A RATE-INDEPENDENT TEST FOR SOLAR FLARE SYMPATHY

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Abstract. Solar flare sympathy is the triggering of a flare in one active region by a flare in another. Statistical tests for flare sympathy have returned varying results. However, existing tests have relied on flaring rates in active regions being constant in time, or else have attempted to model the rate variation, which is a difficult task. A simple test is described which is independent of flaring rates. The test generalizes the approach of Fritzová-Švestková, Chase, and Svěstka (1976), and examines the distribution of flare coincidences in pairs of active regions as a function of coincidence interval τ. The test is applied to available soft X-ray and Hα flare event listings. The soft X-ray events exhibit a deficit of flare coincidences for $\tau \leq 20\,\mathrm{min}$, which is most likely due to an event selection effect whereby the increased soft X-ray emission due to one flare prevents a second flare being identified. The Hα events show an excess of flare coincidences for $\tau \leq 10\,\mathrm{min}$, suggesting flare sympathy. The number of Hα event pairs occurring within 10 minutes of one another is higher than that expected on the basis of random coincidence by a fraction 0.12 ± 0.02. Nearby active regions (spatial separation $< 50^\circ$) show a greater excess of coincidences for $\tau \leq 10\,\mathrm{min}$ than active regions which are far apart (spatial separation $\geq 50^\circ$). However, the active regions which are far apart still show some evidence for an excess of coincidences at very short coincidence intervals ($\tau \leq 2\,\mathrm{min}$), which appears to exclude the possibility of a coronal disturbance propagating from one region to another.

1. Introduction

There is considerable interest in the possibility of solar flare sympathy, which is defined as flaring activity in one active region causing flaring activity in a second active region. In principle observations of flare sympathy could shed light on the flare mechanism, the process of flare initiation, and also on mechanisms of energy transport in flares. Flare sympathy also presents a challenge for models of flare statistics, which tend to assume flares occur as independent events (e.g., Rosner and Vaiana, 1978; Lu and Hamilton, 1991; Wheatland and Craig, 2003).

Flare sympathy according to the stated definition certainly occurs, based on detailed observations of flare-initiated disturbances producing brightenings in remote active regions (for recent accounts, see e.g., Wang *et al.*, 2001; Balasubramaniam *et al.*, 2005). However, statistical studies, which have attempted to pin down the nature and degree of sympathy, have returned mixed results. Fritzová-Švestková, Chase, and Švestka (1976) found some evidence for flare sympathy for active regions within $30°$ of one another based on a random coincidence formula (discussed below) applied to Hα event lists. The sympathetic events occurred within about 20 minutes of one another. Pearce and Harrison (1990) also found an excess of overlap of Hα emission times beyond that expected on the basis of random coincidence for active regions with close spatial separation ($< 50°$). Wheatland, Sturrock, and McTiernan (1998) examined the waiting-time distribution (the distribution of times between events) for solar flare hard X-ray bursts observed by the ICE/ISEE 3 spacecraft. They found an excess of short waiting times ($< 10\,\mathrm{min}$) with respect to a time-dependent Poisson process, using rates estimated from the data. However, Biesecker and Thompson (2000) found no excess of short waiting times in the range 2–80 minutes for hard X-ray flares observed with the Burst and Transient Source Experiment. They also found no increase in the rate of Geostationary Observational Environmental Satellites (GOES) soft X-ray flares associated with the ≈ 2 hour passage of Extreme Ultraviolet Imaging Telescope (EIT) waves across the disk. Moon *et al.* (2001) found the waiting-time distribution for GOES soft X-ray flares to be well described by a time-dependent Poisson process, for waiting times less than 30 hours. In a follow-up study, Moon *et al.* (2002) examined the subset of active region pairs with an excess of flare overlap time (by comparison with random occurrence at a constant rate). They found that flares from this set of active regions show an excess of waiting times shorter than about 4 hours, by comparison with a time-dependent Poisson process.

A specific criticism of the tests for solar flare sympathy used by different authors to date is that they either assume the flaring rate is constant, or else attempt to model the rate in an ad hoc way. For example, the random coincidence formula used by Fritzová-Švestková, Chase, and Švestka (1976) gives the expected number of flare coincidences in two active regions during an interval τ as

$$N_{12} = 2\frac{N_1 N_2 \tau}{T^2}, \tag{1}$$

where N_1 and N_2 are the number of flares observed in each active region during the common observing period T. The factor of two appears because a flare in the second active region is coincident with one in the first if it occurs within an interval of duration 2τ, i.e. during the interval $(t_1 - \tau, t_1 + \tau)$, where t_1 is the time of the event in the first active region. As acknowledged by Fritzová-Švestková, Chase, and Švestka (1976), this formula is strictly valid only if the rates of occurrence of flares in the two active regions are constant. For example, if two active regions both have an increasing rate of flaring during the observing period, then the number of coincidences will in general be higher than the number given by Equation (1). Solar active regions are known to exhibit time variation in flaring rate during a transit of the disk (e.g., Wheatland, 2001), and hence Equation (1) does not provide a basis for a rigorous test for sympathy.

The problem also applies to the other approach to testing for flare sympathy, i.e. examining the waiting-time distribution. If the rate of flaring is constant then flares occur as a Poisson process in time and the waiting-time distribution is exponential. Comparison with this distribution is straightforward. However, if the rate of flaring varies with time, then it is necessary to make a comparison with a time-dependent Poisson process (e.g., Wheatland, Sturrock, and McTiernan, 1998; Biesecker and Thompson, 2000; Moon *et al.*, 2001; Moon *et al.*, 2002). This involves determining the time history of the rate from the data, which is a non-trivial problem (e.g., Scargle, 1998). If intervals when the rate is high are not accurately identified, then the model waiting-time distribution will have a deficit of short waiting times compared with observations, falsely implying flare sympathy.

In this paper a test for flare sympathy is described which applies even when the flaring rate varies in time. The test involves a simple generalization of Equation (1) to the time-dependent case. The test is then applied to available flare event lists, to re-examine claims of evidence for sympathy. The layout of the paper is as follows. In Section 2 the test is described. In Section 3 the test is applied to soft X-ray and Hα event lists, and comparison is made with earlier studies. Section 4 presents a discussion of the results.

2. Method

Consider two model active regions observed for a time T, which are assumed to flare independently in time according to time-dependent Poisson processes. Flaring in the regions is then completely described by the Poisson rates $\lambda_1(t)$ and $\lambda_2(t)$, which give the probabilities per unit time of events occurring in each region (e.g., Cox and Isham, 1980). Given a flare in active region 1 at time t, the expected number of events in region 2 in a coincidence interval $(t-\tau, t+\tau)$ is

$$\int_{t-\tau}^{t+\tau} \lambda_2(t)\, dt \approx 2\tau\lambda_2(t), \tag{2}$$

where the approximation neglects the variation in $\lambda_2(t)$ during the coincidence interval. The probability of a flare occurring in region 1 during the interval $(t, t+dt)$ is $\lambda_1(t)dt$, so the expected number of flare coincidences in the interval $(t, t+dt)$ is $\Delta N_{12}(t,\tau) = 2\tau\lambda_2(t)\times\lambda_1(t)\, dt$. The expected number of coincidences over the total observation period is obtained by integrating this expression over time:

$$N_{12} = \int_0^T \Delta N_{12}(t,\tau)\,dt = 2\tau \int_0^T \lambda_1(t)\lambda_2(t)\,dt. \tag{3}$$

Equation (3) is a time-dependent generalization of Equation (1), valid provided the rates do not vary appreciably over the coincidence interval τ. In common with Equation (1) there is a simple linear dependence on τ, so following Fritzová-Švestková, Chase, and Švestka (1976), differentiating gives the distribution of coincidence numbers as a function of τ, which is independent of τ:

$$\frac{dN_{12}}{d\tau} = 2\int_0^T \lambda_1(t)\lambda_2(t)\,dt. \tag{4}$$

The lack of τ-dependence of the model distribution of coincidence numbers provides a test of the validity of the Poisson model, i.e. a test of the assumption that events in the regions occur independently. Specifically, the left hand side of Equation (4) may be approximately constructed for a number of active region pairs by differencing of observed coincidence numbers, for a range of values of τ. The results may be summed over the active region pairs, for each τ. If the Poisson model is correct, the resulting distribution should be constant in τ. Flare sympathy should show appear as an excess of coincidences at small values of τ.

To formalise the test we consider a chi-square comparison of the observed distribution with the Poisson model. Specifically, we may write the model, for an active region pair labelled i, as $N_{12,i}^{\rm mod} = C_i\tau$, where C_i is a constant [see Equation (3)]. From the observations we can construct the approximation to the coincidence number distribution

$$f_i^{\rm obs} = \frac{N_{12,i}^{\rm obs}(\tau + \frac{1}{2}\Delta\tau) - N_{12,i}^{\rm obs}(\tau - \frac{1}{2}\Delta\tau)}{\Delta\tau}, \tag{5}$$

for a chosen $\Delta\tau$ and for a range of values of τ. (Note that we use f to denote a finite difference approximation to $dN_{12}/d\tau$.) The corresponding model distribution is

$$f_i^{\rm mod} = C_i. \tag{6}$$

The uncertainty associated with the model follows from Poisson counting statistics for numbers of events in each interval in τ:

$$\sigma_i^{\rm mod} = \frac{\left[N_{12,i}^{\rm mod}(\tau + \frac{1}{2}\Delta\tau) - N_{12,i}^{\rm mod}(\tau - \frac{1}{2}\Delta\tau)\right]^{1/2}}{\Delta\tau} = (C_i/\Delta\tau)^{1/2}. \tag{7}$$

Summing over M observed active regions gives

$$\begin{aligned} f^{\rm mod} &= \sum_{i=1}^{M} C_i \\ &\equiv C, \end{aligned} \tag{8}$$

and combining uncertainties in quadrature gives

$$\begin{aligned} \sigma^{\rm mod} &= \left[\sum_{i=1}^{M} \left(\sigma_i^{\rm mod}\right)^2\right]^{1/2} \\ &= (C/\Delta\tau)^{1/2}. \end{aligned} \tag{9}$$

The chi-square statistic for a set of points τ_j $(j = 1, 2, ..., N)$ is then

$$\begin{aligned} \chi^2 &= \sum_{j=1}^{N} \frac{\left(f^{\rm mod} - f_j^{\rm obs}\right)^2}{(\sigma^{\rm mod})^2} \\ &= \frac{\Delta\tau}{C} \sum_{j=1}^{N} \left(C - f_j^{\rm obs}\right)^2, \end{aligned} \tag{10}$$

where $f_j^{\rm obs} = f^{\rm obs}(\tau_j)$. Minimizing χ^2 with respect to C gives

$$C_{\rm min} = \left[N^{-1} \sum_{j=1}^{N} \left(f_j^{\rm obs}\right)^2\right]^{1/2}. \tag{11}$$

A chi-square test (with $N - 1$ degrees of freedom) may be performed using this value of $C_{\rm min}$ (e.g., Press *et al.*, 1992).

3. Results

3.1. Soft X-ray events

The Geostationary Observational Environmental Satellite (GOES) annual listings of soft X-ray flares provided by the U.S. National Oceanic and Atmospheric Administration National Geophysical Data Center (NOAA/NGDC)[1] tabulate soft X-ray events selected from 1-8Å whole-Sun GOES observations for the period 1975 to the present day. Many events in the listings are identified with active regions, based on correlated optical flares. These listings were used for statistical flare sympathy studies by Biesecker and Thompson (2000), Moon *et al.* (2001), and Moon *et al.* (2002), as discussed in Section 1.

[1] Available from `http://www.ngdc.noaa.gov/stp/SOLAR/ftpsolarflares.html`

GOES events are selected by a simple procedure. The start of an event is defined by four consecutive one-minute 1-8Å flux measurements which are monotonically increasing, with the fourth measurement at least 1.4 times the first. The end of an event is defined by the flux returning to half the peak value, where the peak value includes the preflare background. The whole-Sun nature of the observations, together with the event selection procedure, introduces a selection effect into the GOES event lists whereby some flares occurring close in time are missed (Wheatland, 2001). According to the event selection procedure, if two flares overlap in time, the second flare must produce an increase of around 40% in flux above that of the first flare to be identified and included in the GOES lists. Smaller succeeding events are missed as a result, an effect which we will refer to as 'obscuration'.

To apply the sympathy test described in Section 2, a list of active region pairs was generated based on the GOES listings for 1975-2004, including only those pairs of active regions which were on the disk for overlapping periods of time and which have at least 10 listed events each. This gave 1047 active region pairs. The observed coincidence number distribution f^{obs} was constructed for these active region pairs for $0 \leq \tau \leq 1200\,\mathrm{min}$ with $\Delta\tau = 30\,\mathrm{min}$, using the listed start times of events, and only including events with a peak flux above $10^{-6}\,\mathrm{W m^{-2}}$ (C1 events). The range of τ and the restriction to events above C1 was chosen to match the analysis of GOES events by Moon *et al.* (2001) and Moon *et al.* (2002).

Figure 1 shows the results. The observed coincidence number distribution is shown by the diamonds, with representative error bars (estimated based on square roots of numbers of coincidences associated with each point). The solid horizontal line in the figure is the minimum chi-square value C_{min}, from Equation (11). The figure shows a deficit of coincidences for $\tau \leq 60\,\mathrm{min}$, and the chi-square test indicates that the data is incompatible with the constant model. For $\tau > 60\,\mathrm{min}$ the distribution is consistent with a constant model. The deficit at small τ suggests flare anti-sympathy, i.e. that flares are less likely to occur close in time to one another.

The most likely explanation for the observed deficit of events close in time is obscuration. Figure 2 compares the observed coincidence number distribution with the distribution of GOES event durations. The top panel of Figure 2 shows an expanded view of the distribution in Figure 1 for small τ, constructed for $0 \leq \tau \leq 100\,\mathrm{min}$ and with $\Delta\tau = 2\,\mathrm{min}$. This panel shows that the deficit in coincidences applies predominantly for $\tau \leq 20\,\mathrm{min}$. The bottom panel shows the cumulative distribution function (CDF) for event durations, i.e. the fraction of events with longer duration, versus duration. The mean duration of the GOES events is around 17 minutes. The observed distribution of event durations is consistent with obscuration causing the deficit of coincidences seen in the top panel.

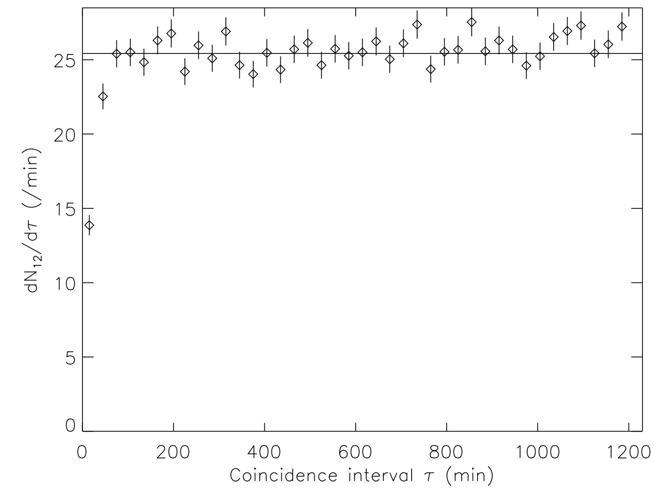

Figure 1. Distribution of coincidence numbers of GOES events for flare-producing active region pairs, for coincidence intervals $0 \leq \tau \leq 1200\,\mathrm{min}$.

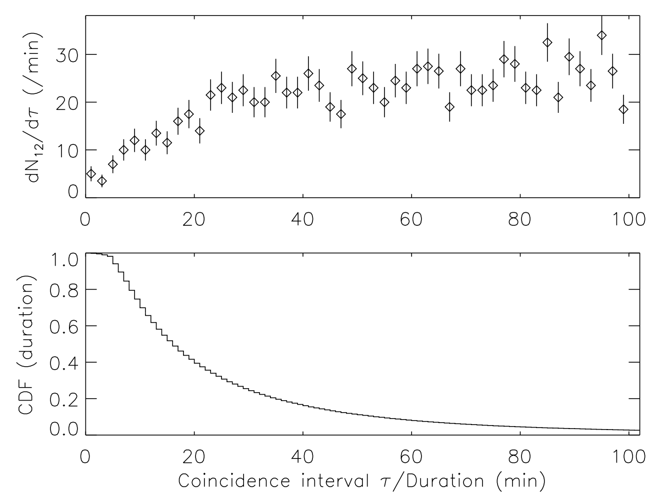

Figure 2. Top panel: Distribution of coincidence numbers of GOES events for flare-producing active region pairs, for coincidence intervals $0 \leq \tau \leq 100\,\mathrm{min}$. Bottom panel: CDF for event durations.

Moon *et al.* (2002) presented statistical evidence for sympathy of GOES events on the timescale of a few hours for certain pairs of active regions. Their method involved identifying active region pairs with an excess of flare overlap times [using a variant of Equation (1)], and then examining the waiting-time distribution for events from the regions. We have applied the test described in Section 2 to the 17 active region pairs identified by Moon

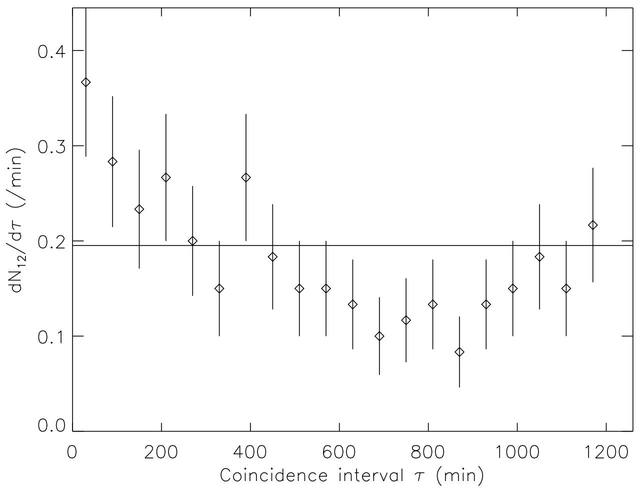

Figure 3. Distribution of coincidence numbers of GOES events for the Moon *et al.* (2002) sample of active regions, for coincidence intervals $0 \leq \tau \leq 1200\,\mathrm{min}$.

et al. (2002) and listed in their Table 2. Figure 3 shows the results, for $0 \leq \tau \leq 1200\,\mathrm{min}$ and $\Delta\tau = 60\,\mathrm{min}$. The horizontal line shows the value of C_{min}. In this case there is no evidence of a deficit at small τ, and indeed there appears to be an excess of coincidences for $\tau \leq 200\,\mathrm{min}$, consistent with the findings of Moon *et al.* (2002). The data is inconsistent with the constant model at the 5% level.

It is interesting to consider why the Moon *et al.* (2002) sample of active regions gives different results from the overall set (compare Figures 1 and 3). We note that Moon *et al.* (2002) have selected regions on the basis of having an excess of flare overlap time. This selection will pick out examples where obscuration has been less important. Relevant situations include when two events occur close in time but are of comparable size, or when two events occur close in time and the second is larger than the first. An inspection of the Moon *et al.* (2002) sample of events indicates that they fit these categories. We also note that the Moon *et al.* (2002) sample only includes 23 event pairs among the 17 active regions pairs. Hence Moon *et al.* (2002) appear to have selected a small, unusual sample.

3.2. Hα events

The NOAA/NGDC provides listings of Hα flare events for the period 1980 to the present, compiled from monthly reports produced by 21 observatories.[2] As discussed in Section 1, a number of statistical studies of flare sympathy

[2] Also available from `http://www.ngdc.noaa.gov/stp/SOLAR/ftpsolarflares.html`

have used Hα event listings (e.g., Fritzová-Švestková, Chase, and Švestka, 1976; Pearce and Harrison, 1990).

The Hα event listings contain event reports compiled from the different observatories, as well as group lines, which condense and average individual reports for the same event, and assign a group number to the distinct events. The identification of distinct events was made by the World Data Center in Boulder.

The sympathy test was applied to the distinct (based on group number) Hα events from the subset of flare-producing active region pairs used in Section 3.1 that were on the disk during 1980-2004, the period of the Hα observations. The listed start times of events were used. The coincidence number distribution was constructed for $0 \leq \tau \leq 100\,\mathrm{min}$ with $\Delta\tau = 2.0\,\mathrm{min}$, chosen to match the range of coincidence times considered by Fritzová-Švestková, Chase, and Sv̌estka (1976). The earlier authors were motivated by estimates of propagation times for coronal disturbances.

Figure 4 shows the result. The distribution (diamonds with error bars) has an excess of coincidence numbers for $0 \leq \tau \leq 10\,\mathrm{min}$, suggesting flare sympathy. The solid horizontal line shows the minimum chi-square model C_{min}. The chi-square test indicates that the data is incompatible with the model. The fractional excess of events for $\tau \leq T$ (by comparison with the model) is given by

$$\epsilon_T = \sum_{\tau_j \leq T} 1 - C/f_j^{\mathrm{obs}}, \tag{12}$$

and it is straightforward to estimate an uncertainty for this fraction, based on the model and observational uncertainties. For $T = 10\,\mathrm{min}$ we find $\epsilon_T = 0.12 \pm 0.02$.

It is of interest to examine whether nearby active regions have a greater excess of coincidences than active regions which are far apart, following Fritzová-Švestková, Chase, and Švestka (1976), and Pearce and Harrison (1990). To do this, the spatial separation of the 792 active region pairs was determined based on linear fitting of heliographic angular positions versus time for each active region as recorded in the United States Air Force/Mt Wilson listings[3]. From these fittings, an angular separation θ was determined at a fixed time, namely the middle of the observation period of the active region with the lower active region number. The USAF/Mt Wilson lists cover the period 1981 to the present day, so angular separations were not determined for the active region pairs producing Hα events in 1980, and these pairs were omitted from the analysis.

The sympathy test was applied to active region pairs with small separations ($\theta < 50°$; 220 pairs) and with large separations ($\theta \geq 50°$; 443 pairs). Figure 5 shows the results, with the coincidence number distribution for

[3] Available from `http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotregions.html`

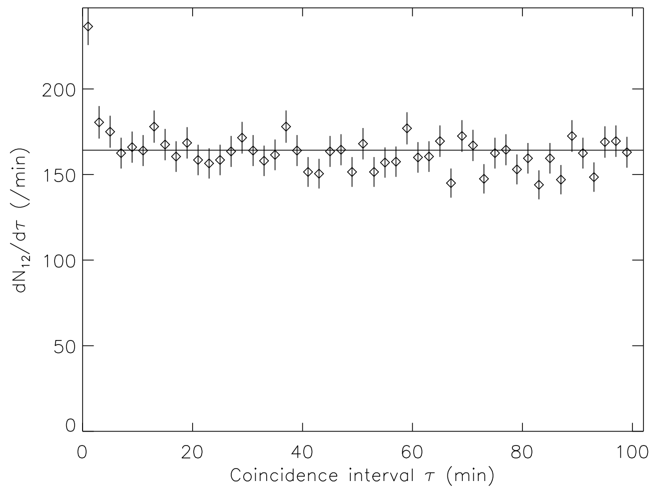

Figure 4. Distribution of coincidence numbers of Hα events for flare-producing active region pairs, for coincidence intervals $0 \leq \tau \leq 100\,\mathrm{min}$.

small separations shown in the top panel, and the distribution for large separations shown in the bottom panel. The two distributions are similar, and in particular both distributions appear to show an excess of coincidences for small τ, although the excess is larger for the pairs with small separations. From Equation (12) the fractions of excess coincidences for intervals less than $T = 10\,\mathrm{min}$ are $\epsilon_T = 0.15 \pm 0.02$ (small separations) and $\epsilon_T = 0.09 \pm 0.02$ (large separations). This is also reflected in the chi-square probabilities. The distribution for small separations is inconsistent with the constant model at a significance $\approx 10^{-6}$. The distribution for large separations is inconsistent with the constant model at a marginal significance (0.07).

Figure 5 shows a greater excess of coincidences at small τ associated with nearby active regions, consistent with the findings of Fritzová-Švestková, Chase, and Švestka (1976), and Pearce and Harrison (1990). However, Figure 5 also indicates some excess of coincidences at very short intervals ($\tau \leq 2\,\mathrm{min}$) for active regions which are widely separated, although the result is marginal (chi-square probability 7%). If the excess is real, it is difficult to interpret in terms of a flare in one region triggering a coronal disturbance which propagates to the second region and produces flaring activity there, because the expected propagation times are too long. For example, a coronal disturbance travelling at $10^6\,\mathrm{ms}^{-1}$ would traverse 50° in about 10 minutes. It is plausible that energy release could occur at an intermediate location, and propagate to two active regions producing almost simultaneous Hα emission. However, this scenario seems unlikely as a general explanation for the apparent effect.

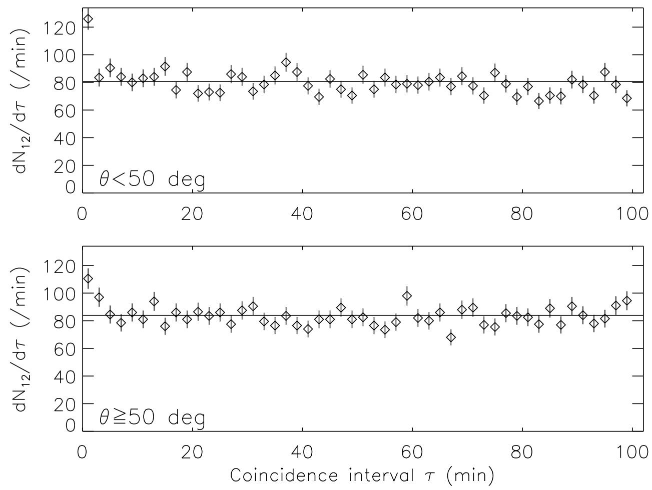

Figure 5. Distribution of coincidence numbers of Hα events for flare-producing active regions with separations $\theta < 50^\circ$ (top panel) and $\theta \geq 50^\circ$ (bottom panel).

An alternative explanation is that the Hα event listings contain selection effects which contribute excess coincidences in widely separated active regions. The listings are produced by compilation of event reports from a number of observatories, and the individual event reports are incomplete and of variable quality. Errors of identification may be present in individual reports, and there may also be errors in the identification of unique events by the World Data Center in Boulder. A specific error which could produce the apparent effect is assignment of some individual event reports to the wrong active region.

4. Discussion

This paper presents a new statistical test for solar flare sympathy, which (in contrast to other tests) is valid when the flaring rate varies in time. However, the test is still susceptible to selection effects present in the data. The test examines the distribution of numbers of event coincidences as a function of coincidence interval τ for a sample of pairs of active regions, and is a generalisation of the approach used by Fritzová-Švestkova, Chase, and Švestka (1976). The test is demonstrated in application to pairs of flare-producing active regions using available listings of flare events based on soft X-ray and Hα observations. Where possible, comparison is made with previous studies.

The GOES X-ray events above C1 class reveal a deficit of coincidences (anti-sympathy) for $\tau \leq 20\,$min, which is attributed to the difficulty of identifying a second flare due to the increase in soft X-ray flux associated with a first flare. This effect has previously been identified in the GOES listings (Wheatland, 2001). Moon *et al.* (2002) reported statistical evidence for sympathy based on GOES events for a specific set of active regions. When the present test is applied to this restricted set, some evidence for sympathy is found.

The Hα events reveal an excess of coincidences (sympathy) for $\tau \leq 10\,\mathrm{min}$, by a fraction 0.12 ± 0.02. Previous studies (e.g., Fritzová-Švestkova, Chase, and Švestka, 1976; Pearce and Harrison, 1990) have presented evidence that sympathy is present for nearby active region pairs, but not for widely separated pairs. We also find a greater excess of coincidences for close (angular separation $< 50°$) versus widely separated ($\geq 50°$) active region pairs. However, the widely separated active region pairs still show some evidence of an excess of coincidences at very short coincidence intervals ($\tau \leq 2\,\mathrm{min}$). It is difficult to understand why active regions separated by more than $50°$ should show an excess of nearly simultaneous Hα events.

The results presented here illustrate the difficulties of statistical identification of solar flare sympathy. In general it may be concluded that, in a statistical sense, flare sympathy is a weak effect. Statistical models describing flare occurrence typically assume that flares occur independently (e.g., Rosner and Vaiana, 1978; Lu and Hamilton, 1991; Wheatland and Craig, 2003), and this appears to be a reasonably accurate assumption. The results for the GOES events also highlight the problem of event selection. Ideally, statistical searches for flare sympathy should use datasets which provide a complete sample of events (above some given threshold). Given the compiled nature of the Hα listings, it is possible that they contain selection effects which contribute some of the apparent observed sympathy.

Finally we note that the method presented here is quite general, and may be applied in any context where it is necessary to identify whether pairs of event sequences are independent. It is possible that there are other applications of the method for identifying event sympathy or anti-sympathy, in astrophysics or more generally.

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