PLASMA COLLECTIVE EFFECTS IN THE PRESENCE OF DUST

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Abstract. Charged dust particles strongly affect collective processes in plasmas. Here we review recent advances in the theoretical study of collective effects in dusty plasmas. In particular, we consider processes of charging of dust by plasma currents, wave propagation and scattering, properties of Alfvén waves, and discuss collective effects in plasma-dust crystals.

1. Introduction

A great deal of interest in studying the physics of dusty plasmas, whose constituents are electrons, ions, and extremely heavy highly charged dust particles, was primarily connected with an increasing worldwide effort to model DC, RF, and microwave plasma discharge devices (where dusty particles grains appear naturally, grow in size, and play a role as natural contaminants in the processes) used in plasma-assisted materials processing for sputtering, deposition and implantation, to produce materials such as diamond films and selective surfaces for solar collectors (Selwyn et al., 1989, 1990). Another example of recent interest is the low temperature edge plasma physics in nuclear fusion devices, where dust grains emitted from walls may strongly influence anomalous transport properties (Benkadda et al., 1995; Tsypin et al., 1997).

Dust grains are usually of micron and submicron size, negatively charged, with many electrons collected on each particulate, and have a mass that is much larger than the positive ion mass. The negative charge of the dust grains could be due to different processes, such as due to bombardment by charged particles from surrounding plasmas, ultraviolet irradiation, sputtering of energetic ions (Northrop, 1992; Allen, 1992). Since the dust particles are almost immediately electrically charged being immersed in the ambient plasma, they must be coupled to each other as well as the plasma via the electric and magnetic fields (Whipple et al., 1985). The presence of chaotically moving charged dust grains has been shown to lead to considerable modification of plasma collective properties strongly affecting dispersion relations and damping of the usual plasma waves, as well as leading to specific oscillation modes associated with dust motion (Tsytovich and Havnes, 1993; Vladimirov, 1994a; Melandsø et al., 1993; Cramer and Vladimirov, 1996a, 1996b; Vladimirov and Cramer, 1996). Interaction of dust particles with

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themselves is also naturally affected by their plasma environment and therefore by plasma collective effects.

Dusty plasmas are also common in a variety of low-temperature plasmas in space environments, such as the lower ionosphere of the Earth, planetary atmospheres, asteroid zones, nebulas, and cometary tails (Goertz, 1989; Mendis and Rosenberg, 1994). Although in space physics research effects of dust grains has been studied for many years (Spitzer, 1978), only recently the role of plasma collective phenomena was recognized due to progress in experiments on comets, planetary rings, and Earth's environment (Northrop, 1992; Mendis and Rosenberg, 1994).

An exciting area of the most recent research is the plasma-dust crystal formation, theoretically predicted in (Ikezi, 1986). The success in creation of crystalline dusty structures (Chu and I, 1994; Thomas et al., 1994; Hayashi and Tachibana, 1994; Melzer et al., 1994) in laboratory radio frequency discharge plasmas makes it possible to study fundamental processes of phase transitions in a Coulomb system of charged particles (Melzer et al., 1996a; Chiang and I, 1996; Thomas and Morfill, 1996), as well as opens new possibilities for formation of new materials from dust with a big binding energy. Thus the Coulomb crystallization of dust grains in a laboratory plasma holds great potential as a model system for studies of structure, dynamics, and phase transitions in condensed matter (Melzer et al., 1996a; Chiang and I, 1996; Thomas and Morfill, 1996). This makes the topic of dusty plasmas of interest to a wide community of researchers.

The macroscopic lattices, named "dust-plasma crystals", are made of highly (negatively) charged particulates of micrometer size levitated in the sheath region above a horizontal negatively biased electrode. The grains in the lattices strongly interact via repulsive Coulomb forces, which is affected by ions and electrons of surrounding plasma. The particle system crystallizes when the interaction potential far exceeds their thermal energy (Ikezi, 1986).

Although the dust-plasma crystal systems are generally three-dimensional, in the most experiments the charged particles are located just in few layers above the horizontal electrode, where gravity is balanced by the sheath electric field. These layers act mainly as two-dimensional systems, with limited out-of-plane particle motion (Thomas and Morfill, 1996; Quinn et al., 1996). On the other hand, it was demonstrated that aligning of dust grains from different layers in the vertical plane is strongly connected with plasma collective mechanisms, namely, with the presence of super-sonic ions flowing towards the negative electrode (Vladimirov and Nambu, 1995; Vladimirov and Ishihara, 1996). It was shown (Melandsø and Goree, 1995; Ishihara and Vladimirov, 1997) that in otherwise uniform plasma, the flow leaves a polarized oscillating wake potential behind a stationary dust particle, with ions focussing to make the plasma potential positive in a local region. This positive region attracts negative particles, promoting the vertical alignment clearly observed experimentally (Melzer et al., 1996a; Chiang and I, 1996; Thomas and Morfill, 1996).

Theoretical studies of waves and instabilities in such dust-plasma crystal systems are just started (Melandsø, 1996; Vladimirov et al., 1997). At the same time, an enhanced level of plasma oscillations has been reported in experiments on dusty crystals (Chu et al., 1994; Praburam and Goree, 1996) thus allowing us to assume a significant role of plasma collective phenomena in these strongly coupled systems.

Here, recent advances in the study of collective effects in dusty plasmas are reviewed. The important point is that dust grains embedded in a plasma change its collective properties considerably and cannot always be considered just as an additional ideal plasma component (when one can directly apply the known results for multi-component plasmas), in particular, since the number of grains in the dust Debye sphere may sometimes be less than unity (Whipple et al., 1985). Moreover, the charges on dust particles are not fixed in processes of their interactions with plasma fields and other dust grains (Tsytovich and Havnes, 1993; Vladimirov, 1994a). Indeed, the floating charges are mostly determined by plasma electron and ion currents on the dust grains (Allen, 1992). The currents are naturally affected by the interactions which lead to a change of dust charges in the interaction process. This leads to qualitatively new effects (Tsytovich and Havnes, 1993; Vladimirov, 1994a, 1994b, 1994c, 1996), especially strong for low-frequency oscillations whose frequencies are comparable to the charging frequency. Below, we discuss some of this effects in more details.

Another important feature of dusty plasmas is that the presence of grains with even constant charges can strongly modify the existing wave spectra; as an example, we consider here modification of linear (Cramer and Vladimirov, 1996a) and nonlinear (Vladimirov and Cramer, 1996) Alfvén waves as well as surface Alfvén waves (Cramer and Vladimirov, 1996b) in a magnetized dusty plasma. Finally, influence of plasma collective processes on arrangements and vibrations of dust particles in the crystal-like Coulomb structures is considered. Note that the wake potential formation which leads to the vertical alignment of dust grains in the experiments, and, as a consequence, to quasi-two-dimensional features of the structures (i.e., hexagonal arrangements) when the number of dust layers is not high (the three-dimensional lattices corresponding to minimum of the potential energy are body-centered-cubic or face-centered-cubic) can also affect the specific lattice modes propagating in such systems.

2. Charging of a Dust Particle

Let us consider an unmagnetized plasma whose constituents are electrons, ions, and massive static ($m_d = \infty$) negatively charged dust grains. The latter are assumed to be point charges (i.e., their sizes are supposed to be much smaller than the effective Debye radius λ_D of the dusty plasma).

The basic charging equation is given by

$$\frac{dq}{dt} = I(q), \tag{1}$$

where q is the charge residing on a dust grain, and the current I on the dust particle is a sum of electron and ion plasma currents

$$I(q) = \sum_{\alpha} \int e_\alpha f_\alpha \sigma_\alpha(v, q) v d\mathbf{v}. \tag{2}$$

Here, the subscript $\alpha = e, i$ describes electrons or ions, e_α and f_α are the corresponding charge and distribution function of plasma particles, respectively, $e_e = -e_i \equiv -e$, $v \equiv |\mathbf{v}|$ is the absolute value of particle speed $\mathbf{v}$, σ_α is the charging cross-section (Spitzer, 1978):

$$\sigma_\alpha = \pi a^2 \left(1 - \frac{2e_\alpha q}{a m_\alpha v^2}\right) \quad \text{if} \quad \frac{2e_\alpha q}{a m_\alpha v^2} < 1,$$
$$\sigma_\alpha = 0 \quad \text{if} \quad \frac{2e_\alpha q}{a m_\alpha v^2} \geq 1, \tag{3}$$

and, finally, m_α is the mass of the plasma electrons or ions. The last inequality in (3) gives restriction on particle charging velocities for electrons (we remind that we consider the negatively charged dust particles, i.e., $q < 0$). Thus we see that only sufficiently fast plasma electrons (which have enough energy to cross the potential barrier) can charge the dust particles.

In the state of equilibrium, we have

$$I^{\text{eq}}(Q) \equiv I_e^{\text{eq}}(Q) + I_i^{\text{eq}}(Q) = 0, \tag{4}$$

where $Q = -Z_d e$ is the equilibrium charge of the dust particulate. The equilibrium electron and ion currents are given by

$$I_e^{\text{eq}} = -e\sqrt{\frac{\pi}{2}}\, a^2 v_{Te} n_e \exp\left(-\frac{Z_d e^2}{a T_e}\right) \tag{5}$$

and

$$I_i^{\text{eq}} = e\sqrt{\frac{\pi}{2}}\, a^2 v_{Ti} n_i \left(1 + \frac{Z_d e^2}{a T_i}\right). \tag{6}$$

Here, $v_{T\alpha} = (T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity, n_α is the number density, and T_α is the corresponding temperature of plasma particles. Thus the equilibrium charge can be found as a solution of the equation following from Equation (4):

$$\frac{\omega_{pe}^2}{v_{Te}} \exp\left(-\frac{Z_d e^2}{a T_e}\right) = \frac{\omega_{pi}^2}{v_{Ti}} \left(\frac{T_i}{T_e} + \frac{Z_d e^2}{a T_e}\right), \tag{7}$$

where $\omega_{p\alpha} = (4\pi n_\alpha e_\alpha^2/m_\alpha)^{1/2}$ is the plasma frequency (electron or ion).

Furthermore, it is convenient to introduce the dimensionless variables

$$\tau \equiv \frac{T_i}{T_e} , \quad z \equiv \frac{Z_d e^2}{a T_e} . \tag{8}$$

For hydrogen plasmas, Equation (7) gives $z \simeq 2.5$ if $\tau = 1$ and $z \simeq 1.9$ if $\tau = 0.1$. Usually, Z_d is of order 10^3–10^4, but the total negative charge on the dust particles is close to (and does not exceed as a rule) the total charge of electrons. Thus, the following dimensionless parameter is of order unity

$$\mu \equiv \frac{Z_d n_d}{n_e} \sim 1, \tag{9}$$

where n_d is the dust density. However, more accurate calculations (which were done for a dust cloud in a thermal plasma, see Havnes et al., 1984) give that the following parameter is of order unity

$$P \equiv \frac{n_d}{n_e} \frac{a T_e}{e^2} = \frac{\mu}{z} . \tag{10}$$

When $z \sim 1$, these two conditions coincide.

The charging dissipative process is characterized by the charging frequency ν_d

$$\nu_d \equiv - \left. \frac{\partial I(q)}{\partial q} \right|_{q=-Z_d e} = \frac{1}{\sqrt{2\pi}} \frac{\omega_{pi}^2 a}{v_{Ti}} (1 + \tau + z). \tag{11}$$

Note that the charging frequency describes the process of charging of the dust particles for small deviations from the equilibrium; it naturally contains contributions from both electrons and ions. To obtain the last expression on the right-hand side of Equation (11), Equation (7) has been used. Comparison of the rate (11) with the plasma collision frequencies (Tsypin et al., 1997; Tsytovich and Havnes, 1993) demonstrates that this process can be the most important dissipative process for dusty plasmas. For our purposes, the frequency characterizing the rate of capture of plasma electrons by dust particles ν_{ed}^{eq} is also interesting

$$\begin{aligned} \nu_{ed}^{\text{eq}} &= \frac{n_d}{n_e} \int f_e \sigma_e v d\mathbf{v} = \nu_d P \frac{\tau + z}{1 + \tau + z} \\ &= 2\omega_{p0} \sqrt{\frac{P n_d a^3}{2}} \sqrt{\frac{n_e}{n_{e0}}} \exp\left(- \frac{\mu_0}{P} \frac{n_{e0}}{n_e} \right), \end{aligned} \tag{12}$$

where $\omega_{p0} = (4\pi n_{e0} e^2/m_e)^{1/2}$, $\mu_0 = Z_d n_{d0}/n_{e0}$, and n_{e0} and n_{d0} are, respectively, the electron and dust densities at the initial moment. The latter expression on the right-hand side of Equation (12) is used when studying wave amplification processes in non-stationary dusty plasmas (Vladimirov, 1994c).

3. Dielectric Permittivity of a Dusty Plasma

We introduce the distribution function of dust particles as (Vladimirov, 1994a)

$$f_d = f_d(q, \mathbf{r}, t), \tag{13}$$

where the charge q is the additional independent variable. The corresponding kinetic equation is given by (we remind that the dust particles have infinite masses and, consequently, $v_d = 0$)

$$\frac{\partial f_d}{\partial t} + \frac{\partial}{\partial q} I(q) f_d = 0. \tag{14}$$

The state of equilibrium corresponds to the equilibrium distribution function f_d^{eq} of charges on dust particles. We have

$$Q = \frac{1}{n_d} \int q f_d^{\mathrm{eq}} dq. \tag{15}$$

The kinetics of electrons and ions are described by usual distribution functions f_α:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{e_\alpha}{m_\alpha} \mathbf{E} \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = - \int \sigma_\alpha v (f_d f_\alpha - f_d^{\mathrm{eq}} f_\alpha^{\mathrm{eq}}) dq. \tag{16}$$

We assume that in the equilibrium state the electron and ion capture by the dust is compensated by external sources and the equilibrium electron (ion) distribution f_α^{eq} is isotropic. Term with magnetic field has been neglected on the left-hand side of Equation (16).

Presence of wave gives rise to small perturbations of the above distribution functions

$$f_d = f_d^{\mathrm{eq}} + \delta f_d, \quad f_\alpha = f_\alpha^{\mathrm{eq}} + \delta f_\alpha, \tag{17}$$

where $|\delta f_d| \ll |f_d|$ and $|\delta f_\alpha| \ll |f_\alpha|$. Furthermore, we linearize kinetic equations (14) and (16) with respect to these perturbations. Here, we note that, in principle, the ionization process can either be perturbed by δf_e (e.g., if the ionization is due to electron impact) or can affect dust (e.g., if the ionization is by radiation). For simplicity, we exclude such possibilities in the present investigation.

Thus, we obtain for the dust particle distribution function

$$\frac{\partial \delta f_d}{\partial t} + \frac{\partial}{\partial q} [I^{\mathrm{eq}}(q) \delta f_d] + \frac{\partial}{\partial q} [\delta I(q) f_d^{\mathrm{eq}}] = 0, \tag{18}$$

where

$$I(q) = I^{\mathrm{eq}}(q) + \delta I(q), \qquad \int I^{\mathrm{eq}}(q) f_d^{\mathrm{eq}} dq = 0,$$

$$\delta I(q) = \sum_\alpha \int d\mathbf{v} e_\alpha \sigma_\alpha v \delta f_\alpha. \tag{19}$$

The Fourier-component of the linear perturbation of the equilibrium charge on dust particles is

$$\delta Q_{\mathbf{k}\omega} = \frac{1}{n_d}\int q\delta f_d dq = \frac{i}{\omega + i\nu_d^{\mathrm{eq}}}\,\frac{1}{1+G_{\mathbf{k}\omega}}\,\frac{1}{n_d}\int I_{\mathbf{k}\omega}^{(1)}(q) f_d^{\mathrm{eq}} dq, \tag{20}$$

and that of the linear perturbation of the electron (ion) distribution function is given by

$$\begin{aligned}\delta f_{\alpha,\mathbf{k}\omega} \;=\; & -\frac{ie_\alpha/m_\alpha}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{\alpha d}^{\mathrm{eq}}(v)}\,\mathbf{E}_{\mathbf{k}\omega}\cdot\frac{\partial f_\alpha^{\mathrm{eq}}}{\partial \mathbf{v}} \\ & + \frac{1}{\omega + i\nu_d^{\mathrm{eq}}}\,\frac{1}{1+G_{\mathbf{k}\omega}}\,\frac{\sigma_\alpha' v f_\alpha^{\mathrm{eq}}}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{\alpha d}^{\mathrm{eq}}(v)}\int I_{\mathbf{k}\omega}^{(1)}(q) f_d^{\mathrm{eq}} dq. \end{aligned} \tag{21}$$

In Equations (20) and (21), the following notations have been used:

$$I_{\mathbf{k}\omega}^{(1)}(q) = \sum_\alpha \int_{V_d} d\mathbf{v}\frac{-ie_\alpha\sigma_\alpha v}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{\alpha d}^{\mathrm{eq}}(v)}\left(\frac{e_\alpha}{m_\alpha}\,\mathbf{E}_{\mathbf{k}\omega}\cdot\frac{\partial f_\alpha^{\mathrm{eq}}}{\partial \mathbf{v}}\right), \tag{22}$$

$$G_{\mathbf{k}\omega} = \frac{-1}{\omega + i\nu_d^{\mathrm{eq}}}\sum_\alpha \int_{V_d} d\mathbf{v}\frac{\nu_{\alpha d}^{\mathrm{eq}}(v) e_\alpha \sigma_\alpha' v f_\alpha^{\mathrm{eq}}}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{\alpha d}^{\mathrm{eq}}(v)}\,, \tag{23}$$

where

$$\sigma_\alpha^{\mathrm{eq}} = \frac{1}{n_d}\int \sigma_\alpha(q) f_d^{\mathrm{eq}} dq = \sigma_\alpha(-Z_d e), \quad \sigma_\alpha' \equiv \frac{\partial\sigma_\alpha(q)}{\partial q}\,. \tag{24}$$

Note that σ_α' is also the step function on v, which should be taken into account by the same phase-space volume V_d of integration over $\mathbf{v}$ as in integration of expressions containing the charging cross-section (3).

To find the dielectric permittivity of longitudinal waves, we use Poisson equation. In the case of the thermal particle distributions and sufficiently high frequency $[\omega \sim \omega_{pe} \gg \max(kv_{Te}, \nu_d^{\mathrm{eq}}, \nu_{ed}^{\mathrm{eq}})]$ of the waves, we have the following approximate expression (Vladimirov, 1994a)

$$\begin{aligned}\varepsilon_{\mathbf{k}\omega}^{(l)} \;\simeq\; & 1 + \frac{4\pi}{\omega^2 k^2}\sum_\alpha \frac{-e_\alpha^2}{m_\alpha v_{T\alpha}^2}\int d\mathbf{v}\left\{1 - i\frac{\nu_{\alpha d}^{\mathrm{eq}}(v)}{\omega} - \frac{[\nu_{\alpha d}^{\mathrm{eq}}(v)]^2}{\omega^2}\right\}(\mathbf{k}\cdot\mathbf{v})^2 f_\alpha^{\mathrm{eq}} \\ =\; & 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 - i\frac{2}{3}(2+z)\frac{\nu_{ed}^{\mathrm{eq}}}{\omega} - \sqrt{\pi z}A\mathrm{e}^z\left(\frac{\nu_{ed}^{\mathrm{eq}}}{\omega}\right)^2\right], \end{aligned} \tag{25}$$

where

$$A \equiv \frac{5}{4} - \frac{z}{6} + \left(\frac{5}{4} - z + \frac{z^2}{3}\right)\int_1^\infty d\tau\, \exp[-(\tau^2 - 1)z]. \tag{26}$$

Therefore, the charging process leads to appearing of additional imaginary and real parts in the high-frequency longitudinal dielectric permittivity. These additional terms depend on relation between the charging frequency and electron plasma frequency. For Langmuir waves, the damping due to the charging process is given by

$$\gamma_d^L \simeq -\frac{1}{3}\,(2+z)\nu_{ed}^{\text{eq}} = -\frac{1}{3}\,P(2+z)\frac{\tau+z}{1+\tau+z}\,\nu_d^{\text{eq}}. \tag{27}$$

For the transversal dielectric function we use Maxwell equations and find

$$\begin{aligned}\varepsilon_{\mathbf{k}\omega}^{(t)} &\simeq 1+\frac{2\pi}{\omega^2 k^2}\sum_\alpha \frac{-e_\alpha^2}{m_\alpha v_{T\alpha}^2}\int d\mathbf{v}\left\{1-i\frac{\nu_{\alpha d}^{\text{eq}}(v)}{\omega}-\frac{[\nu_{\alpha d}^{\text{eq}}(v)]^2}{\omega^2}\right\}(\mathbf{k}\times\mathbf{v})^2 f_\alpha^{\text{eq}} \\ &= 1-\frac{\omega_{pe}^2}{\omega^2}\left[1-i\frac{2}{3}\,(2+z)\frac{\nu_{ed}^{\text{eq}}}{\omega}-\sqrt{\pi z}A\mathrm{e}^z\left(\frac{\nu_{ed}^{\text{eq}}}{\omega}\right)^2\right]\end{aligned} \tag{28}$$

which exactly coincides with expression (25) for the high-frequency longitudinal dielectric permittivity. Equation (28) gives the following damping rate for electromagnetic waves due to the charging effects on dust particles

$$\gamma_d^{EM} \simeq -\frac{1}{3}\,\frac{\omega_{pe}^2}{\omega^2}\,(2+z)\nu_{ed}^{\text{eq}}. \tag{29}$$

If the frequency of electromagnetic waves does not far exceed ω_{pe}, when the phase velocity is significantly influenced by the conduction current, the damping rate (29) is of the order of the electron capture rate ν_{ed}^{eq}. Note the real corrections to the electron plasma frequency which are also effective for electromagnetic waves.

4. Wave Scattering

The usual Thomson scattering of a wave on a test particle of charge q and mass m takes place when the incident wave causes particle oscillation in its field. The scattered power in this case is proportional to m^{-2}, and because of the very large mass of the dust particle m_d the effect decreases despite the large charge residing on the dust particle. Thus because of the large masses of dust grains comparing with masses of electrons and ions, the wave scattering on the plasma particles forming their Debye spheres can dominate over the Thomson scattering on the the test “bare” dust grain.

To find the scattering of electromagnetic waves on a dust particle, we use the procedure elaborated in (Vladimirov, 1994b). In particular, the scattering cross-section is given by

$$\sigma^{em} = \sigma^T \frac{3c^3}{16\pi}\int d\mathbf{k}\left(1+\cos^2\Theta\right)\frac{|Z_{\mathbf{k}-\mathbf{k}_0}^{\text{eff}}|^2}{\omega_0^2}\,\delta\left(\omega_0-\sqrt{\omega_{pe}^2+k^2c^2}\right), \tag{30}$$

where σ^T is the cross-section of the Thomson scattering,

$$\sigma^T = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^3} \,, \tag{31}$$

c is the light speed, Θ is the angle between incident and scattered waves, subscript 0 corresponds to the incident wave with frequency $\omega_0 = (\omega_{pe}^2 + k_0^2 c^2)^{1/2}$, and $Z_{\mathbf{k}}^{\text{eff}} = Q_{\mathbf{k}}/e$ is Fourier component of the "effective charge" residing on the dust particle. In the linear approximation, the effective charge is given by

$$Z_{\mathbf{k}}^{\text{eff}} = Z_d \frac{\varepsilon_{\mathbf{k}}^{(e)} - 1}{\varepsilon_{\mathbf{k}}} \,, \tag{32}$$

where $-Z_d e$, as elsewhere in this paper, is the equilibrium charge of the dust particle, and

$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}}^{(e)} + \varepsilon_{\mathbf{k}}^{(i)} - 1 \tag{33}$$

is the complete static dielectric permittivity (i.e., $\varepsilon_{\mathbf{k}}^{(\alpha)}$ corresponds to the dielectric function of α plasma component) taking into account perturbations of the charging process.

Using the kinetic theory introduced above, we obtain in the static approximation

$$\varepsilon_{\mathbf{k},\omega=0} = 1 + \frac{1}{k^2\lambda_{De}^2}\frac{1+\tau}{\tau}\left(1 + P\frac{\tau+z}{1+\tau+z}F_{\mathbf{k}}\right), \tag{34}$$

where $\lambda_{De} = v_{Te}/\omega_{pe}$ is the electron Debye length and the formfactor $F_{\mathbf{k}\omega}$ is defined by

$$F_{\mathbf{k}} = \frac{1 + \Gamma_{\mathbf{k},\omega=0}}{1 + G_{\mathbf{k},\omega=0}} \,. \tag{35}$$

In this equation, the factor $G_{\mathbf{k}\omega}$ is given by Equation (23) and

$$\Gamma_{\mathbf{k}\omega} = \sum_\alpha \int d\mathbf{v}\frac{-ie_\alpha\sigma'_\alpha v f_\alpha^{\text{eq}}}{\omega - \mathbf{k}\cdot\mathbf{v} + i\nu_{\alpha d}^{\text{eq}}(v)} \,. \tag{36}$$

We see that the charging process leads to appearing of the additional real part in the static longitudinal dielectric permittivity. In particular, this additional term depends on the parameter P. Note also that the factor before the function $F_{\mathbf{k}}$ in the right-hand side of Equation (34) is exactly equal to $\nu_{ed}^{\text{eq}}/\nu_d^{\text{eq}}$, see Equation (12).

In the case of thermal particle distributions and static longitudinal perturbations $[\omega \ll \max(kv_{T\alpha}, \nu_d^{\text{eq}}, \nu_{\alpha d}^{\text{eq}})]$, we have the following expressions:

$$\Gamma_{\mathbf{k},\omega=0} = \frac{\pi n_d a^2 \mathrm{e}^{-z}}{Pk}\left[\frac{v_{Te}}{v_{Ti}}\frac{1}{\tau+z} + \sqrt{\frac{4z}{\pi}}\int_1^{+\infty} dx\,\mathrm{e}^{-x^2(z-1)}\right], \tag{37}$$

and

$$G_{\mathbf{k},\omega=0} = -\frac{\pi}{2}\frac{\tau+z}{1+\tau+z}\frac{\pi n_d a^2}{k}\left\{\int_0^{+\infty} dx \frac{x\mathrm{e}^{-x}}{x+z} + \frac{1}{1+\tau}\right.$$
$$\left.\times \left[1 + \frac{2z}{\pi\tau}\int_0^{+\infty} dx \frac{\mathrm{e}^{-x}}{x} \arctan\left(\frac{k}{\pi n_d a^2}\frac{\tau x}{\tau x + z}\right)\right]\right\}. \tag{38}$$

Therefore, for the electron part of the longitudinal dielectric permittivity, which is present in Equation (32), we can easily obtain [29]

$$\varepsilon^{(e)}_{\mathbf{k},\omega=0} = 1 + \frac{1}{k^2\lambda_{De}^2}\left(1 + P\frac{\tau+z}{1+\tau+z}F_{\mathbf{k}}\right). \tag{39}$$

Finally, for the effective charge (32) we find

$$Z_{\mathbf{k}}^{\mathrm{eff}} = Z_d\left(1 + \frac{1}{\tau} + \frac{k^2\lambda_{De}^2}{1 + PF_{\mathbf{k}}(\tau+z)/(1+\tau+z)}\right)^{-1}. \tag{40}$$

Thus, we have found that the effective cross-section of scattering of electromagnetic waves on a dust particle depends not only on the scattering parameter $1/k\lambda_{De}$ but also on the parameters characterizing the charging process, in particular P.

Finally, we note that here, for the sake of simplicity, we presented only the linear theory of the scattering on the isolated dust grain. In general, nonlinear effects can be important for the process. Some of these effects (although not taking into account dust charge fluctuations) have been considered in (Tsytovich, 1992).

5. Alfvén Waves in a Magnetized Dusty Plasma

We consider here the influence on the dispersion properties of Alfvén waves of the collection of electrons and ions from the background plasma by the charged grains. This affects the equilibrium state when the electron and ion charge balance necessarily includes the charge and density of the particulates

$$-en_e + en_i - Z_d en_d = 0. \tag{41}$$

For many dusty plasmas, an appreciable proportion of the negative charge in the plasma may reside on the dust particles. In that case the dispersion properties of Alfvén in the plasma are strongly modified by the dust, because the ion Hall current is not compensated by the electron Hall current at low frequencies as well as at frequencies comparable with the ion-cyclotron frequency, so that ion cyclotron effects extend to frequencies much less than the ion-cyclotron frequency (Cramer and Vladimirov, 1996a; Vladimirov and Cramer, 1996). The waves at frequencies comparable to the ion-cyclotron frequency are also circularly polarized modes, as in the dust-free case. The Alfvén resonance (Cramer and Vladimirov, 1996a) is strongly modified in the presence of dust grains because of the imbalance of electron and ion charges.

We invoke the standard two-fluid MHD model which includes the fluid momentum equations for the plasma (singly charged cold) ions and (inertialess) electrons, the ion continuity equation, as well as Maxwell's equations ignoring the displacement current. The background magnetic field $\mathbf{B}_0$ is in the z-direction, and the dust grains have infinite masses (we thus exclude dust dynamics from consideration). Furthermore, we introduce the parameter $\delta = n_e/n_i$ which measures the charge imbalance in the plasma. The total system is supposed to be neutral with the remainder of the charge residing on the dust particles according to Equation (41).

The starting equations are Euler equations for the ion and electron velocities $\mathbf{v}_i$ and $\mathbf{v}_e$, Maxwell equations for the wave electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ (the latter also includes the background field $\mathbf{B}_0$), and continuity equations for the ion and electron densities n_i and n_e. The electron inertia is neglected. In the one-dimensional case (being interested in evolution parallel to the external magnetic field), we introduce the right(left)-circularly polarized magnetic field components $B_\pm = B_x \pm iB_y$ and find

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_z)}{\partial z} = 0, \tag{42}$$

$$\frac{\partial B_\pm}{\partial t} = -\frac{\partial}{\partial z}\left[\frac{1}{\delta}\,(v_z B_\pm - v_\pm)\right] \mp \frac{i v_A^2}{\Omega_i}\,\frac{\partial}{\partial z}\left(\frac{1}{\rho\delta}\,\frac{\partial B_\pm}{\partial z}\right), \tag{43}$$

$$\frac{dv_\pm}{dt} = \mp i\Omega_i \frac{1-\delta}{\delta}\,(v_z B_\pm - v_\pm) + \frac{v_A^2}{\rho\delta}\,\frac{\partial B_\pm}{\partial z}\,, \tag{44}$$

$$\frac{dv_z}{dt} = \mp\frac{i\Omega_i}{2}\,\frac{1-\delta}{\delta}\,(v_\pm B_\pm^* - v_\pm^* B_\pm) - \frac{v_A^2}{2\rho\delta}\,\frac{\partial}{\partial z}\,(|B_\pm|^2), \tag{45}$$

where the right(left)-circularly polarized velocity component has been introduced analogously to the magnetic field component: $v_\pm = v_x \pm i v_y$. In Equations (42–45), we have introduced the dimensionless magnetic field amplitude normalized as $\mathbf{B} = \mathbf{B}/B_0$, where $B_0 = |\mathbf{B}_0|$, and omitted the subscript i for the ion velocity. Furthermore, $\rho = m_i n_i/\rho_0$ is the normalized density of the ion component of the plasma, where $\rho_0 = m_i n_{i0}$ and n_{i0} corresponds to the equilibrium ion concentration, $\Omega_i = eB_0/m_i$ is the ion gyrofrequency, and $v_A = (B_0^2/\mu_0\rho_0)^{1/2}$ is the Alfvén velocity.

Linearizing Equations (42–45) with respect to the wave magnetic field, we find

$$\left(\frac{\partial^2}{\partial t^2} \mp i\Omega_i \frac{1-\delta_0}{\delta_0}\,\frac{\partial}{\partial t} - \frac{v_A^2}{\delta_0}\,\frac{\partial^2}{\partial z^2} \pm i\,\frac{v_A^2}{\Omega_i\delta_0}\,\frac{\partial^3}{\partial t\,\partial z^2}\right) B_\pm = 0, \tag{46}$$

where $\delta_0 = n_{e0}/n_{i0}$ measures the unperturbed electron and ion number density imbalance. First, we consider the case

$$k v_A \ll \Omega_i \frac{1-\delta_0}{\sqrt{2(1+\delta_0)}} < \Omega_i, \tag{47}$$

(region A) and find the following approximate solutions:

$$\omega_{-1} \equiv \omega_{HF} \simeq \Omega_m \left[1 + \frac{k^2 v_A^2}{(1-\delta_0)^2 \Omega_i^2}\right], \quad \omega_{+1} \equiv \omega_{LF} \simeq \frac{k^2 v_A^2}{(1-\delta_0)\Omega_i}, \tag{48}$$

where

$$\Omega_m = \Omega_i \frac{1-\delta_0}{\delta_0}. \tag{49}$$

If we consider the higher wavenumber range defined by

$$\Omega_i \frac{1-\delta_0}{\sqrt{2(1+\delta_0)}} \ll k v_A \ll \Omega_i, \tag{50}$$

(region B) then we obtain

$$\omega_{HF,LF} \simeq k v_A \sqrt{\frac{1+\delta_0}{2}} \pm \frac{\Omega_i}{2}\left(1 - \delta_0 + \frac{k^2 v_A^2}{\Omega_i^2}\right). \tag{51}$$

i.e. an almost linear dispersion relation, with a displacement in frequency due to the charge imbalance and ion cyclotron dispersion, of opposite sign for the two modes.

The linear electromagnetic modes do not involve perturbations of ρ or v_z. Furthermore, the fluctuations in ρ and v_z are assumed to be of small amplitude [of order $|B_\pm|^2$ from Equation (45)], so we can linearize Equations (42–45) in these quantities and find

$$\begin{aligned} &\frac{\partial}{\partial t}\left[\frac{\partial B_\pm}{\partial t} \mp i\Omega_m B_\pm \pm \frac{i v_A^2}{\delta_0 \Omega_i}\frac{\partial^2 B_\pm}{\partial z^2}\right. \\ &\quad \left. + \frac{\partial}{\partial z}\left(\mp \frac{i v_A^2 \Delta\rho}{\delta_0 \Omega_i}\frac{\partial B_\pm}{\partial z} + \frac{v_z B_\pm}{\delta_0} + \frac{v_\pm \Delta\rho}{\delta_0^2}\right)\right] \\ &\quad = \frac{1}{\delta_0}\frac{\partial^2}{\partial z^2}\left(v_A^2 B_\pm - v_z v_\pm\right). \end{aligned} \tag{52}$$

The higher frequency mode is assumed to propagate at a frequency close to Ω_m, so that we introduce the envelope amplitudes of the wave magnetic field and ion velocity perturbations as $B_{-1} = b_1 \exp(-i\Omega_m t)$ and $v_{-1} = v_1 \exp(-i\Omega_m t)$. Retaining the largest dispersion and nonlinear terms, we thus find

$$i\frac{\partial b_1}{\partial t} + \frac{v_A^2}{\delta_0^2 \Omega_m}\frac{\partial^2 b_1}{\partial z^2} + i\frac{1+\delta_0}{\delta_0}\frac{\partial (v_z b_1)}{\partial z} + \Omega_m \Delta\rho b_1 = 0. \tag{53}$$

For the parallel component of the ion velocity perturbation, we have

$$\frac{\partial v_z}{\partial t} = -\frac{i\Omega_m}{2}\,(v_1^* b_1 - v_1 b_1^*) - \frac{v_A^2}{2\delta_0}\,\frac{\partial(|b_1|^2)}{\partial z}\,. \tag{54}$$

Equations (53) and (54), together with the linearized continuity equation (42), form the set of equations describing evolution of the higher frequency mode amplitude.

For the lower frequency mode, we find that the nonlinear contribution in the approximation used is negligible containing high orders of field derivatives. We also should mention that for the lower-frequency mode, the dust dynamics can be of large importance, so that the starting equations should be modified to include dust dynamic terms. For the range of wavenumbers corresponding to the region B, defined by Equation (50), the same method of stretched coordinates as used for the dust-free case can be invoked. As a result, there are no qualitatively new features comparing with the dust-free case, although in the resulting DNLS equation the dispersion and nonlinear coefficients are affected by the presence of dust.

We now seek wave solutions of the weakly nonlinear equations for the higher frequency mode of region A. We have the linear solution of Equations (53) and (54) in the form $b_1 = b_{10}\exp(-i\Omega t + iKz)$ and $v_1 = v_{10}\exp(-i\Omega t + iKz)$, where b_{10} and v_{10} are constants, and $\Omega \simeq K^2 v_A^2/\delta_0^2\Omega_m$ is the dispersion correction to the frequency Ω_m. Furthermore, we allow for a slow dependence of the amplitudes b_{10} and v_{10}, as well as their phase Θ, on t and z and consider a propagating solution only, where all functions (including phases) depend on $Z = z - v_0 t$. In this case, for substitution in Equation (54) we have $v_1(Z) = -\delta_0 v_0 b_1(Z)$. This is an important simplification which holds only under the assumption of the dependence of both amplitudes and phase on Z.

Assuming that all perturbations vanish at $Z \to \pm\infty$, we can write Equation (53) in the form of a mixed DNLS–NLS equation

$$-iv_0\frac{db_1}{dZ} + D\frac{d^2 b_1}{dZ^2} + i\mu\frac{d}{dZ}\,(|b_1|^2 b_1) + \nu|b_1|^2 b_1 = 0, \tag{55}$$

where the coefficients are $D = v_A^2/\delta_0^2\Omega_m$, $\mu = (1+\delta_0)v_A^2/2\delta_0 v_0$, and $\nu = \Omega_m v_A^2/2\delta_0^2 v_0^2$. Furthermore, separating the imaginary part we find $K(Z) = (v_0/2D) - (3\mu b_{10}^2/4D)$, and $\Omega(Z) = K(Z)v_0$. The equation for the real part of (55) can be written as

$$\frac{d^2 b_{10}}{dZ^2} = -\frac{\partial U(b_{10})}{\partial b_{10}}\,, \tag{56}$$

where the nonlinear potential is

$$U(b) = \frac{v_0^2}{8D^2}b^2 + \left(\frac{\nu}{4D} - \frac{\mu v_0}{8D^2}\right)b^4 + \frac{\mu^2}{32D^2}\,b^6. \tag{57}$$

The potential (57) describes nonlinear conoidal waves; the corresponding solutions can be written in terms of elliptic functions. Note that when $\nu < \mu v_0/2D$ or $v_A^2 <$

$(1 + \delta_0)\delta_0^2 v_0^2$, a metastable equilibrium state with $b_{10} \neq 0$ appears. A separatrix dividing the quasiperiodic oscillations in this case corresponds to a localized wave packet. Also, we find that no solutions in the form of standard soliton solutions occur if the conditions opposite to the above are applicable.

6. Formation of Dust Crystals

The possibility of formation of Coulomb quasilattices (Ikezi, 1986) involving the micro-meter sized highly charged dust particulates was first time demonstrated experimentally in works (Chu and I, 1994; Thomas et al., 1994; Hayashi and Tachibana, 1994; Metzer et al., 1994). In the experiments, the dust-crystal structure is observed in the sheath region of a radio-frequency discharge plasma where there is balance between the gravitational and electrostatic forces. The distance between the dust grains is of order Debye length λ_D.

It is well known that in the sheath region strong ion flow is established; according to the Bohm criterion the speed of the flow exceeds the ion acoustic velocity (Vladimirov and Nambu, 1995; Vladimirov and Ishihara, 1996). The drag force on a test particle in a plasma with finite flows necessarily includes collective effects. It was shown that if the speed of the flow exceeds the velocity of ion oscillations in the flow, an oscillating stationary wake is formed behind the static test particle (Vladimirov and Nambu, 1995; Vladimirov and Ishihara, 1996; Melandsø and Goree, 1995; Ishihara and Vladimirov, 1997). The effect is similar to the Cooper pairing of electrons in superconductors, and was earlier studied, e.g., for two-component electron-ion plasmas (Nambu and Akama, 1985). We note that the collective mechanism can be responsible for the oscillatory potential in the direction parallel (Vladimirov and Nambu, 1995; Ishihara and Vladimirov, 1997) as well as perpendicular (Vladimirov and Ishihara, 1996; Ishihara and Vladimirov, 1997) to the flow, and the attraction due to the wake potential can overcome the static Coulomb repulsion. The characteristic spacing in this case is of order Debye length λ_D in agreement with the experiments.

We consider the cylindrical geometry (ρ, φ, z); the plasma ion flow is in the $-z$-direction with velocity v_{i0}. The test dust particle of the charge Q is placed on the position $(0, 0)$. Furthermore, we calculate the potential behind the test particle downstream the flow within the wake cone: $|z| > \rho\sqrt{M^2 - 1}$, where $M = v_{i0}/v_s$ is the Mach number and $v_s = (T_e n_i / m_i n_e)^{1/2}$ is the sound velocity. Note that because we are interested in static dust particulates placed in the ion flow, the Cherenkov angle depends only on the speed of the flow. The static Coulomb repulsion between the dust particles is strong if the distance between them is less than the Debye length. Here, we note that the main contribution to the effective Debye length λ_D in a typical discharge conditions Chu and I, 1994; Thomas et al., 1994; Hayashi and Tachibana, 1994; Melzer et al., 1994, 1996a; Chiang and I, 1996) is due to the plasma electrons: $\lambda_D = \lambda_{De} \equiv (T_e/4\pi n_e e^2)^{1/2}$. Below, we are interested in the case where dust particles are subject to the Debye screening potential as well as the wake potential which arises when the ion flow velocity exceeds the critical speed.

We write the electrostatic potential of the static dust particle as

$$\Phi(\mathbf{r}) = Q \int \frac{d\mathbf{k}}{2\pi^2 k^2} \frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{\varepsilon(\mathbf{k}, -k_z v_{i0})}, \tag{58}$$

where $\mathbf{k} = (\mathbf{k}_\perp, k_z)$. The dielectric response function of the plasma in the presence of the finite ion flow with the speed v_{i0} ($v_s < v_{i0} \ll v_{Te}$, where $v_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity and m_e is the electron mass) is calculated under condition

$$k v_{Ti} \ll |k_z v_{i0}| \ll k v_{Te}, \tag{59}$$

where $v_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal velocity, T_i is the ion temperature, and, as we stated already, the flow is in $-z$-direction. For the plasma dielectric response function, we have

$$\varepsilon(\mathbf{k}, -k_z v_{i0}) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pi}^2}{(-k_z v_{i0} + i0)^2}\ , \tag{60}$$

where $\omega_{pi} = (4\pi n_i e^2/m_i)^{1/2}$ is the ion plasma frequency. Note the imaginary part $+i0$ (appearing due to causality) which is important when integrating over k_z.

Furthermore, we separate in Equation (58) (using the inverse of function (60)) the static Debye and oscillating wake potentials and obtain outside the Mach cone

$$\Phi(\mathbf{r}) = \Phi_D(\mathbf{r}), \tag{61}$$

where

$$\Phi_D(\mathbf{r}) = \frac{Q}{|\mathbf{r}|} \exp(-|\mathbf{r}|/\lambda_D), \tag{62}$$

and inside the Mach cone

$$\Phi(\mathbf{r}) = \Phi_W(\mathbf{r}), \tag{63}$$

where $\Phi_W(\mathbf{r})$ involves the collective effects caused by the oscillations in the ion flow:

$$\Phi_W(\mathbf{r}) = Q \int \frac{d\mathbf{k}}{2\pi^2 k^2} \frac{k^2\lambda_D^2\omega_{\mathbf{k}}^2 \exp(i\mathbf{k}\cdot\mathbf{r})}{(1 + k^2\lambda_D^2)[(-k_z v_{i0} + i0)^2 - \omega_{\mathbf{k}}^2]}. \tag{64}$$

Here, $\omega_{\mathbf{k}} = k v_s/(1 + k^2\lambda_D^2)^{1/2}$ is the characteristic frequency of the oscillations in the flow; it naturally appears when we equal to zero the plasma response function (60). The potential (64) describes the strong resonant interaction between the oscillations in the ion flow and the test particulate when $|k_z v_{i0}|$ is close to $\omega_{\mathbf{k}}$.

In dimensionless units normalized by the inverse of the Debye length, we can write Equation (64) as

$$\Phi_W(\mathbf{r}) = \frac{Q}{\lambda_D M^2} \int \frac{dk_z d\mathbf{k}_\perp}{2\pi^2} \frac{k^2}{1+k^2} \frac{\exp(i\mathbf{k}\cdot\mathbf{r}/\lambda_D)}{[(k_z - i0)^2 + k_0^2][(k_z - i0)^2 - k_1^2]}, \quad (65)$$

where $k^2 = k_z^2 + k_\perp^2$ and $k_{0,1}^2 = \pm(1 - M^{-2} + k_\perp^2)/2 + [k_\perp^2 M^{-2} + (1 - M^{-2} + k_\perp^2)^2/4]^{1/2}$. Note that the contribution from the poles at $k_z = \pm ik_0$ provides the non-oscillating part which modifies the static Debye shielding scale (62) in plasmas with finite ion flows.

After integration over k_z in (65) we find

$$\Phi_W(\mathbf{r}) = -\frac{Q}{\lambda_D M^2} \int \frac{d\mathbf{k}_\perp}{2\pi|k_1|} \frac{k_\perp^2 + k_1^2}{k_0^2 + k_1^2} \frac{\exp(ik_\perp\rho\cos\varphi/\lambda_D)}{1 + k_\perp^2 + k_1^2} \sin(|k_1 z|/\lambda_D). \quad (66)$$

Taking into account the cylindrical symmetry of the wake cone, we obtain

$$\Phi_W(\rho, z) = -\frac{Q}{\lambda_D M^2} \int_0^{+\infty} \frac{k_\perp dk_\perp}{|k_1|} \frac{k_\perp^2 + k_1^2}{k_0^2 + k_1^2} \frac{J_0(k_\perp\rho/\lambda_D)}{1 + k_\perp^2 + k_1^2} \sin(|k_1 z|/\lambda_D), \quad (67)$$

where J_0 is the Bessel function of zero order.

It is difficult to find analytically the mathematically exact expression for the potential (67). Thus, below we present an estimation to evaluate the asymptotic behavior of the function. The main contribution to the integral (67) comes from $k_\perp \sim 1$. Then for the distance small in the perpendicular direction, $k_\perp\rho \ll 1$, and for $|z| > \lambda_D\sqrt{M^2 - 1}$, the main contribution to the stationary wake potential is given by

$$\Phi_W(\rho = 0, z) \approx \frac{q_t}{|z|} \frac{2\cos(|z|/L_s)}{1 - M^{-2}}\,, \quad (68)$$

where $L_s = \lambda_D\sqrt{M^2 - 1}$ is the effective length.

From Equation (68), we can conclude that the wake potential is attractive for $\cos(|z|/L_s) < 0$. On the other hand, for the distance $\rho > \lambda_D$ and $|z| > \lambda_D\sqrt{M^2 - 1}$ we find the following approximate expression for the wake potential:

$$\Phi_W(\rho, z) \simeq \frac{Q}{1 - M^{-2}} \sqrt{\frac{\lambda_D}{2\pi\rho}} \times \left\{ \frac{\cos[(\pi/4) + (z_-/\lambda_D\sqrt{M^2 - 1})]}{z_-} + \frac{\cos[(\pi/4) - (z_+/\lambda_D\sqrt{M^2 - 1})]}{z_+} \right\}, \quad (69)$$

where $z_\pm \equiv |z| \pm \rho\sqrt{M^2 - 1} > 0$ (we remind that the oscillating potential exists only in the wake of the test particle, i.e., for $z < 0$ and $|z| > \rho\sqrt{M^2 - 1}$). Obviously, this function oscillates as we change ρ or z. Because oscillating potential

(69) is proportional to the same dust particle charge Q as the static Debye potential (62), and contains no screening exponential, there are regions in space which correspond to the change of the effective potential sign and, hence, to the most probable positions of the particulates. We stress that these regions are not only on the line $\rho = 0$. The effective periodic spacing in the plane perpendicular to the flow is of order Debye length; note that this can be seen not only from approximate expression (69), but also from more exact formula (67).

Thus, we have demonstrated that collective effects can provide the oscillating potential on the line as well as in the plane perpendicular to the direction of the ion flow downstream the dust particle. Since in the experiments the most probable effective spacing is of order Debye length (the estimation is $\sim 0.2 \div 1.7\lambda_D$), we see that the polygon structure can appear in plasma crystal formation. Indeed, because the wake potential cannot change the sign of the effective potential at the distances less than the λ_D, the dust particulates are not expected to be arranged with the distances less than the Debye length. At the same time, the characteristic spacing of the polar radius-vector in the plane perpendicular to the flow is also of order λ_D. Therefore, we can expect the particulates on the equal distances λ_D on the periphery of the circle of the radius of order λ_D. This may correspond to polygon of order not higher than hexagon. Note that hexagonal structures were observed practically in all the experiments on dust crystallization (Chu and I, 1994; Thomas et al., 1994; Hayashi and Tachibana, 1994; Melzer et al., 1994, 1996a; Chiang and I, 1996).

The proposed mechanism can provide a qualitative explanation of the arrangement of particles downstream the test dust particulate. For quantitative picture, it is necessary to calculate the potential of ensemble of dust particulates as well as contribution of other factors such us gravitation, plasma inhomogeneity, etc. Moreover, the regular arrangement of the particles in the first (with regard to the ion flow) layer of the plasma crystal needs more investigations; we mention only that if the regular quasicrystal structure is established in downstream layers, this may also affect the arrangement of particles in the first layer.

7. Vibrational Lattice Modes in the Dust Crystals

The considered above dust-crystal structures are observed in the sheath region where there is balance between the gravitational and electrostatic forces, and in many experiments it consists of just a few layers of dust particles levitating above the horizontal negatively biased electrode (Chu and I, 1994; Thomas et al., 1994; Hayashi and Tachibana, 1994; Melzer et al., 1994, 1996a, 1996b; Chiang and I, 1996; Peters et al., 1996).

Recently, it was demonstrated (Melandsø, 1996; Vladimirov et al., 1997) that lattice waves can propagate in the one-dimensional chain of strongly coupled dust particles. The waves include motion of dust particles in horizontal direction (which is the simplest situation) (Melandsø, 1996) as well as in the vertical direction (when the grain oscillations depend also on the shape of the external potential of the electrode) (Vladimirov et al., 1997). Note that motion of the dust grains in the vertical direction can provide a useful tool for determining the grain charge (Melzer et al., 1996b; Peters et al., 1996).

We demonstrate here that oscillations of the dust grains in the vertical plane can lead to a novel low-frequency mode. We note that excitation of this mode can also be responsible for phase transitions in the system which are the subject of particular recent interest, see, e.g., (Thomas and Morfill, 1996; Quinn et al., 1996). The mode is characterized by an optic-mode-like inverse dispersion (i.e., its frequency decreases with the growing wave number) if $kr_0 \ll 1$ where k is the wave number, r_0 is the interparticle distance, and only nearest-neighbour interactions are taken into account.

Consider vibrations of a one-dimensional horizontal chain of particulates of equal mass M separated by the distance r_0. We assume that the interaction potential between particles can be approximated by the Debye law

$$\Phi = \frac{Q}{r} \exp\left(-\frac{r}{\lambda_D}\right). \tag{70}$$

In addition to the electrostatic Debye shielded force, the gravitational force Mg and the sheath electrostatic force $F_e = QE(z)$ act on the dust grains in the vertical direction z. The balance of forces in the linear approximation with respect to small perturbations δz of the equilibrium at $z = 0$ gives the equation for vertical oscillations

$$M\frac{d^2\delta z_n}{dt^2} = \frac{Q^2}{r_0^3}\, \mathrm{e}^{-r_0/\lambda_D}(1 + r_0/\lambda_D)(2\delta z_n - \delta z_{n-1} - \delta z_{n+1}) - Mg + F_e. \tag{71}$$

Here

$$F_e - Mg = -\gamma\delta z_n, \tag{72}$$

where γ is a constant assuming linear variation of the sheath electric field, and δz_n is the vertical deviation of a particle number n from its equilibrium position. We note that although in general particles oscillate in the vertical as well as in horizontal direction, in the linear approximation their transverse vibrations (which are the subject of this study) and longitudinal vibrations are not coupled. Substituting $\delta z_n = A\exp(-i\omega t + iknr_o)$ into Equation (71) gives the dispersion of the vertical oscillations

$$\omega^2 = \frac{\gamma}{M} - \frac{4Q^2}{Mr_0^3}\, \mathrm{e}^{-r_0/\lambda_D}(1 + r_0/\lambda_D)\sin^2\frac{kr_0}{2}. \tag{73}$$

We see that for $k = 0$ the characteristic frequency is given by $\omega^2 = \gamma/M$, decreasing with growing wave number when $kr_0 \ll 1$.

To estimate the effective width of the potential well γ, we consider the standard model of the sheath (Vladimirov et al., 1997), which considers Boltzmann distributed electrons. For simplicity, we ignore the influence the dust grains may cause on the field distribution in the sheath region. The ion continuity equation gives the ion density n_i in terms of the density n_0 in the plasma bulk

$$n_i(z) = n_0 \left[1 - \frac{2e\phi(z)}{m_i v_0^2}\right]^{-1/2}, \tag{74}$$

where v_0 is the speed of the ion flow towards the negatively charged electrode, m_i is the ion mass, and $\phi(z)$ is the sheath potential. From Poisson's equation, we then obtain

$$\frac{d^2\phi(z)}{dz^2} = 4\pi e n_0 \left[\exp\left(\frac{e\phi(z)}{T_e}\right) - \left(1 - \frac{2e\phi(z)}{m_i v_0^2}\right)^{-1/2}\right]. \tag{75}$$

This equation can be integrated once to give [applying the boundary conditions $E(\infty) = \phi(\infty) = 0$]

$$E^2(z) = 8\pi n_0 T_e \left\{\exp\left(\frac{e\phi(z)}{T_e}\right) - 1 + \frac{v_0^2}{v_s^2}\left[\left(1 - \frac{2e\phi(z)}{T_e}\frac{v_s^2}{v_0^2}\right)^{1/2} - 1\right]\right\}, \tag{76}$$

where $v_s^2 = T_e/m_i$ is the squared sound speed.

Linearizing Equation (76) with respect to small potential and field variations, we find

$$\delta E \approx \frac{4\pi n_0 T_e}{E_0}\left[\exp\left(\frac{e\phi_0}{T_e}\right) - \left(1 - \frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]\frac{e\delta\phi}{T_e}, \tag{77}$$

where the electric field E_0 is at $z = 0$, and for the dust grains it is balanced by the gravity

$$|QE_0| = Mg. \tag{78}$$

Now, we assume that the sheath electric field near the position of the dust grains can be linearly approximated. Thus we find

$$\delta E \approx -4\pi n_0 e\left[\exp\left(\frac{e\phi_0}{T_e}\right) - \left(1 - \frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]\delta z, \tag{79}$$

and the effective width of the potential well is given by

$$\gamma = 4\pi e|Q|n_0\left[\exp\left(\frac{e\phi_0}{T_e}\right) - \left(1 - \frac{2e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2}\right]. \tag{80}$$

Equations (76) and (78) for the sheath potential ϕ_0 in the equilibrium position can be solved numerically. When $e\phi_0 \ll T_e$, the characteristic frequency is approximately given by

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\gamma}{M}} \approx \frac{1}{2\pi}\sqrt{\frac{g(1 - v_s^2/v_0^2)}{\lambda_D}} \simeq 20\ \mathrm{Hz}, \tag{81}$$

where we assumed $\lambda_D \approx (T_e/4\pi n_0 e^2)^{1/2} \sim 2 \times 10^{-2}$ cm and $v_0^2/v_s^2 \sim 1.5$.

Thus, the vertical oscillations of a one-dimensional chain of dust grains levitating in the sheath field of a horizontal negatively biased electrode can give rise to a specific low-frequency mode which is characterized by inverse optic-mode-like dispersion when the wave lengths far exceed the intergrain distance. Excitation of the mode may stimulate phase transitions in the system. We note that considering the one dimensional chain we ignored the ion drag force which is not important in this situation. However, for several layers of dust grains in the vertical direction, the ion drag force which in a plasma with finite flows necessarily includes such collective effects as generation of the wake fields studied in the preceding section must be taken into account.

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