



Figure 3. Matter power spectra at $z = 60$ for interacting DE models with constant coupling as computed by `CMBEASY`.

quantities are usually computed by setting up a random-phase realization of the power spectrum of the studied cosmological model according to the Zel'dovich approximation (Zel'dovich 1970). The normalization amplitude of the power spectrum is adjusted such that the linearly evolved rms-fluctuations σ_8 on a top-hat scale of $8 h^{-1} \text{Mpc}$ at a given redshift z_{norm} (usually chosen to be $z_{\text{norm}} = 0$) have a prescribed amplitude.

The coupling between DE and CDM can have a strong impact on the transfer function of matter density fluctuations, as first pointed out by Mainini & Bonometto (2007b). For this reason, we compute the required initial power spectrum directly with the modified Boltzmann code `CMBEASY` because the phenomenological parametrizations of the matter power spectrum available for the Λ CDM cosmology (e.g. Bardeen et al. 1986; Eisenstein & Hu 1998) would not be accurate enough. The resulting effect on the power spectrum is shown in Fig. 3 for the different models considered in our set of simulations.

Once the desired density field has been realized with this procedure, the displacements of the particles from the grid points need to be rescaled with the linear growth factor D_+ for the cosmological model under investigation between the redshifts z_{norm} and z_i in order to set the correct amplitude of the power spectrum at the starting redshift of the simulation. Also, the velocities of the particles are related to the local overdensities according to linear perturbation theory, via the following relation, here written in the Fourier space:

$$v(\mathbf{k}, a) = i f(a) a H \delta(\mathbf{k}, a) \frac{\mathbf{k}}{k^2}, \quad (39)$$

where the growth rate $f(a)$ is defined as

$$f(a) \equiv \frac{d \ln D_+}{d \ln a}. \quad (40)$$

This requires an accurate calculation of the linear growth function $D_+(z)$ for the coupled model, which we again compute numerically with `CMBEASY`.

We note that a phenomenological parametrization of the growth function for coupled DE models with constant coupling to dark matter has recently been made available by Di Porto & Amendola (2008). However, it is only valid for models with no admixture of uncoupled matter, whereas in our case we also have a baryonic component. Also, in the Λ CDM cosmology, the total growth rate is well approximated by a power of the total matter density $\frac{\gamma}{M}$, with $\gamma = 0.55$, roughly independently of the cosmological constant density (Peebles 1980). This is however no longer true in coupled cosmologies, as we show in Fig. 4. We find that, for our set of

coupled DE models, a different phenomenological fit given by

$$f(a) \sim \frac{\gamma}{M} \left(1 + \gamma \frac{\gamma_{\text{CDM}}}{M} q_c \beta_c^2 \right), \quad (41)$$

with $\gamma = 0.56$ (as previously found in Amendola & Quercellini 2004) and $q_c = 2.4$ works well. The fit (41) reproduces the growth rate with a maximum error of ~ 2 per cent over a range of coupling values between 0 and 0.2 and for a cosmic baryon fraction $b//_m$ at $z = 0$ in the interval 0.0–0.1 for the case of the potential slope $\alpha = 0.143$ (corresponding to the slope of the RP1–RP5 models). For a value of $\alpha = 2.0$ (corresponding to the slope assumed for RP6), the maximum error increases to ~ 4 per cent in the same range of coupling and baryon fraction. In Fig. 4, we plot both the fitting formulae together with the exact $f(a)$. For our initial conditions setup, we in any case prefer to use the exact function $f(a)$ directly computed for each model with `CMBEASY`, rather than any of the phenomenological approximations.

3.2 Tests of the numerical implementation: the linear growth factor

As a first test of our implementation, we check whether the linear growth of density fluctuations in the simulations is in agreement with the linear theory prediction for each coupled DE model under investigation. To do so, we compute the growth factor from the simulation outputs of the low-resolution simulations described in Table 2 by evaluating the change in the amplitude of the matter power spectrum on very large scales, and we compare it with the solution of the system of coupled equations for linear perturbations (19), numerically integrated with `CMBEASY`. The comparison is shown in Fig. 5 for all the constant coupling models. The accuracy of the linear growth computed from the simulations in fitting the theoretical prediction is of the same order for all the values of the coupling, and the discrepancy with respect to the numerical solution obtained with our modified version of `GADGET-2` never exceeds a few per cent.

3.3 Our set of N -body simulations

In our simulations, we are especially interested in the effects that the presence of a coupling between DE and CDM induces in the properties of collapsed structures, and we would like to understand which of these effects are due to linear features of the coupled theory, and which due to the modified gravitational interaction in the dark sector. This goal turns out to be challenging due to the presence of several different sources of changes in the simulation outcomes within our set of runs. To summarize this, let us briefly discuss in which respect, besides the different gravitational interactions, the high-resolution simulations listed in Table 2 are different from each other.

(i) The initial conditions of the simulations are generated using a different matter power spectrum for each model, i.e. the influence of the coupled DE on the initial power spectrum is taken into account and this means that every simulation will have a slightly different initial power spectrum shape.

(ii) The amplitude of density fluctuations is normalized at $z = 0$ for all the simulations to $\sigma_8 = 0.796$, but due to the different shapes of the individual power spectra the amplitude of density fluctuations at the present time will not be the same in all simulations at all scales.

(iii) The initial displacement of particles is computed for each simulation by scaling down the individual power spectrum