Modeling a falling slinky

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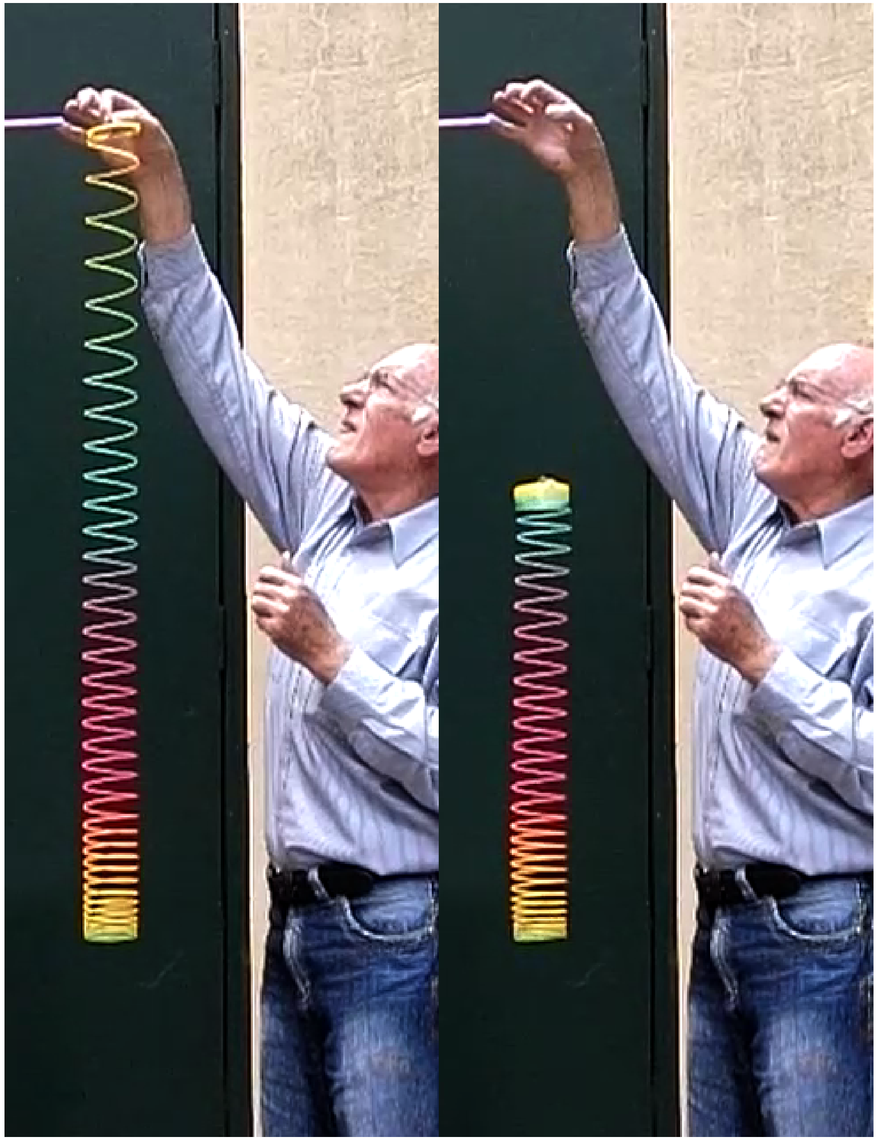
Research Bite
7 June 2012



The fall of a plastic rainbow-coloured slinky.

Overview

Background

Slinky physics and the fall

Waves on slinkies

Modeling the fall

Improving an existing model

Comparison with real slinkies

Conclusions

Background: Slinky physics

- Slinkies are useful physics demonstration devices
  - hanging configuration (e.g. Mak 1993)
  - vertical modes of oscillation when suspended (e.g. Young 1993)
  - wave propagation/dispersion (e.g. Crawford 1987; Vandegrift et al. 1989)
  - peculiar dynamics when falling (e.g. Calkin 1993; Aguirregabiria et al. 2007)
- Slinkies are tension springs[1]
  - under tension subject to Hooke's law but not compression
  - they collapse to a state with turns in contact
  - tension is required to separate collapsed turns
- A slinky suspended from its top and dropped ...
  - collapses from the top down
  - the bottom remains hanging ($\approx 0.3\,\mathrm{s}$) during the collapse!

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[1] Compression springs (the other type of spring) may be under compression or tension according to $F = -kx$. They have separated turns in a relaxed state, with zero tension.

For movies, see: www.physics.usyd.edu.au/~wheat/slinky/.

Background: Waves on slinkies

- Collapse of tension occurs from the top down
  - a (tension) wave propagates down the slinky
    - the tension remains ahead of the wave front
    - turns collapse behind the wave front
- Uncollapsed slinky turns are described by a wave equation:

$$m\frac{\partial^2 x}{\partial t^2} = k\frac{\partial^2 x}{\partial \xi^2} + mg \tag{1}$$

  - m is slinky mass, k is spring constant
  - $x = x(\xi, t)$ is the vertical location of a point on slinky
  - coordinate ξ defines mass fraction: $dm = m\,d\xi$ and $0 \leq \xi \leq 1$
    - so $\xi_i = i/N$ is the end of turn i (for an N-turn slinky)
- Eq. (1): waves in turn spacing propagate
  - characteristic propagation time along slinky:[2]

$$t_p = \sqrt{m/k} \tag{2}$$

[2] For typical slinkies $m \approx 0.2\,\mathrm{kg}$ and $k \approx 0.8\,\mathrm{N/m}$ giving a characteristic time $t_p \approx 0.5\,\mathrm{s}$.

Modeling the fall: Improving an existing model

- Solving the equation of motion directly is tricky
  - Eq. (1) applies until turns collide
  - the tension-spring behaviour complicates the description
- An earlier model used a semi-analytic approach: (Calkin 1993)
  - wave front assumed to be at $\xi_c = \xi_c(t)$ at time t
  - behind the front: the turns are collapsed
  - ahead of the front: the hanging configuration (right)
  - calculate the total momentum implied by this
  - set equal to the impulse mgt at time t and solve for ξ_c
  - total collapse time (bottom starts to fall):[a]

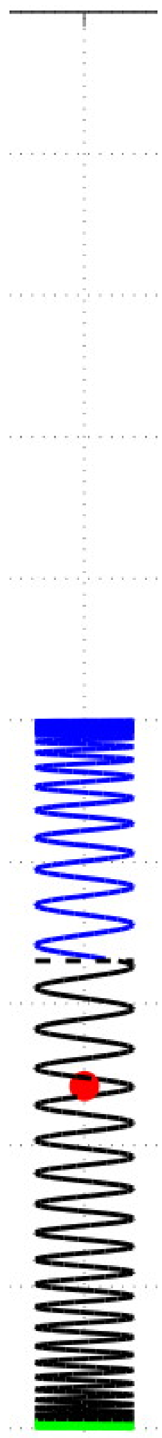
$$t_c = \sqrt{\frac{m}{3k}\xi_1^3} = \sqrt{\tfrac{1}{3}\xi_1^3}\, t_p \tag{3}$$

    - $1 - \xi_1$ is fraction of hanging slinky collapsed at bottom

[a] For typical slinky parameters, with $\xi_1 = 0.9$, the total collapse time is $t_c \approx 0.24\,\mathrm{s}$.

- Problem: turns collapse instantly at the front in the model
  - for real slinkies: a finite time for turns to come together
- An improved model: (Cross & Wheatland 2012)
  - same semi-analytic approach but ...
  - ... including a finite time for collapse behind the front
  - tension relaxes linearly over a fixed number of turns
  - the total collapse time is unchanged
- Movies of the slinky fall and the model:

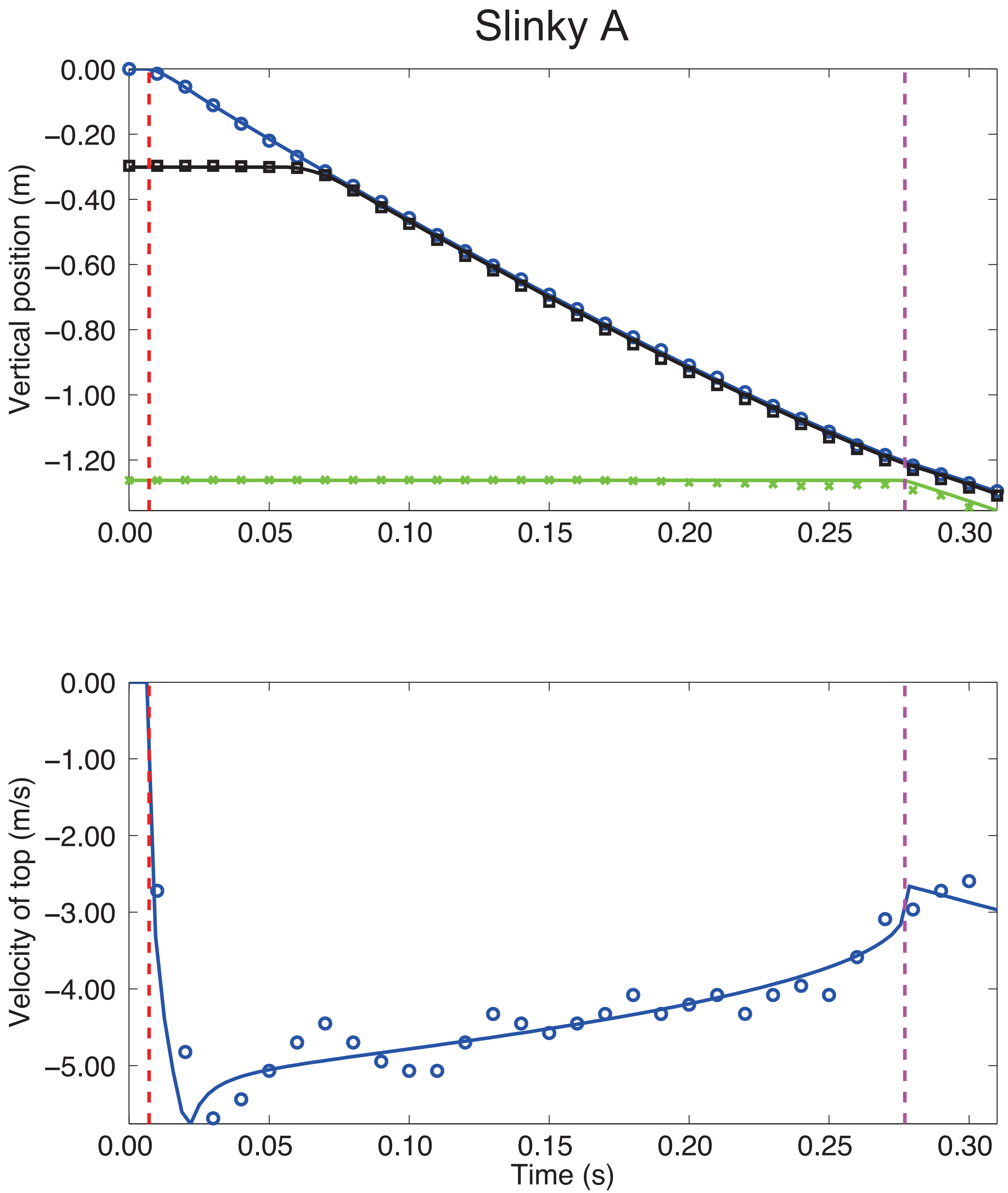
  http://www.physics.usyd.edu.au/~wheat/slinky



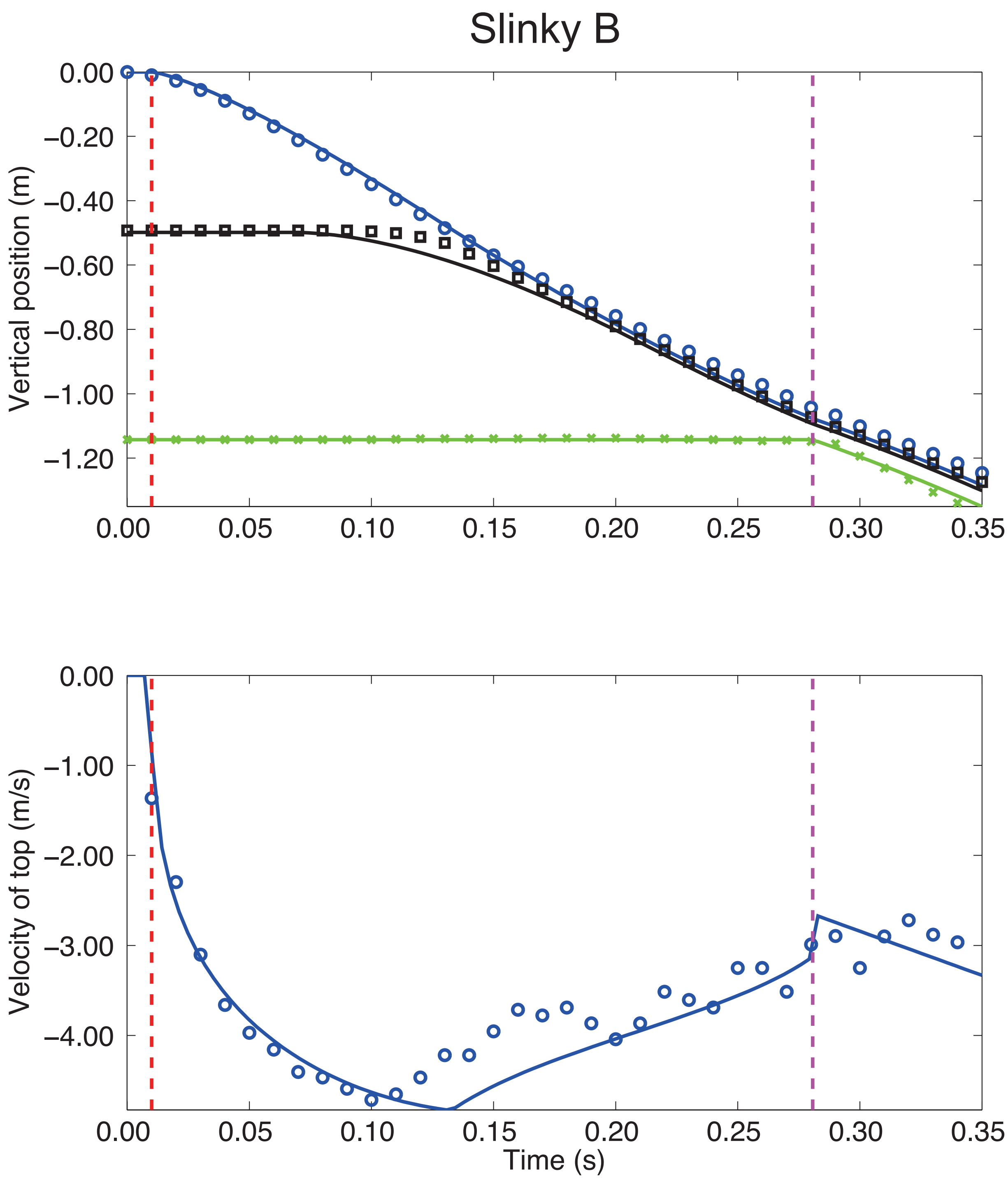
Modeling the fall: Comparison with real slinkies

- Rod dropped slinkies and filmed them at 300 frames/s
  - position of turns with time extracted from frames
  - position of top fitted to model to determine parameters:
    - spring constant, collapse parameter, time of release
- Two slinkies with different properties considered
  - slinky A is a metal slinky
  - slinky B is the plastic rainbow-coloured slinky

	Slinky A	Slinky B
Mass (g)	215.5	48.7
Collapsed length (mm)	58	66
Stretched length (m)	1.26	1.14
Number of turns	86	39



Slinky A
Vertical position (m)
0.00
−0.20
−0.40
−0.60
−0.80
−1.00
−1.20
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Velocity of top (m/s)
0.00
−1.00
−2.00
−3.00
−4.00
−5.00
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Time (s)


Slinky B
Vertical position (m)
0.00
−0.20
−0.40
−0.60
−0.80
−1.00
−1.20
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
Velocity of top (m/s)
0.00
−1.00
−2.00
−3.00
−4.00
Time (s)

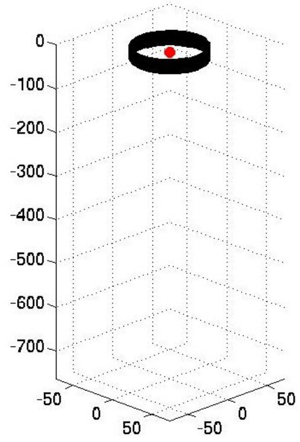
- An independent test of the model parameters:
  - the spring constant k is a fitted parameter
  - fundamental period T_0 depends on this: (Young 1993)

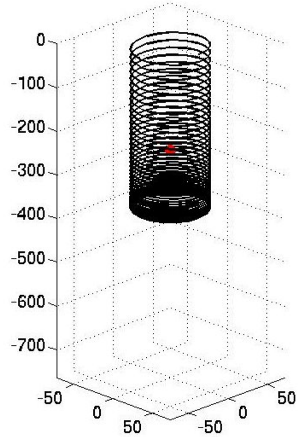
$$T_0 = 4\sqrt{\frac{m}{k}} = 4t_p \qquad (4)$$

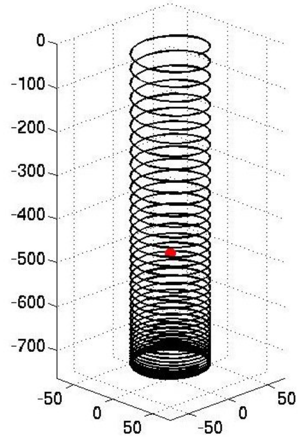
- Rod measured T_0 for each slinky ...

	Slinky A	Slinky B
Model T_0 (s)	2.23	1.88
Observed T_0 (s)	2.18	1.77

- Consistent after taking uncertainties into account







Conclusions

- Slinkies are useful physics demonstration devices
- Falling slinkies exhibit peculiar physics
  - tension collapses from the top down
  - the bottom remains suspended until the top hits it
  - a wave must propagate downwards before the bottom falls
- An improved model of the fall is developed (Cross & Wheatland 2012)
  - based on an existing semi-analytic model ... (Calkin 1993)
  - ... modified so the collapse of slinky turns takes a finite time
- The new model is fitted to data from high-speed movies
  - good qualitative fit to data achieved
  - values of spring constant checked independently