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Cooperative behavior of colloidal particles in a complex plasma

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Abstract

Plasma wave propagation and particle wave motions in complex "dusty" plasmas are considered. The presence of dust affects plasma waves, in particular, by introducing a specific damping associated with the dust charging; on the other hand, plasma collective processes influence the arrangements and vibrations of colloidal "dust" particles in the dust clouds including Coulomb lattice structures.

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1. Introduction

Cooperative phenomena in dusty plasmas attract considerable interest related with numerous applications in low temperature and strongly coupled dusty plasmas, as well as space dusty plasmas. For typical laboratory conditions, dust grains are usually of micron and submicron size, negatively charged due to plasma currents on the grain surface (typically, with excess electrons collected on each particulate due to higher mobility of plasma electrons), and have a mass that is much larger than the heaviest ion mass. Since the dust particles are almost immediately electrically charged being immersed in the plasma, they must be coupled to each other as well as the plasma via the electric and magnetic fields [1]. The presence of chaotically moving charged dust grains has been shown to lead to considerable modification of plasma collective properties strongly affecting dispersion relations and damping of plasma waves, as well as leading to specific oscillation modes associated with dust motion [2–7]. Interaction

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of dust particles with the plasma and with themselves is also naturally affected by the plasma environment and therefore by the plasma collective effects [8–16].

Here, the most significant advances in the theory of collective effects in dusty plasmas are reviewed. The important point is that dust grains embedded in a plasma change its collective properties considerably and although dust can sometimes be considered as an additional ideal plasma component (when one can apply the known results for multi-component plasmas), in general, this assumption does not work, e.g., when the number of grains in the dust Debye sphere is less than one [1]. Moreover, the charges on dust particles are not fixed in processes of their interactions with plasma fields and other dust grains [2,3]. Indeed, the floating charges are mostly determined by plasma electron and ion currents on the dust grains. The currents are naturally affected by the interactions which lead to a change of dust charges in the interaction process; this leads to qualitatively new effects [2,3,8,9]. Below, we discuss some of these effects in more detail.

Another important feature of dusty plasmas is that the presence of grains with even constant charges can strongly modify the existing wave spectra; as an example, we consider here modification of linear [5] Alfvén waves (note that similar effects were also considered for nonlinear [6] as well as surface [7] Alfvén waves in a magnetized dusty plasma). Finally, lattice wave of dust particles in the crystal-like Coulomb structures can propagate [10,11]. Here, we note that the wake potential formation [12–14] which leads to the vertical alignment of dust grains in the experiments, and, as a consequence, to quasi-two-dimensional features of the structures (i.e., hexagonal arrangements) when the number of dust layers is not high (the three-dimensional lattices corresponding to minimum of the potential energy are body-centered-cubic or face-centered-cubic) also affects the specific lattice modes propagating in such systems when the number of layers in the crystal is more than one: the asymmetry of the interaction potential changes the dispersion characteristics of the lattice modes [15] and can sometimes lead to specific instabilities in the chain of dust particles [16].

2. The charging dissipative process

Consider an unmagnetized plasma whose constituents are electrons, ions, and massive static ($m_d = \infty$) negatively charged dust grains. The latter are assumed to be point charges (i.e., their sizes are supposed to be much smaller than the effective Debye radius λ_D of the dusty plasma). The basic charging equation is given by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = I(q)\,, \tag{1}$$

where Q is the charge residing on a dust grain, and the current I on the dust particle is a sum of electron and ion plasma currents. In the state of equilibrium, we have

$$I^{\mathrm{eq}}(Q^{\mathrm{eq}}) \equiv I_e^{\mathrm{eq}}(Q^{\mathrm{eq}}) + I_i^{\mathrm{eq}}(Q^{\mathrm{eq}}) = 0\,. \tag{2}$$

For Maxwellian electrons and ions, the equilibrium charge $Q^{\mathrm{eq}} = Z_d e$ can be found as a solution of the equation following from Eq. (2):

$$\frac{\omega_{pe}^2}{v_{Te}} \exp\left(-\frac{Z_d e^2}{aT_e}\right) = \frac{\omega_{pi}^2}{v_{Ti}} \left(\frac{T_i}{T_e} + \frac{Z_d e^2}{aT_e}\right) , \tag{3}$$

where a is the grain radius, $\omega_{pe(i)} = (4\pi n_{e(i)} e^2/m_{e(i)})^{1/2}$ is the electron (ion) plasma frequency, e is the electron charge (ions are assumed to be single charged, i.e., the ion charge is $-e$), and $v_{Te(i)} = (T_e/m_{e(i)})^{1/2}$ is the electron (ion) thermal velocity. Usually, the dimensionless dust charge Z_d is of order 10^3–10^4, but the total negative charge on the dust particles is close to (and does not exceed as a rule) the total charge of electrons. The following dimensionless parameter is typically of order unity:

$$P \equiv \frac{n_d}{n_e} \frac{aT_e}{e^2} . \tag{4}$$

The charging dissipative process is characterized by the charging frequency ν_d

$$\nu_d \equiv - \left.\frac{\partial I(Q)}{\partial Q}\right|_{Q=Z_d e} = \frac{1}{\sqrt{2\pi}} \frac{\omega_{pi}^2 a}{v_{Ti}} \left(1 + \frac{T_i}{T_e} + \frac{Z_d e^2}{aT_e}\right) . \tag{5}$$

The charging frequency describes the process of charging of the dust particles for small deviations from the equilibrium; it naturally contains contributions from both electrons and ions. Comparison of the rate (5) with the plasma collision frequencies [2,3] demonstrates that this process can be the most important dissipative process for dusty plasmas. Another important characteristic is the frequency characterizing the rate of capture of plasma electrons by dust particles ν_{ed}^{eq}:

$$\nu_{ed}^{\mathrm{eq}} = \nu_d \frac{n_d}{n_e} \frac{aT_e}{e^2} \left(\frac{T_i}{T_e} + \frac{Z_d e^2}{aT_e}\right) \left(1 + \frac{T_i}{T_e} + \frac{Z_d e^2}{aT_e}\right)^{-1} . \tag{6}$$

This characteristic frequency enters equations when the ion contribution can be neglected [3].

3. Plasma waves in the presence of the dust charging

To find the dielectric properties of a dusty plasma, a kinetic theory is developed [2,3]. In the case of Maxwellian plasma particle distributions and sufficiently high frequency of the propagating wave [$\omega \sim \omega_{pe} \gg \max(kv_{Te}, \nu_d, \nu_{ed}^{\mathrm{eq}})$], the following approximate expression [3] for the plasma dielectric permittivity taking into account the dust charging effects can be obtained:

$$\varepsilon_{\mathbf{k}\omega}^{(l)} \simeq 1 - \frac{\omega_{pe}^2}{\omega^2} \left[1 - i\,\frac{2}{3}(2+z)\frac{\nu_{ed}^{\mathrm{eq}}}{\omega} - \sqrt{\pi z} A \mathrm{e}^z \left(\frac{\nu_{ed}^{\mathrm{eq}}}{\omega}\right)^2\right] , \tag{7}$$

where $A \equiv (\frac{5}{4}) - (z/6) + [(\frac{5}{4}) - z + (z^2/3)] \int_1^\infty \mathrm{d}x \exp[-(x^2-1)z]$ and $z = Z_d e^2/aT_e$. Therefore, the charging process leads to appearing of additional imaginary and real parts in the high-frequency longitudinal dielectric permittivity. These additional terms depend on relation between the charging frequency and electron plasma frequency. For

Langmuir waves, the damping due to the charging process is directly proportional to the effective rate of capture of plasma electrons

$$\gamma_d^L \simeq -\tfrac{1}{3}(2+z)\nu_{ed}^{\mathrm{eq}} . \tag{8}$$

For the transverse dielectric function, the corresponding expression coincides with expression (7) for the high-frequency longitudinal dielectric permittivity. Thus, for the electromagnetic waves the damping due to the charging effects on dust particles is given by

$$\gamma_d^{EM} \simeq -\frac{1}{3}\frac{\omega_{pe}^2}{\omega^2}(2+z)\nu_{ed}^{\mathrm{eq}} . \tag{9}$$

If the frequency of electromagnetic waves does not far exceed ω_{pe}, when the phase velocity is significantly influenced by the conduction current, the damping rate (9) is of the order of the electron capture rate ν_{ed}^{eq}. Note the real corrections to the electron plasma frequency which are also effective for electromagnetic waves.

4. Lattice waves in dust crystals

In the experiments, the dust-crystal Coulomb lattice structures are usually observed in the sheath region of a radio-frequency discharge plasma where there is balance between the gravitational and electrostatic forces [17]; the distance between the dust grains is typically of the order of the plasma Debye length λ_D. Recently, it was demonstrated [10,11,15] that lattice waves can propagate in the one-dimensional chain of strongly coupled dust particles. The waves include motion of dust particles in horizontal direction (which is the simplest situation) [10] as well as in the vertical direction (when the grain oscillations depend also on the shape of the external potential of the electrode) [11,15]. Note that motion of the dust grains in the vertical direction can provide a useful tool for determining the grain charge as well as other dust and plasma sheath characteristics, see, e.g., in Ref. [18].

We demonstrate here that oscillations of the dust grains in the vertical plane can lead to a novel low-frequency mode. We note that excitation of this mode can also be responsible for phase transitions in the system. The mode is characterized by an optic-mode-like inverse dispersion (i.e., its frequency decreases with the growing wave number) if $kr_0 \ll 1$ where k is the wave number, r_0 is the interparticle distance, and only nearest-neighbor interactions are taken into account.

Consider vibrations of a one-dimensional horizontal chain of particulates of equal mass M separated by the distance r_0. We assume that the interaction potential between particles can be approximated by the Debye law:

$$\Phi = \frac{Q}{r}\exp\left(-\frac{r}{\lambda_D}\right) . \tag{10}$$

In addition to the electrostastic Debye shielded force, the gravitational force Mg and the sheath electrostatic force $F_e = QE(z)$ act on the dust grains in the vertical direction z. The balance of forces in the linear approximation with respect to small perturbations

δz of the equilibrium at $z=0$ gives the equation for vertical oscillations:

$$M\,\frac{\mathrm{d}^2\delta z_n}{\mathrm{d}t^2}=\frac{Q^2}{r_0^3}\,\mathrm{e}^{-r_0/\lambda_D}(1+r_0/\lambda_D)(2\delta z_n-\delta z_{n-1}-\delta z_{n+1})-Mg+F_e\,. \tag{11}$$

Here,

$$F_e-Mg=-\gamma\delta z_n\,, \tag{12}$$

where γ is a constant assuming linear variation of the sheath electric field, and δz_n is the vertical deviation of a particle number n from its equilibrium position. We note that although in general particles oscillate in the vertical as well as in horizontal direction, in the linear approximation their transverse vibrations (which are the subject of this study) and longitudinal vibrations are not coupled. Substituting $\delta z_n=A\exp(-\mathrm{i}\omega t+\mathrm{i}knr_0)$ into (11) gives the dispersion of the vertical oscillations:

$$\omega^2=\frac{\gamma}{M}-\frac{4Q^2}{Mr_0^3}\,\mathrm{e}^{-r_0/\lambda_D}(1+r_0/\lambda_D)\sin^2\frac{kr_0}{2}\,. \tag{13}$$

We see that for $k=0$ the characteristic frequency is given by $\omega^2=\gamma/M$, decreasing with growing wave number when $kr_0\ll 1$.

Equations for the sheath potential ϕ_0 in the equilibrium position can be solved numerically. When $e\phi_0\ll T_e$, the characteristic frequency is approximately given by

$$f_0=\frac{1}{2\pi}\sqrt{\frac{\gamma}{M}}\approx\frac{1}{2\pi}\sqrt{\frac{g(1-v_s^2/v_0^2)}{\lambda_D}}\simeq 20\ \mathrm{Hz}\,, \tag{14}$$

where v_0 is the speed of the ion flow, $v_s=(T_e/m_i)^{1/2}$ is the ion-sound velocity, and we assumed $\lambda_D\approx(T_e/4\pi n_0e^2)^{1/2}\sim 2\times 10^{-2}$ cm and $v_0^2/v_s^2\sim 1.5$.

Thus, the vertical oscillations of a one-dimensional chain of dust grains levitating in the sheath field of a horizontal negatively biased electrode can give rise to a specific low-frequency mode which is characterized by inverse optic-mode-like dispersion when the wave lengths far exceed the intergrain distance. Excitation of the mode may stimulate phase transitions in the system. We note that considering the one-dimensional chain we ignored the ion drag force which is not important in this situation. It is well known, however, that in the sheath region strong ion flow is established; according to the Bohm criterion the speed of the flow exceeds the ion acoustic velocity [12,13]. The ion drag force on a test particle in a plasma with finite flows necessarily includes collective effects. It was shown that if the speed of the flow exceeds the velocity of ion oscillations in the flow, an oscillating stationary wake is formed behind the static test particle [12–14]. The effect is similar to the Cooper pairing of electrons in superconductors. We note that the collective mechanism can be responsible for the oscillatory potential in the direction parallel [12,14] as well as perpendicular [13,14] to the flow, and the attraction due to the wake potential can overcome the static Coulomb repulsion. The characteristic spacing in this case is of order Debye length λ_D which is in agreement with the experimental data. Thus, for several layers of dust grains in the vertical direction, the ion drag force which in a plasma with finite flows necessarily includes such collective effects as generation of the wake fields studied in the preceding section must be taken into account. The relevant consideration [15] shows that the second mode of vibrations, appearing (in addition to the first mode described above) in the case of two vertically arranged horizontal chains, is strongly affected by the asymmetry of the vertical potential associated with the plasma wake formation; moreover, the plasma wake can induce some instabilities [16] in the lattice wave propagation even in the one-dimensional chain.

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References

[1] E.C. Whipple, T.G. Northrop, D.A. Mendis, J. Geophys. Res. 90 (1985) 7405.
[2] V.N. Tsytovich, O. Havnes, Comments, Plasma Phys. Control Fusion 15 (1993) 267.
[3] S.V. Vladimirov, Phys. Plasmas 1 (1994) 2762.
[4] F. Melandsø, T.K. Aslaksen, O. Havnes, Planet. Space Sci. 41 (1993) 321.
[5] N.F. Cramer, S.V. Vladimirov, Phys. Scripta 53 (1996) 586.
[6] S.V. Vladimirov, N.F. Cramer, Phys. Rev. E 54 (1996) 6762.
[7] N.F. Cramer, S.V. Vladimirov, Phys. Plasmas 3 (1996) 4740.
[8] S.V. Vladimirov, Phys. Rev. E 50 (1994) 1422.
[9] S.V. Vladimirov, Phys. Rev. E 49 (1994) R997.
[10] F. Melandso, Phys. Plasmas 3 (1996) 3890.
[11] S.V. Vladimirov, P.V. Shevchenko, N.F. Cramer, Phys. Rev. E 56 (1997) R74.
[12] S.V. Vladimirov, M. Nambu, Phys. Rev. E 52 (1995) R2172.
[13] S.V. Vladimirov, O. Ishihara, Phys. Plasmas 3 (1996) 444.
[14] O. Ishihara, S.V. Vladimirov, Phys. Plasmas 4 (1997) 69.
[15] S.V. Vladimirov, P.V. Shevchenko, N.F. Cramer, Phys. Plasmas 5 (1998) 4.
[16] A.V. Ivlev, G. Morfill, Phys. Rev. E 63 (2001) 016409.
[17] H.M. Thomas, G.E. Morfill, Nature (London) 379 (1996) 806.
[18] S.V. Vladimirov, N.F. Cramer, Phys. Rev. E 62 (2000) 2754.