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Collective behaviour and particle motions in a dusty plasma

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Abstract. Recent advances in the study of collective effects and particle motions in plasma-dust crystals are presented. Plasma collective processes influence arrangements and vibrations of dust particles in these Coulomb structures. The important effect is the wake potential formation leading to the vertical alignment of dust grains in the experiments. Oscillations of dust particles in the lattices are strongly affected by collective processes in the ambient plasma, in particular, by the wake. Modes associated with vertical vibrations of dust grains, which can provide a useful tool for diagnostics in the sheath region, are identified, and their dispersion characteristics are discussed.

Physics of dusty plasmas, whose constituents are electrons, ions and extremely heavy highly charged dust particles, has recently attracted considerable interest connected, in particular, with an increasing worldwide effort to model direct current, radio frequency, and microwave plasma discharge devices used in plasma-assisted materials processing. Dusty plasmas are also common in a variety of low-temperature plasmas in space environments, such as the lower ionosphere of the Earth, planetary atmospheres, asteroid zones, nebulae, and cometary tails. Another example of recent interest is low temperature edge plasma physics in nuclear fusion devices, where dust grains emitted from walls may strongly influence anomalous transport properties.

An exciting area of the most recent research is plasma-dust crystal formation [1]. The macroscopic lattices, named 'dust-plasma crystals', are made of highly (negatively) charged particulates of micrometer size levitated in the sheath region above a horizontal negatively biased electrode. Although the dust-plasma crystal systems are generally three-dimensional, in most experiments the charged particles are located just a few layers above the horizontal electrode, where gravity is balanced by the sheath electric field. These layers act mainly as two-dimensional systems, with limited out-of-plane particle motion [1]. On the other hand, it was demonstrated that aligning of dust grains from different layers in the vertical plane is strongly connected with plasma collective mechanisms, namely, with the presence of supersonic ions flowing towards the negative electrode. It was shown in [2] that in otherwise uniform plasma, the flow leaves a polarized oscillating wake potential behind a stationary dust particle, with ions focussing to make the plasma potential positive in a local region. This positive region attracts negative particles, promoting vertical alignment clearly observed experimentally [1].

Here, recent advances in the study of collective effects and particle motions in dusty plasmas are presented. In particular, the influence of plasma collective processes on arrangements and vibrations of dust particles in crystal-like coulomb structures is considered. Note that the wake potential formation which leads to the vertical alignment of dust grains in the experiments, and, as a consequence, to quasi-two-dimensional features of the structures (i.e. hexagonal arrangements) when the number of dust layers is not high (the three-dimensional

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lattices corresponding to the minimum of the potential energy are body-centred-cubic or face-centred-cubic) can also affect the specific lattice modes propagating in such systems.

The drag force on a test particle in a plasma with finite flows necessarily includes collective effects. If the speed of the flow exceeds the velocity of ion oscillations in the flow an oscillating stationary wake is formed behind the static test particle [2]; the effect is similar to Cooper pairing of electrons in superconductors. We note that the collective mechanism can be responsible for the oscillatory potential in the direction parallel as well as perpendicular to the flow, and the attraction due to the wake potential can overcome the static coulomb repulsion. The characteristic spacing in this case is of the order of the Debye length $\lambda_{\rm D}$ in agreement with the experiments.

Consider the cylindrical geometry (ρ, φ, z); the plasma ions flow in the $-z$ direction with velocity $v_{\rm i0}$; the test dust particle of the charge Q is placed on the position $(0, 0)$. We calculate the potential behind the test particle downstream the flow within the wake cone: $|z| > \rho(M^2 - 1)^{1/2}$, where $M = v_{\rm i0}/v_s$ is the Mach number and $v_{\rm s} = (T_{\rm e} n_{\rm i}/m_{\rm i} n_{\rm e})^{1/2}$ is the sound velocity ($T_{\rm e}$ and $n_{\rm e}$ are the electron temperature and number density, and $m_{\rm i}$ and $n_{\rm i}$ are the ion mass and number density). The electrostatic potential of the static dust particle outside the Mach cone is

$$\Phi(\boldsymbol{r}) = \Phi_D(\boldsymbol{r}) = \frac{Q}{|\boldsymbol{r}|}\exp(-|\boldsymbol{r}|/\lambda_{\rm D}) \qquad (1)$$

where $\lambda_{\rm D} = (T_{\rm e}/4\pi n_{\rm e} e^2)^{1/2}$, while inside the Mach cone the potential involves the collective effects caused by the oscillations in the ion flow

$$\Phi(\boldsymbol{r}) = \Phi_{\rm W}(\boldsymbol{r}) = Q\int \frac{{\rm d}\boldsymbol{k}}{2\pi^2 k^2}\frac{k^2\lambda_{\rm D}^2\omega_{\boldsymbol{k}}^2\exp({\rm i}\boldsymbol{k}\cdot\boldsymbol{r})}{(1+k^2\lambda_{\rm D}^2)((-k_z v_{\rm i0}+{\rm i}0)^2-\omega_{\boldsymbol{k}}^2)}. \qquad (2)$$

Here, $\omega_{\boldsymbol{k}} = k v_{\rm s}/(1+k^2\lambda_{\rm D}^2)^{1/2}$ is the characteristic frequency of the oscillations in the flow. The potential (2) describes the strong resonant interaction between the oscillations in the ion flow and the test particulate when $|k_z v_{\rm i0}|$ is close to $\omega_{\boldsymbol{k}}$.

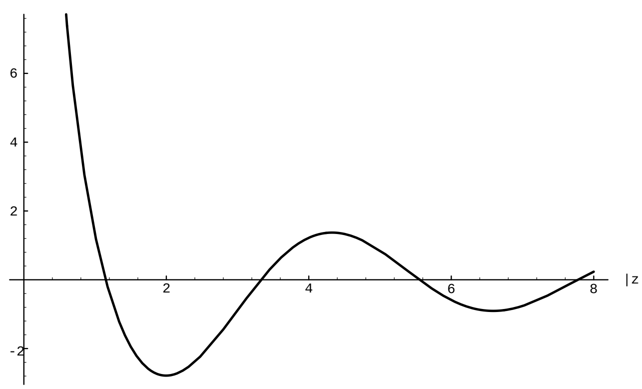

Figure 1. The wake potential on the line behind a test dust particle.

For distances $k_\perp\rho \ll 1$ and $|z| > \lambda_{\rm D}(M^2-1)^{1/2}$, the main contribution to the stationary wake potential is given by

$$\Phi_{\rm W}(\rho = 0, z) \approx \frac{q_t}{|z|}\frac{2\cos(|z|/L_s)}{1-M^{-2}} \qquad (3)$$

where $L_s = \lambda_{\rm D}(M^2-1)^{1/2}$ is the effective length. This potential is presented on figure 1. From equation (3), we can conclude that the wake potential is attractive for $\cos(|z|/L_s) < 0$.

On the other hand, for distances $\rho > \lambda_D$ and $|z| > \lambda_D(M^2-1)^{1/2}$ we find

$$\Phi_W(\rho, z) \simeq \frac{Q}{1-M^{-2}}\sqrt{\frac{\lambda_D}{2\pi\rho}}\left\{\frac{\cos[(\pi/4)+(z_-/\lambda_D\sqrt{M^2-1})]}{z_-} + \frac{\cos[(\pi/4)-(z_+/\lambda_D\sqrt{M^2-1})]}{z_+}\right\} \quad (4)$$

where $z_\pm \equiv |z| \pm \rho(M^2-1)^{1/2} > 0$. Because the oscillating potentials are proportional to the same dust particle charge Q as the static Debye potential, and contain no screening exponential, there are regions in space which correspond to the change of the effective potential sign and, hence, to the most probable positions of the particulates. We stress that these regions are not only on the line $\rho = 0$. The effective periodic spacing in the plane perpendicular to the flow is of the order of the plasma Debye length. Behaviour of the potential in the Mach cone is presented on figure 2 (see also Ishihara and Vladimirov [2]).

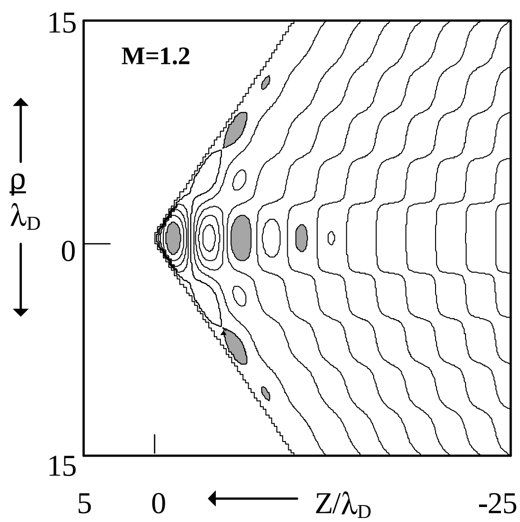

Figure 2. The wake potential in the Mach cone behind a test dust particle.

Thus, plasma collective effects can provide the oscillating potential on the line as well as in the plane perpendicular to the direction of the ion flow downstream of the dust particle. Indeed, because the wake potential cannot change the sign of the effective potential at the distances less than the λ_D, the dust particulates are not expected to be arranged with the distances less than the Debye length. At the same time, the characteristic spacing of the polar radius-vector in the plane perpendicular to the flow is also of order λ_D. Therefore, we can expect the particulates on the equal distances λ_D on the periphery of the circle of the radius of order λ_D. This may correspond to a polygon of order not higher than hexagon. Hexagonal structures were observed practically in all the experiments on dust crystallization [1].

The lattice waves in the dust crystal include motion of dust particles in horizontal direction as well as in the vertical direction. Consider vibrations of the two one-dimensional horizontal chains of particulates of equal mass M separated by the distance r_0 in the horizontal direction and d in the vertical direction [3], see figure 3.

The potential in the vertical direction acting on the lower particle due to the upper one is given by (3). The potential acting on the upper particle due to the lower particle is the simple Debye repulsive potential (1). The balance of forces in the vertical direction, in addition to the

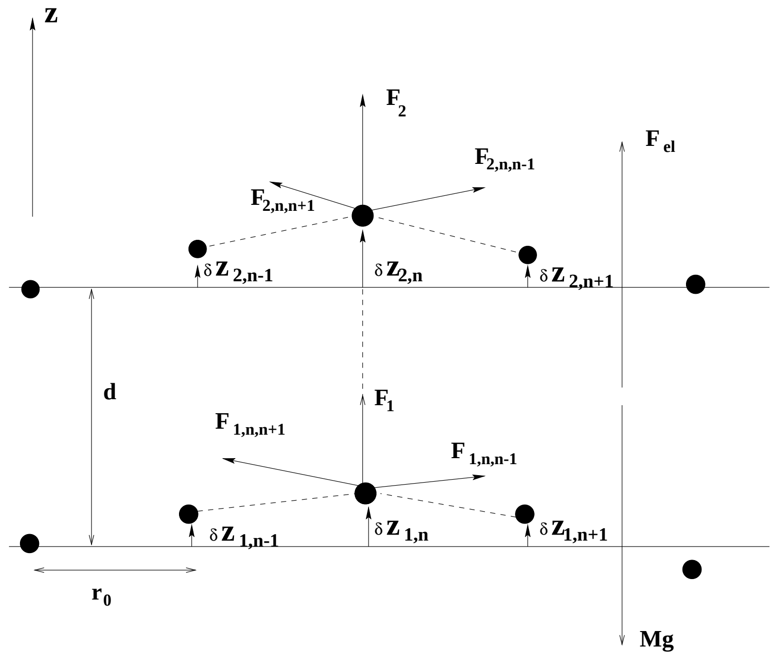

Figure 3. Forces acting on dust particles in two one-dimensional horizontal chains: the gravitational Mg and the sheath electric $F_{\rm el}$ external fields, as well as the wake F_1 and Debye F_2 interaction fields.

electrostatic Debye and the wake potential forces, includes the gravitational force $F = Mg$ as well as the sheath electrostatic force $F_{\rm el} = QE(z)$ acting on the dust grains.

In equilibrium, since the interchain distance d is assumed to be small compared with the distance z_{01} of the lower chain from the electrode (as well as small compared with the width of the sheath), we can assume that the sheath electric field in the range of distances z_{01} to $z_{02} = z_{01} + d$ can be linearly approximated as $F_{\rm el} - Mg = -\gamma(z - z_0)$, where γ is a constant and z_0 is the equilibrium position of a particle of mass M due to the forces Mg and $F_{\rm el}$ only. We stress that the actual equilibrium positions of particles in lower and upper chains are z_{01} and z_{02}, respectively. The equilibrium balance of the forces in the vertical direction acting on the lower chain and the upper chain can be written as

$$F_{\rm el,1}(z_{01}) - Mg + F_1^0(z_{02} - z_{01}) = 0 \qquad F_{\rm el,2}(z_{02}) - Mg + F_2^0(z_{02} - z_{01}) = 0 \tag{5}$$

where $F_{1,2}^0$ are the forces of interaction between the chains due to the potentials (3) and (1). Here, since we have $z_{02} = z_{01} + d$

$$F_1^0(z_{02} - z_{01}) = Q\frac{{\rm d}\Phi_1(|z|)}{{\rm d}|z|}\bigg|_{|z|=d} \qquad F_2^0(z_{02} - z_{01}) = -Q\frac{{\rm d}\Phi_2(|z|)}{{\rm d}|z|}\bigg|_{|z|=d}. \tag{6}$$

The equilibrium distance d can be found from equations (5): $F_2^0(d) - F_1^0(d) = \gamma d$.

By introducing small perturbations $\delta z_{i,n}$ of the equilibrium at z_{0i}, where $i = 1, 2$ for the lower and upper chains, respectively, including interactions with nearest neighbours in each chain, and substituting $\delta z_{i,n} = A_{\rm i} \exp(-{\rm i}\omega t + {\rm i}knr_0)$ we obtain

$$\omega_1^2 = \frac{\gamma}{M} - \frac{4Q^2}{Mr_0^3}\,{\rm e}^{-r_0/\lambda_{\rm D}}(1 + r_0/\lambda_{\rm D})\sin^2\frac{kr_0}{2} \qquad A_1 = A_2 \tag{7}$$

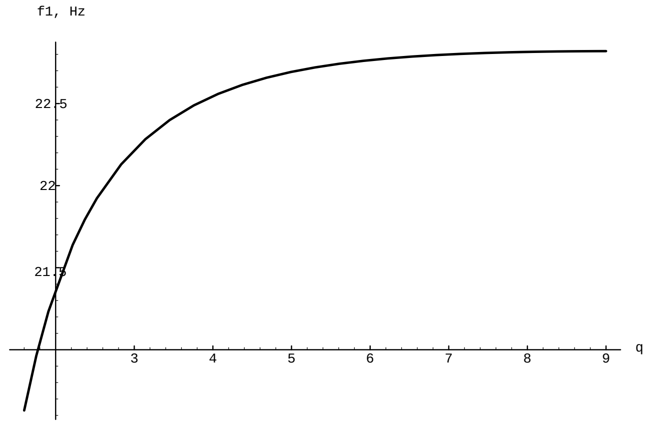

Figure 4. Dependence of frequency f_1 of the first mode (the amplitudes are the same and in phase for the vertically arranged grains in the first and in the second chains) against dimensionless grain charge $q = |Q/e| \times 10^{-4}$.

$$\omega_2^2 = \frac{\gamma + \gamma_1 - \gamma_2}{M} - \frac{4Q^2}{Mr_0^3} e^{-r_0/\lambda_D}(1 + r_0/\lambda_D) \sin^2 \frac{kr_0}{2} \qquad A_1 = A_2 \frac{\gamma_1}{\gamma_2} \tag{8}$$

where

$$\gamma_1 = Q \frac{d^2\Phi_1(|z|)}{d|z|^2}\bigg|_{|z|=d} \qquad \gamma_2 = -Q \frac{d^2\Phi_2(|z|)}{d|z|^2}\bigg|_{|z|=d}. \tag{9}$$

We see that for $k = 0$ the characteristic frequencies are given by $\omega_1^2 = \gamma/M$ and $\omega_2^2 = (\gamma - \gamma_1 - \gamma_2)/M$, and they decrease with growing wavenumber when $kr_0 \ll 1$.

To estimate the effective width of the electrode potential well γ, we employ the standard model of the sheath, which considers Boltzmann distributed electrons. For simplicity, we ignore the influence the dust grains may have on the field distribution in the sheath region. From Poisson's equation, integrated once and linearizing, we obtain the effective width of the electrode potential well

$$\gamma = 4\pi e|Q|n_0 \left(\exp\left(\frac{e\phi_0}{T_e}\right) - \left(1 - \frac{2\,e\phi_0}{T_e}\frac{v_s^2}{v_0^2}\right)^{-1/2} \right). \tag{10}$$

Solutions of equations for the sheath potential ϕ_0 in the equilibrium position and for the equilibrium distance d in the vertical direction can be found numerically. Assuming $\lambda_D \approx 2 \times 10^{-2}$ cm, $v_0^2/v_s^2 \approx 1.5$, $Q/e = 2 \times 10^4$, $M = 0.6 \times 10^{-9}$ g, the equilibrium vertical distance is given by $d = 1.75\lambda_D$, and $\gamma_1/\gamma_2 \approx -25.3$. The characteristic frequencies of the two modes are approximately

$$f_1(k = 0) = \frac{1}{2\pi}\sqrt{\frac{\gamma}{M}} \approx 21.3 \text{ Hz} \qquad f_2(k = 0) = \frac{1}{2\pi}\sqrt{\frac{\gamma + \gamma_1 - \gamma_2}{M}} \approx 63.5 \text{ Hz}. \tag{11}$$

Their dependencies on Q are presented on figures 4 and 5 (see also Vladimirov, Shevchenko, and Cramer [3]).

Since the equilibrium distance d between the chains is almost independent of Q, the frequency f_2 is approximately directly proportional to Q. Note that the amplitude of dust grain oscillations in the lower chain for the second mode (when the grains oscillate with opposite phases) is much smaller than the amplitude of the oscillations in the upper chain,

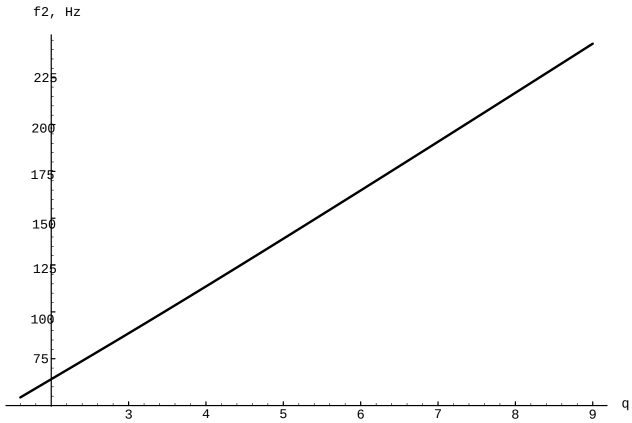

Figure 5. Dependence of frequency f_2 of the second mode (the amplitudes are different and in counter-phase for the vertically arranged grains in the first and in the second chains) against dimensionless charge $q = |Q/e| \times 10^{-4}$.

$|A_1/A_2| = |\gamma_1/\gamma_2| \approx 25.3$. For the first mode, the amplitudes are the same for the upper and lower particles.

Thus, the vertical oscillations of the two one-dimensional chains of dust grains levitating in the sheath field of a horizontal negatively biased electrode can give rise to two specific low-frequency modes which are characterized by an inverse optic-mode-like dispersion when the wavelengths far exceed the intergrain distance. Excitation of these modes can be responsible for phase transitions in the system. Note also that the second mode may be especially useful for diagnostic purposes because its dispersion is almost directly proportional to the dust grain charge Q which in turn is a function of plasma parameters.

Acknowledgment

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References

[1] Thomas H M and Morfill G E 1996 *Nature* **379** 806
[2] Vladimirov S V and Nambu M 1995 *Phys. Rev.* E **52** R2172
Vladimirov S V and Ishihara O 1996 *Phys. Plasmas* **3** 444
Ishihara O and Vladimirov S V 1997 *Phys. Plasmas* **4** 69
[3] Vladimirov S V, Shevchenko P V and Cramer N F 1997 *Phys. Rev.* E **56** R74
Vladimirov S V, Shevchenko P V and Cramer N F 1997 *Phys. Plasmas* **5** 4