilon_K(t)}{\partial\Omega^2}\frac{d\Omega_{\mathbf{K}}(t)}{dt}+i\gamma_{\mathbf{K}}(t)\frac{\partial\varepsilon_K(t)}{\partial\Omega}\right]_{\Omega=\Omega_{\mathbf{K}}(t)} = 0. \quad (45)$$

We stress that up to this point, Eq. (45) is quite general, in the sense that no specific resonant conditions have been used to derive it. If we specify that the time evolution is due to particle collisions, the resonant conditions, as was used to derive (31), should be applied.

Now, we take into account resonance (1) and calculate imaginary part of all the dielectric functions in the left hand side of Eq. (45). First of all, we note that because of absence of the scattering resonance, see condition (3), we have

$$\mathrm{Im}\varepsilon_K^{SP(3)} = 0. \quad (46)$$

Then, we consider contributions from imaginary parts of $\varepsilon_K^{SP(1)}$ and $\varepsilon_K^{SP(2)}$. After integration in parts in Eqs. (41) and (42) we find

$$\mathrm{Im}(\varepsilon_K^{SP(1)}+\varepsilon_K^{SP(2)}) = \int \frac{d\mathbf{p}dk}{(2\pi)^3} \frac{2e^3}{\pi m_e|\mathbf{k}|} \frac{\mathbf{K}\cdot(\mathbf{K}-\mathbf{k})}{|\mathbf{K}||\mathbf{K}-\mathbf{k}|\varepsilon_{K-k}} \frac{1}{(\Omega-\mathbf{K}\cdot\mathbf{v})^2} \times\left[\mathrm{Re}\frac{S_{k,K-k}}{\varepsilon_k}+\mathrm{Re}\frac{S_{K,-k}}{\varepsilon_{-k}}\right]\Phi_{\mathbf{p}}\delta(\omega-\mathbf{k}\cdot\mathbf{v}). \quad (47)$$

To calculate the expression in the square brackets in the right hand side of this equation, we first note that

$$\varepsilon_k = \varepsilon_{-k}^{*}, \quad (48)$$

where the asterisk means a complex conjugate. Furthermore, the second-order plasma response, defined as in (23), under conditions (1)–(3), has the following symmetry:

$$S_{k,K-k} = -S_{K,-k}^{*}. \quad (49)$$

Therefore, we conclude that contributions from imaginary parts of $\varepsilon_K^{SP(1)}$ and $\varepsilon_K^{SP(2)}$ cancel each other:

$$\mathrm{Im}(\varepsilon_K^{SP(1)}+\varepsilon_K^{SP(2)}) = 0. \quad (50)$$

Similar consideration allows us to establish the zero contribution from the polarizational term ε^{RP} given by Eq. (40):

$$\mathrm{Im}\ \varepsilon_K^{RP} = 4\int dk\frac{|E^{(0)}|_k^2}{\varepsilon_{K-k}}\ \mathrm{Im}(S_{k,K-k}S_{K,-k}) = 0. \quad (51)$$

Note that this cancellation is due to the same symmetry as in the case of the above-thermal level of resonant oscillations, see, e.g., Ref. 8. Thus we have shown that only ε^D and $\varepsilon^{SP(4)}$ effectively contribute to the nonlinear imaginary part of dispersion equation (45). Also, there is no imaginary contribution from the linear dielectric permittivity $\varepsilon_K(t)$ because of the absence of the linear Čerenkov resonance; see condition (2).

VI. CHANGE OF THE NONRESONANT WAVE AMPLITUDE, NUMBER OF QUANTA, AND ENERGY

Using results of the previous section, we can write the following expression for the growth (or damping) rate of the nonresonant wave amplitude $E_{\mathbf{K}}^{(a)}$

$$\gamma_{\mathbf{K}}(t) = -\left\{\frac{1}{\partial\varepsilon_K(t)/\partial\Omega}\left[\mathrm{Im}(\varepsilon_K^D+\varepsilon_K^{SP(4)})+\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t} + \frac{1}{2}\frac{\partial^2\varepsilon_K(t)}{\partial\Omega^2}\frac{d\Omega_{\mathbf{K}}(t)}{dt}\right]\right\}_{\Omega=\Omega_{\mathbf{K}}(t)}. \quad (52)$$

The number of the nonresonant quanta can be introduced as

$$N(t) = \int\frac{d\mathbf{K}}{(2\pi)^3}N_{\mathbf{K}}(t) = \int\frac{d\mathbf{K}}{(2\pi)^3}\pi^2|E^N|_{\mathbf{K}}^2(t)\left[\frac{\partial\varepsilon_K(t)}{\partial\Omega}\right]_{\Omega=\Omega_{\mathbf{K}}(t)}. \quad (53)$$

Then the wave energy of the nonresonant waves is defined by

$$W(t) = \int\frac{d\mathbf{K}}{(2\pi)^3}\Omega_{\mathbf{K}}(t)N_{\mathbf{K}}(t). \quad (54)$$

For change in time of number of quanta (53) we obtain

$$\frac{dN(t)}{dt} = \int\frac{d\mathbf{K}}{(2\pi)^3}\frac{dN_{\mathbf{K}}(t)}{dt}, \quad (55)$$

where

$$\frac{dN_{\mathbf{K}}(t)}{dt}=\pi^2|E^N|^2_{\mathbf{K}}(t)\left[\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t}+\frac{\partial^2\varepsilon_K(t)}{\partial\Omega^2}\frac{d\Omega_{\mathbf{K}}(t)}{dt}+2\gamma_{\mathbf{K}}(t)\frac{\partial\varepsilon_K(t)}{\partial\Omega}\right]_{\Omega=\Omega_{\mathbf{K}}(t)}. \tag{56}$$

If we introduce the rate of change of the number of quanta,

$$\frac{dN_{\mathbf{K}}(t)}{dt}=\Gamma_{\mathbf{K}}(t)N_{\mathbf{K}}(t), \tag{57}$$

then from (52) we find

$$\Gamma_{\mathbf{K}}(t)=-\left\{\frac{1}{\partial\varepsilon_K(t)/\partial\Omega}\left[2\,\mathrm{Im}(\varepsilon_K^D+\varepsilon_K^{SP(4)})+\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t}\right]\right\}_{\Omega=\Omega_{\mathbf{K}}(t)}. \tag{58}$$

Change of the nonresonant wave energy is given by

$$\frac{dW(t)}{dt}=\int\frac{d\mathbf{K}}{(2\pi)^3}\left[\frac{d\Omega_{\mathbf{K}}(t)}{dt}N_{\mathbf{K}}(t)+\Omega_{\mathbf{K}}(t)\frac{dN_{\mathbf{K}}(t)}{dt}\right]. \tag{59}$$

Now, we calculate the contribution from $\varepsilon_K^D+\varepsilon_K^{SP(4)}$. Using Eqs. (24), (32), (39), and (44), and taking into account resonance (1), we can write

$$\begin{aligned}\mathrm{Im}[\varepsilon_K^D+\varepsilon_K^{SP(4)}]&=\int\frac{d\mathbf{p}dk}{(2\pi)^3}\frac{4\pi^2e^4}{|\mathbf{K}|^2|\mathbf{k}|^2}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}}\left(\mathbf{K}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\delta(\omega-\mathbf{k}\cdot\mathbf{v})\\&\quad\times\left[|E^{(0)}|^2_k\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right)-\frac{2e^2\Phi_{\mathbf{p}}}{\pi|\mathbf{k}|^2|\varepsilon_k|^2}\int\frac{d\mathbf{p}'}{(2\pi)^3}\delta(\omega-\mathbf{k}\cdot\mathbf{v}')\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}\right)\right]\\&=2e^4\int\frac{d\mathbf{p}d\mathbf{p}'d\mathbf{k}}{(2\pi)^6|\mathbf{k}|^4}\frac{4\pi e^2}{m_e^2}\frac{3(\mathbf{K}\cdot\mathbf{k})}{(\Omega-\mathbf{K}\cdot\mathbf{v})^4}\frac{\delta(\mathbf{k}\cdot\mathbf{v}-\mathbf{k}\cdot\mathbf{v}')}{|\varepsilon_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}}|^2}\left[\Phi_{\mathbf{p}'}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}\right)-\Phi_{\mathbf{p}}\left(\mathbf{k}\cdot\frac{\partial\Phi_{\mathbf{p}'}}{\partial\mathbf{p}'}\right)\right]\\&=-\frac{4\pi e^2}{m_e}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{1}{(\Omega-\mathbf{K}\cdot\mathbf{v})^3}\frac{\partial\Phi_{\mathbf{p}}}{\partial t}=-\frac{1}{2}\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t},\end{aligned} \tag{60}$$

where we have also used Eq. (31).

Thus, an important theorem is proved: in the systems considered, the nonlinear contribution is balanced by the reverse absorption effect due to Coulomb collisions. The balance leads to conservation of the nonresonant quanta, as can be easily seen from Eq. (58),

$$\Gamma_{\mathbf{K}}(t)=0. \tag{61}$$

For the nonresonant wave amplitude we have from (52),

$$\begin{aligned}\gamma_{\mathbf{K}}(t)&=-\left\{\frac{1}{2\,\partial\varepsilon_K(t)/\partial\Omega}\left[\frac{\partial^2\varepsilon_K(t)}{\partial\Omega\,\partial t}+\frac{\partial^2\varepsilon_K(t)}{\partial\Omega^2}\frac{d\Omega_{\mathbf{K}}(t)}{dt}\right]\right\}_{\Omega=\Omega_{\mathbf{K}}(t)}\\&=-\frac{1}{2}\frac{d}{dt}\left[\ln\left(\frac{\partial\varepsilon_K(t)}{\partial\Omega}\right)\right]_{\Omega=\Omega_{\mathbf{K}}(t)}.\end{aligned} \tag{62}$$

The change of the nonresonant wave energy, calculated from (59) taking into account the conservation of the plasmon number, is proportional to the change of the eigenfrequency of the waves:

$$\frac{dW(t)}{dt}=\int\frac{d\mathbf{K}}{(2\pi)^3}\frac{d\Omega_{\mathbf{K}}(t)}{dt}N_{\mathbf{K}}(t). \tag{63}$$

Thus the wave energy (54), unlike the nonresonant quanta, is not conserved in the presence of the resonant wave fluctuations in closed plasma systems. We note that the conservation of the nonresonant quanta is a consequence of the general theorem of adiabatic invariants in closed systems[34] or gauge invariance under changes of phase in the system's Lagrangian.[35] It is well known from the course of mechanics that when the frequency of the pendulum is adiabatically changed in time, its energy also changes as in Eq. (63). As has been noted in Refs. 11 and 36, the same physical mechanism leads to change of the wave energy in slowly varying closed adiabatic plasma systems.

VII. REGULAR NONRESONANT WAVES

Above, we considered propagation of turbulent nonresonant waves in the presence of electrostatic resonant oscillations. Since the effects studied are linear in the nonresonant waves, the case of a regular test wave is described by the same rates (62) and (63) for the change of the wave amplitude and energy. The number of the nonresonant quanta is also conserved in this case [i.e., Eq. (61) is observed which is actually clear from general considerations for closed plasma systems]. However, the perturbation equations are different in the case of the regular nonresonant waves:

$$\langle E\rangle=E^N. \tag{64}$$

Below, we present the corresponding series necessary to derive the dispersion equation.

In contrast to (6), we now divide the distribution function into the regular and fluctuating parts in which perturbations of the regular distribution due to the regular test wave field are considered:

$$f_{\mathbf{p}}=\langle f_{\mathbf{p}}\rangle+\delta f_{\mathbf{p}}, \quad \langle f_{\mathbf{p}}\rangle=\Phi_{\mathbf{p}}+\sum_{j\geqslant 1} f_{\mathbf{p}}^{(j)}, \tag{65}$$

where $f_{\mathbf{p}}^{(j)}$ is proportional to the corresponding order of the regular fields. For the fluctuating field $\delta f_{\mathbf{p}}$ we, as before, use expansion (7).

The regular perturbations of the distribution function, $f_{\mathbf{p}}^{(j)}$, can be divided into the part $f_{\mathbf{p}}^{R(j)}$, which is proportional to the zeroth-order regular distribution function $\Phi_{\mathbf{p}}$, and part $f_{\mathbf{p}}^{S(j)}$, which is proportional to the zeroth-order spontaneous fluctuations $\delta f_{\mathbf{p}}^{(0)}$ of particle distribution. Therefore, we have

$$-i(\omega-\mathbf{k}\cdot\mathbf{v})f_{\mathbf{p},k}^{R(1)}=-e\mathbf{E}^N\cdot\frac{\partial\Phi_{\mathbf{p}}}{\partial\mathbf{p}}, \tag{66}$$

$$-i(\omega-\mathbf{k}\cdot\mathbf{v})f_{\mathbf{p},k}^{R(j+1)}=-e\int d^{(2)}\left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial\mathbf{p}}(\delta f_{\mathbf{p},k_2}^{R(j)}+f_{\mathbf{p},k_2}^{R(j)})\right\rangle, \tag{67}$$

$$-i(\omega-\mathbf{k}\cdot\mathbf{v})f_{\mathbf{p},k}^{S(1)}=-e\int d^{(2)}\left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial\mathbf{p}}\delta f_{\mathbf{p},k_2}^{(0)}\right\rangle, \tag{68}$$

$$-i(\omega-\mathbf{k}\cdot\mathbf{v})f_{\mathbf{p},k}^{S(j+1)}=-e\int d^{(2)}\left\langle \mathbf{E}_{k_1}\cdot\frac{\partial}{\partial\mathbf{p}}(\delta f_{\mathbf{p},k_2}^{S(j)}+f_{\mathbf{p},k_2}^{S(j)})\right\rangle. \tag{69}$$

For further calculations, we need to consider terms up to $f_{\mathbf{p}}^{R(3)}$ and $f_{\mathbf{p}}^{S(3)}$. In the first order, we have perturbation (66) which contributes to the linear dielectric permittivity of the nonresonant waves, and perturbation (68) which contributes to the dispersion equation for the test nonresonant wave only if the electric field in it is the virtual one E^V. Note that the term containing combination $E^{(0)}\delta f^{(0)}$ in (68) gives a contribution to the Balescu–Lenard collision integral (31). Thus from (68) we have, instead of (26), the following equation:

$$\rho_k^{S(1)}=\int\frac{d\mathbf{p}dk_1}{(2\pi)^3}\frac{e}{i|\mathbf{k}_1|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times\langle(E_{k_1}^{RV}+E_{k_1}^{SV})\delta f_{\mathbf{p},k-k_1}^{(0)}\rangle, \tag{70}$$

where virtual fields E^{RV} and E^{SV} satisfy Eqs. (25) and (28), respectively. Note that for the process considered, no regular virtual fields appear.

In the second order, we have contributions from $f_{\mathbf{p}}^{R(2)}$ and $f_{\mathbf{p}}^{S(2)}$. The term containing $f_{\mathbf{p}}^{R(2)}$ gives rise to the second-order charge density $\rho^{R(2)}$ which can be written as

$$-\frac{4\pi ie}{|\mathbf{k}|}\rho_k^{R(2)}=\int d^{(2)}S_{1,2}\langle(E_1^{RV}+E_1^{SV})E_2^{(0)}\rangle. \tag{71}$$

The term containing $f_{\mathbf{p}}^{S(2)}$ is responsible for

$$\rho_k^{S(2)}=-\int\frac{d\mathbf{p}dk_1dk_2}{(2\pi)^3}\frac{e^2}{|\mathbf{k}_1||\mathbf{k}_2|}\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\times\frac{1}{\omega_2-\mathbf{k}_2\cdot\mathbf{v}+i0}\left(\mathbf{k}_2\cdot\frac{\partial}{\partial\mathbf{p}}\right)\langle E_{k_1}^{(0)}E_{k_2}^{N}\delta f_{\mathbf{p},k-k_1-k_2}^{(0)}\rangle \tag{72}$$

[compare with Eq. (27)]. In the third order, we have just contribution from $f_{\mathbf{p}}^{R(3)}$ which is responsible for the third-order charge density perturbation $\rho^{R(3)}$:

$$-\frac{4\pi ie}{|\mathbf{k}|}\rho_k^{R(3)}=\int d^{(3)}\Sigma_{1,2,3}\langle E_1^{(0)}E_2^NE_3^{(0)}\rangle. \tag{73}$$

Thus we obtain the general nonlinear equation, analogous to (29), which is written for the regular test nonresonant wave E^N:

$$\begin{aligned}\varepsilon_K E_K^N=&2\int d^{(3)}\Sigma_{1,2,3}E_2^N\langle E_1^{(0)}E_3^{(0)}\rangle+2\int d^{(2)}S_{1,2}\langle(E_{k_1}^{RV}+E_{k_1}^{SV})E_{k_2}^{(0)}\rangle-\int\frac{d\mathbf{p}dk_1}{(2\pi)^3}\frac{4\pi e^2}{|\mathbf{K}||\mathbf{k}_1|}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}+i0}\left(\mathbf{k}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)\\&\times\langle(E_{k_1}^{RV}+E_{k_1}^{SV})\delta f_{\mathbf{p},K-k_1}^{(0)}\rangle+\int\frac{d\mathbf{p}dkdK_1}{(2\pi)^3}\frac{4\pi ie^3}{|\mathbf{K}||\mathbf{K}_1||\mathbf{k}|}\frac{1}{\Omega-\mathbf{K}\cdot\mathbf{v}+i0}\left(\mathbf{k}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\\&\times\frac{1}{\Omega_1-\mathbf{K}_1\cdot\mathbf{v}+i0}\left(\mathbf{K}_1\cdot\frac{\partial}{\partial\mathbf{p}}\right)E_{K_1}^N\langle E_k^{(0)}\delta f_{\mathbf{p},K-k-K_1}^{(0)}\rangle.\end{aligned} \tag{74}$$

From this equation, we can see that the contributions due to the nonlinear charge densities lead to the same terms in the dielectric function of the regular test nonresonant wave which are described by Eqs. (39)–(44).

The procedure of accounting for the slow nonstationarity effects in the dispersion equation for the test nonresonant wave is based on expansion of the expression for the displacement,

$$\mathbf{D}_{\mathbf{K}}(t)=\int_{-\infty}^{t}\frac{d\tau}{2\pi}\varepsilon_{\mathbf{K}}(\tau,t-\tau)\mathbf{E}_{\mathbf{K}}(t-\tau); \tag{75}$$

it uses also geometrical optics approximation (33) (with regular amplitude $E_{\mathbf{K}}^{(a)}$), and finally gives the same result (34) (for details, see, e.g., Ref. 8). Thus, for the case of the regular nonresonant wave the final dispersion equation has the form (45), and therefore all the above results on change of the nonresonant wave amplitude, number of quanta, and energy are also applicable to the case of a regular nonresonant wave. And, finally, we note that if the system includes regular resonant waves, qualitatively new features such as more effective energy exchange can appear as a result of the resonant wave coherency.[10,37]

VIII. CONCLUSION

In summary, we have shown that spontaneous emission effects influence the propagation of nonresonant waves in plasmas. The resonant particle fluctuations play an essential role in the evolution of the test waves which are not in resonance with plasma particles.

Averaging over the statistical ensemble, we proceed to a description of "observable" quantities. The motion of plasma particles consists of fluctuating and averaged (slow changing in time) parts. The quantities with slow time variation, such as zeroth-order averaged particle distribution function, define the slow time evolution of the system. In the absence of resonant turbulence, the time evolution of the averaged particle distribution is due to the particle Coulomb collisions, and is described by the Balescu–Lenard collision integral. The system's nonstationarity, in turn, leads to a slow change in time of all the parameters, including eigenfrequency of the nonresonant waves.

However, the nonstationarity is not the only process affecting the propagation of the nonresonant waves. We demonstrated that the nonlinear coupling of the nonresonant wave field with the fluctuating resonant fields leads to effects of the same order as the nonstationarity. Therefore, the complete dispersion equation for the test nonresonant wave includes the direct coupling of the wave field with the resonant fluctuations as well as the so-called polarization contribution (which physically corresponds to processes of transition scattering[38] of the test wave on strongly inhomogeneous particle density fluctuations). In the systems with a high degree of symmetry, such as closed plasma systems in the absence of energy exchange with external sources/sinks and in the absence of external fields, the total polarization contribution is zero. The effect is similar to that in closed systems with a high above-thermal level of resonant plasma turbulence. Moreover, the direct nonlinear coupling of the test wave field with the resonant plasma fluctuations in these systems is balanced by the reversed absorption effects due to the system's nonstationarity. Thus the conservation of the nonresonant wave quanta is observed. However, the test wave amplitude as well as energy are changing due to the system's nonstationarity.

Finally, we note that by choosing the corresponding resonant conditions in (29), we can describe different collective effects in wave propagation. Thus if we assume the scattering resonance to be satisfied [i.e., when (3) is no longer valid], we can obtain quite straightforwardly the scattering cross-sections, see, e.g., Refs. 27 and 31. Analogously, plasma bremsstrahlung can be calculated.[19]

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[1] V.N. Tsytovich, L. Stenflo, and H. Wilhelmsson, Phys. Scr. **11**, 251 (1975).
[2] M. Nambu, Phys. Rev. Lett. **34**, 387 (1975).
[3] D.B. Melrose, Austr. J. Phys. **35**, 67 (1982).
[4] A. Hirose, Comments Plasma Phys. Controlled Fusion **8**, 117 (1984).
[5] M. Nambu, J. Phys. Soc. Jpn. **53**, 1594 (1984).
[6] D.B. Melrose and J. Kuijipers, J. Plasma Phys. **32**, 239 (1984).
[7] O. Ishihara and A. Hirose, Bull. Am. Phys. Soc. **32**, 1859 (1987).
[8] V.S. Krivitsky, V.N. Tsytovich, and S.V. Vladimirov, Phys. Rep. **218**, 141 (1992).
[9] M. Nambu, S.V. Vladimirov, and H. Schamel, Phys. Lett. A **178**, 400 (1993).
[10] S.V. Vladimirov, Phys. Lett. A **193**, 387 (1994).
[11] S.V. Vladimirov and M. Nambu, Phys. Plasmas **2**, 1140 (1995).
[12] V.S. Krivitsky and S.V. Vladimirov, Phys. Lett. A **143**, 329 (1990).
[13] V.N. Tsytovich, Sov. J. Plasma Phys. **6**, 608 (1980).
[14] S.V. Vladimirov and V.S. Krivitsky, Sov. Phys. Radiophys. Quantum Electron. **33**, 674 (1990).
[15] S.I. Popel. S.V. Vladimirov, and M.Y. Yu, Phys. Scr. **47**, 239 (1993).
[16] S.V. Vladimirov, M.Y. Yu, and S.I. Popel, Contrib. Plasma Phys. **33**, 1 (1993).
[17] S.B. Isakov, V.S. Krivistky, and V.N. Tsytovich, Sov. Phys. JETP **63**, 545 (1986).
[18] M. Nambu and T. Hada, Phys. Fluids B **5**, 742 (1993).
[19] V.N. Tsytovich, Usp. Phys. Nauk **165**, 89 (1995) [Sov. Phys. Usp. **38**, 87 (1995)].
[20] O. Ishihara and A. Hirose, Phys. Rev. Lett. **72**, 4090 (1994).
[21] O. Ishihara, Current Top. Phys. Fluids **1**, 45 (1994).
[22] O. Ishihara, A. Hirose, and A.B. Langdom, Phys. Rev. Lett. **44**, 1404 (1980).
[23] O. Ishihara, A. Hirose, and A.B. Langdom, Phys. Fluids **24**, 452 (1981); **25**, 610 (1982).
[24] O. Ishihara and A. Hirose, Phys. Rev. Lett. **46**, 771 (1981); **50**, 1783 (1983).
[25] O. Ishihara and O. Hirose, Phys. Fluids **26**, 100 (1983); **27**, 364 (1984).
[26] B.B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965).
[27] V.N. Tsytovich, *Theory of Turbulent Plasma* (Consultants Bureau, New York, 1977), Chap. 2.
[28] S.V. Vladimirov, V.N. Tsytovich, S.I. Popel, and F.Kh. Khakimov, *Modulational Interactions in Plasmas* (Kluwer Academic, Dordrecht, 1995), Chap. 2.
[29] O. Ishihara and A. Hirose, Phys. Fluids **27**, 2429 (1984).
[30] R. Balescu, Phys. Fluids **3**, 52 (1960); A. Lenard, Ann. Phys. NY **3**, 390 (1960).
[31] V.N. Tsytovich, Phys. Rep. **178**, 261 (1989).
[32] A.N. Kaufman, J. Plasma Phys. **8**, 1 (1972).
[33] O. Ishihara, Phys. Rev. A **35**, 1219 (1987).
[34] L.P. Pitaevsky, Sov. Phys. JETP **12**, 1008 (1961).
[35] R.L. Dewar, J. Plasma Phys. **7**, 267 (1972).
[36] K. Katou, J. Phys. Soc. Jpn. **50**, 642 (1981).
[37] S.V. Vladimirov, Phys. Scr. **50**, 561 (1994).
[38] V.L. Ginzburg and V.N. Tsytovich, *Transition Radiation and Transition Scattering* (Adam Hilger, London, 1990), Chap. 4.