Modelling the coronal magnetic field using Hinode (and future) data

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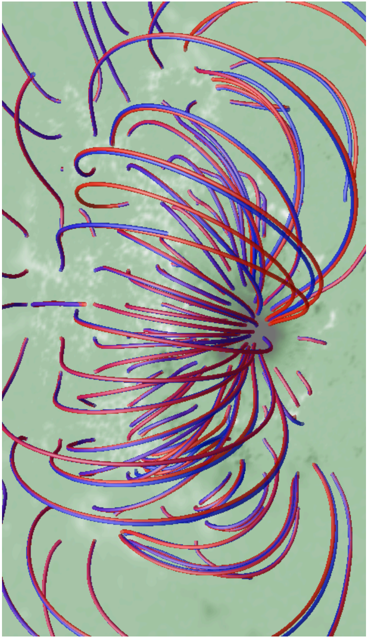

Overview

Background

Solar flares

Vector magnetograms

The nonlinear force-free model

Force-free modelling fails for solar data!

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Self-consistency method

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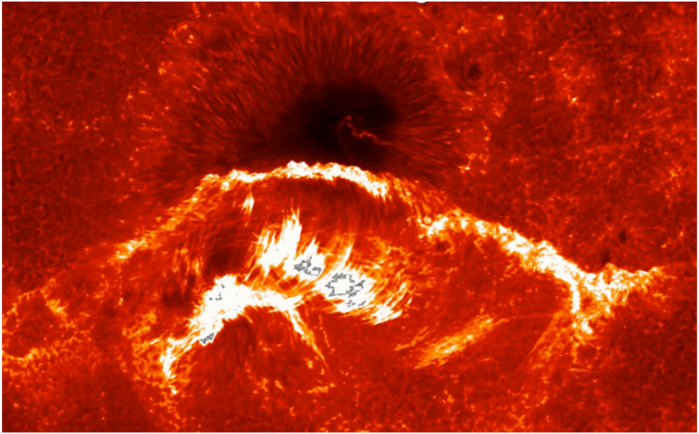
Background: Solar flares

- Magnetic explosions in the Sun's corona
  - large flares influence local space weather
- Motivate need to accurately model the coronal field



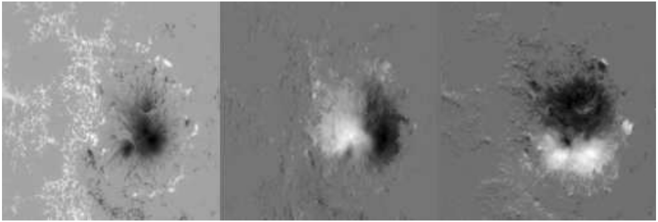
Data: Hinode/SOT (AR 10930, 12 Dec 2006)

Background: Vector magnetograms

"Nobody can measure physical quantities of the solar atmosphere"
(del Toro Iniesta & Ruiz Cobo 1996)

- Polarisation state of photospheric lines measured
- Vector magnetic field inferred (via "inversion")
  - map of **B** at photosphere ("vector magnetogram")
  - an inference rather than a measurement
    (del Toro Iniesta & Ruiz Cobo 1996)
- Problems:
  - instrumental uncertainties
  - validity/reliability of the inversion
  - 180 degree ambiguity in transverse field

- New generation of high resolution instruments
  - Hinode/SOT: satellite launched in 2006
  - SOLIS/VSM: ground based, full disk
  - SDO/HMI: to be launched in 2009
- In principle, boundary conditions for coronal field modelling
  - Hinode/SOT inferred B_z, B_x, B_y



Data: Hinode/SOT (AR 10953 30 Apr 2007)

Background: The nonlinear force-free model

- Force-free field $\mathbf{B}$ satisfies

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{1}$$

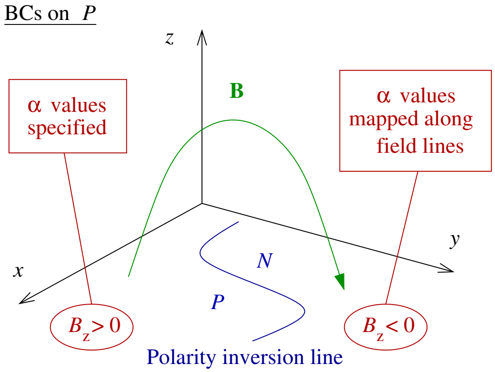
  - "zeroth order" model for the coronal magnetic field (Syrovatskii 1978)
  - current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is parallel to $\mathbf{B}$
  - coupled nonlinear PDEs

- Alternative form:

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \quad \text{and} \quad \mathbf{B} \cdot \nabla \alpha = 0 \tag{2}$$

  - force-free parameter α is constant along field lines

- Boundary conditions (Grad & Rubin 1958):
  - B_n in boundary
  - α in boundary over region where $B_n > 0$ *or* where $B_n < 0$
    - over "one polarity"
    - we label the polarities P and N respectively



- Force-free equations are hard to solve
  - variety of iterative numerical methods (Wiegelmann 2008)
  - demonstrated to work on test cases (Schrijver et al. 2006)
  - some methods use vector **B** in boundary as BCs
- Current-field iteration (Grad & Rubin 1958)
  - at iteration k, solve (linear) system

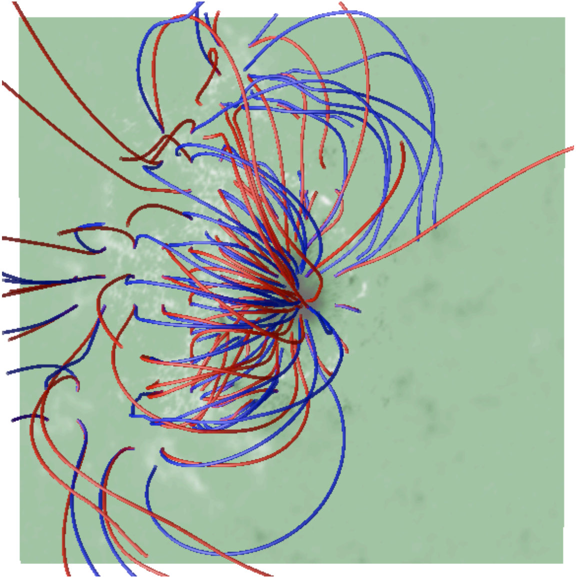
$$\begin{aligned} \nabla \times \mathbf{B}^{(k+1)} &= \alpha^{(k)}\mathbf{B}^{(k)} \\ \mathbf{B}^{(k+1)} \cdot \nabla\alpha^{(k+1)} &= 0 \end{aligned} \tag{3}$$

- Fast current-field iteration (Wheatland 2007)
  - Fourier solution of (3a) ensuring $\nabla \cdot \mathbf{B}^{(k)} = 0$
  - Solution of (3b) by field line tracing ensuring $\nabla \cdot \mathbf{J}^{(k)} = 0$
  - method order N^4 (grid with N^3 points)

Background: Force-free modelling fails for solar data!

- Workshops on application of force-free model to Hinode data
  - 2007: AR 10930, 12-13 Dec 2006 (Schrijver et al. 2008)
  - 2008: AR 10953, 30 April 2007 (DeRosa et al. 2009)
- Failure 1: different methods produce different solutions
  - in particular, energy estimates do not agree
  - impossible to reliably estimate free energy!
- Failure 2: individual solutions not self-consistent
  - there are two choices (P and N) for BCs on α
  - the P and N choices produce different solutions
- Nevertheless, nonlinear force-free modelling is being used...

- AR 10953 on 30 April 2007 (DeRosa et al. 2009)
  - P solution (blue) and N solution (red)

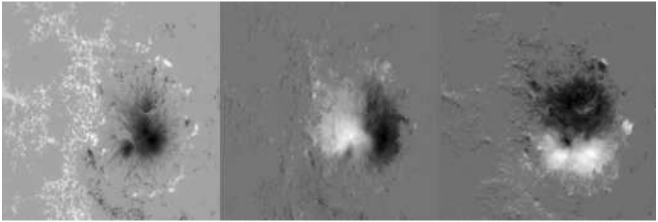


Background: The problem – inconsistency

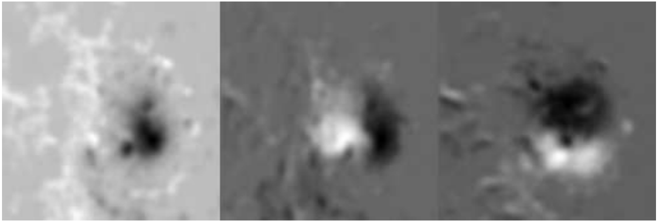
- Boundary conditions inconsistent with force-free model
  - errors in field determination
  - field at photospheric level is forced (Metcalf et al. 1995)
- Necessary conditions for a force-free field (Molodenskii 1969)
  - boundary integrals representing net force, torque
    - zero for a force-free field
  - non-zero for solar boundary data
- "Preprocessing" is used to enforce these conditions... (Wiegelmann et al. 2006)
  - ...but the conditions are necessary, not sufficient
  - preprocessed BCs remain inconsistent with force-free model (DeRosa et al. 2009)
  - solutions still disagree, still inconsistent (DeRosa et al. 2009)
  - and preprocessing smooths the data...

Background: Preprocessing – an unsatisfactory procedure

- Data from 30 April 2007
  - Hinode/SOT inferred B_z, B_x, B_y



- *Data from 30 April 1967?*
  - preprocessed data used at 2008 workshop (DeRosa et al. 2009)

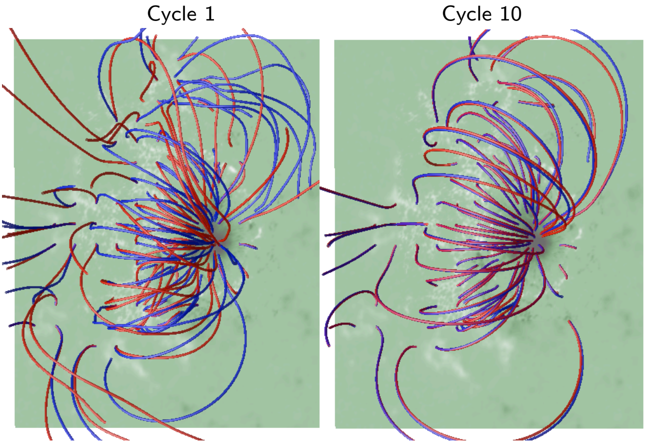


Successful force-free modelling: Self-consistency method

- Alternative approach:
  - Find the "closest" force-free solution to the observed data
- Self-consistency procedure (Wheatland & Régnier 2009, ApJ **700** L88)
  - 1. Construct P and N solutions (current-field iteration)
    - P solution maps boundary values $\alpha_0 \pm \sigma_0$ from $P \to N$
    - N solution maps boundary values $\alpha_0 \pm \sigma_0$ from $N \to P$
    - the two mappings define new boundary values $\alpha_1 \pm \sigma_1$
  - 2. Apply Bayesian decision making
    - given $\alpha_0 \pm \sigma_0$ and $\alpha_1 \pm \sigma_1$, decide most probable value $\alpha_2 \pm \sigma_2$
    - Bayes's theorem: α_2 is an uncertainty-weighted average value
    - α_2 values should be closer to consistency
  - 3. Iterate
    - construct P and N solution starting with $\alpha_2 \pm \sigma_2$
    - each iteration is a "self consistency cycle"
    - should achieve consistency (P and N solutions agree)

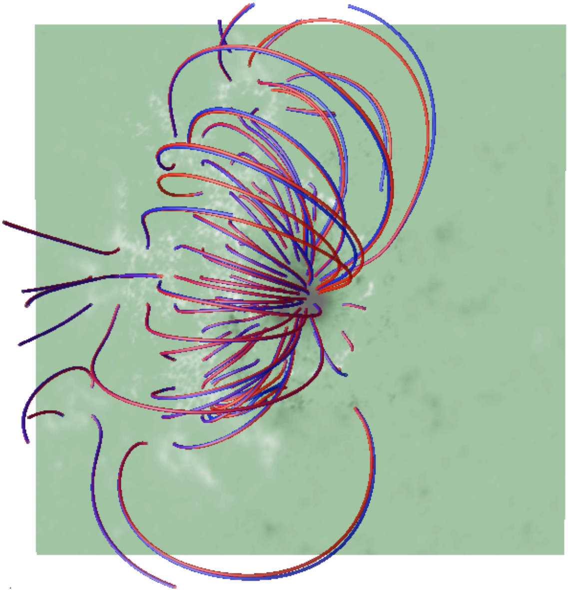
Successful force-free modelling: Application to Hinode

- AR 10953 on 30 April 2007
  - 10 self-consistency cycles
  - 20 current-field iterations for each solution
  - unpreprocessed boundary data

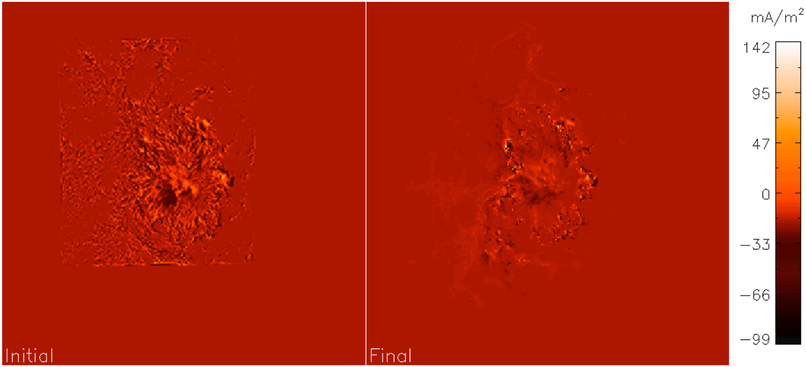


P solutions are in blue, *N* solutions in red

- Self-consistency achieved!
  - quantitative measures confirm convergence



- Currents reduced in magnitude overall by averaging
  - but basic structures remain



- Application a "proof of concept"
  - uncertainties were not assigned
  - Hinode data was embedded in MDI data: undesirable

Magneto-hydrostatic modelling: Grad-Rubin method

- Vector magnetogram data imply non-magnetic forces
  - pressure gradients, flows, gravity forces
- Magneto-hydrostatics is next simplest model

$$\nabla p = \mathbf{J} \times \mathbf{B} \quad \text{and} \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{4}$$

  - inclusion of pressure force
- Spectro-polarimetric data provides thermodynamic information
  - possible to infer p values
    (e.g. Ruiz Cobo & del Toro Iniesta 1992; Degl'Innocenti & Landolfi 2004)
- Grad-Rubin iteration may be applied to (4) (Grad & Rubin 1958)
  - generalisation of current field iteration
  - not substantially more difficult in principle
  - boundary conditions B_n plus p and J_n over one polarity
  - a code is being developed
- Also possible to include a gravity force

Summary

- Vector magnetograms enable coronal field modelling
- Nonlinear force-free model appropriate in the corona
  - but photospheric boundary data is not force-free
  - inconsistency between model, data
  - nonlinear force-free modelling fails!
  - preprocessing is not a solution to the problem
- Self consistency method: successful force-free modelling
  - calculate two possible (inconsistent) solutions
  - use solutions and Bayes's theorem to decide on new BCs on α
  - iterate to achieve consistency
  - demonstrated to work on Hinode/SOT data
- Magneto-hydrostatic modelling
  - solar boundary conditions on pressure may be obtained
  - Grad-Rubin iteration may be applied