Understanding Solar Flare Statistics

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Overview

Observations
- Flare size distribution
- Waiting-time distribution
- Observational difficulties

Physical models for flare statistics
- Classes of models
- A reconnection-based model

Solar flare prediction
- Existing methods of flare prediction
- Event statistics method

Summary

Flare size distribution

- Frequency-size distribution $N(S)dS$: number of flares with size between S and $S+dS$ and per unit time
  - size S: peak flux in X-ray, EUV, or estimated energy
- Power law (Drake 1971):

$$N(S) = AS^{-\gamma}$$

  - $A = A(t)$ describes overall flaring rate
  - $\gamma = 1.5 - 2$ (depends on specific choice of S)
  - observed over > 8 decades in S (Aschwanden et al. 2000)
- A universal law? (See e.g. Bai 1993; Kucera et al. 1997; Sammis 1999; Wheatland 2000)

Waiting-time distribution

- Waiting-time distribution $P(\tau)d\tau$: fraction of times between flares in the range τ to $\tau + d\tau$
- For independent events with a constant mean rate λ (Poisson process):

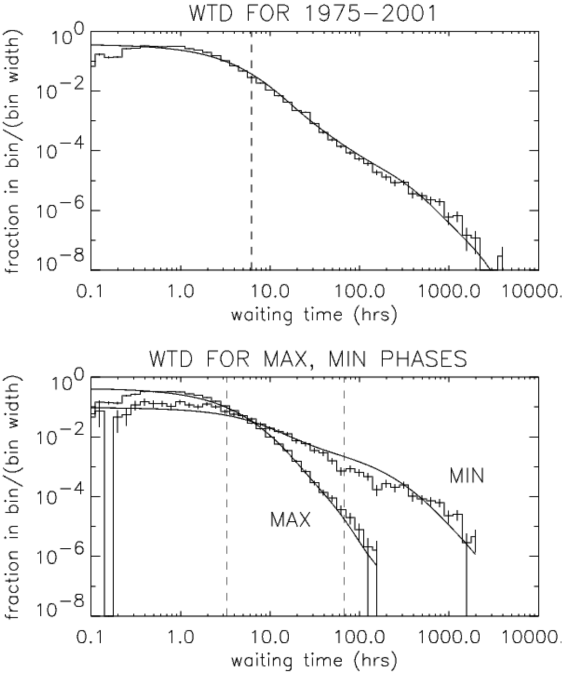
$$P(\tau) = \lambda e^{-\lambda\tau}$$

- Time dependent Poisson process: independent events with a time varying mean rate $\lambda(t)$
  - for a slowly varying rate described by the distribution $f(\lambda)$:

$$P(\tau) = \frac{1}{\bar{\lambda}} \int_0^\infty f(\lambda)\lambda^2 e^{-\lambda\tau}\, d\lambda$$

(Wheatland & Litvinenko 2002)

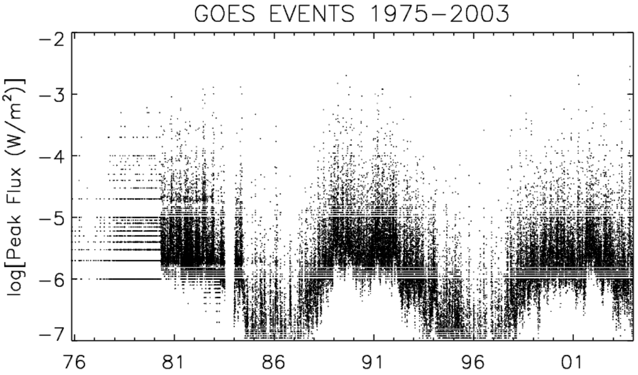
- Departure from Poisson models: evidence for flare sympathy, anti-sympathy (e.g. Moon et al. 2002)
  - also selection effects
- Boffetta et al. (1999): power law $\sim \tau^{-\alpha}$ for long waiting times ($\tau > 10$ hrs)
- Wheatland (2000): power-law tail can be accounted for in terms of a time-varying Poisson process, with the oberved $f(\lambda)$
- Wheatland & Litvinenko (2002): solar cycle variation of the power-law index
  - argues against universality
- Wheatland (2003): similar results for LASCO CMEs



From: Wheatland & Litvinenko (2002)

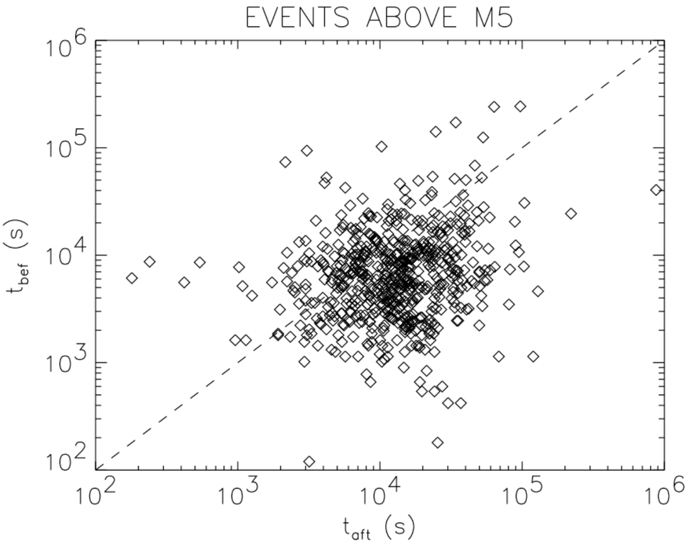
Observational difficulties

- Definitions of events, etc. (e.g. Baeisi et al. 2005)
- Lack of background subtraction for GOES
- GOES selection effects. Event definition: 40% enhancement in flux
  - at times of high background, small events missed
  - time-varying rollover of $N(S)$ at low S



From: Wheatland & Litvinenko (2002)

- obscuration: following a big event, fewer small events (Wheatland 2001)

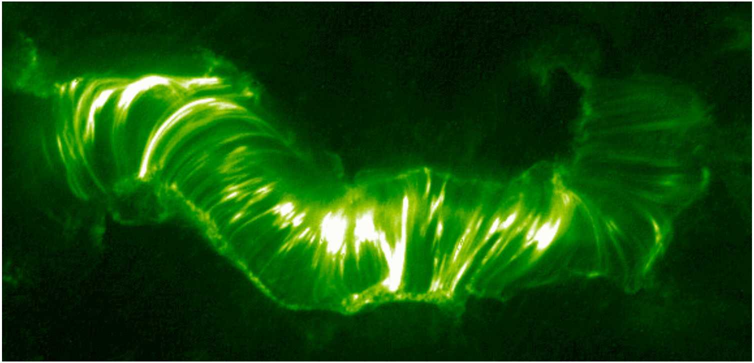


From: Wheatland (2001)

Classes of models

- *Rosner & Vaiana (1978)*: exponential build-up of energy plus random release
  - first order Fermi mechanism
  - modified versions (Litvinenko 1994, 1996; Wheatland & Glukhov 1998)
- *Avalanche models* (Lu & Hamilton 1991; Lu et al. 1993)
  - cellular automaton models — self organised criticality
  - modified versions (e.g. Vassiliadis et al. 1998; Longcope & Noonan 2000; Hughes et al. 2003)
- *MHD turbulence models* (e.g. Longcope & Sudan 1994; Einaudi et al. 1996; Boffetta et al. 1999; Buchlin et al. 2003)
  - natural origin for power-law behavior
  - originally proposed for coronal heating; also argued to apply to flare statistics

- Models divorced from developing ideas of 3D magnetic reconnection (e.g. Lau & Finn 1990)
  - reconnection associated with null points, separators
- Large-scale reconnection explains basic features of large flares (e.g. Somov et al. 2002)



From: http://solar.physics.montana.edu/YPOP/Nuggets/2000/000714/

A reconnection-based model

(Craig & Wheatland 2002, Sol. Phys. 211, 275-287; Wheatland & Craig 2003, ApJ 595, 458-464)

- Multiplicity of flaring sites corresponding to separators
  - individual separator has length scale ℓ
- Mean rate of flaring $\nu(\ell) = v_A/(q\ell)$, $q \sim 10^4$
  - Alfvén transit time governs accumulation of energy in vicinity of separator
- Mean energy $\mathcal{E}(\ell) = Q\ell^2$
  - scaling from flux pile-up separator reconnection solutions (e.g. Litvinenko & Craig 2000)
- Energy distribution at a separator $P(E|\ell)$
  - determined by $\mathcal{E}(\ell)$, $\nu(\ell)$

- Universal distribution of separator lengths $P(\ell)$. For N_s separators, overall frequency-energy distribution

$$N(E) = N_s \int_\ell \nu(\ell)P(E|\ell)P(\ell)d\ell$$

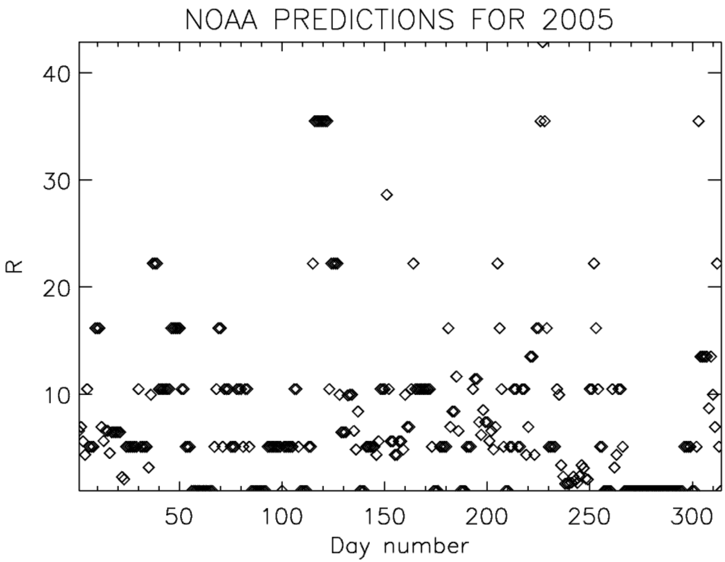
  - separators flare independently
- Reproducing $N(E) \sim E^{-\gamma}$ requires $P(\ell) \sim \ell^{-2(\gamma-1)}$
  - for $\gamma = 1.5$, $P(\ell) \sim \ell^{-1}$
- Model naturally accounts for Poisson waiting times
  - power-law achieved with $N_s = N_s(t)$
- Events at a separator are homologous flares
  - energy distribution $\nu(\ell)P(E|\ell)$ may be observationally testable

Existing methods of flare prediction

- Space weather effects motivate flare prediction
- Existing prediction methods based e.g. on sunspot classification (e.g. McIntosh 1990)
- Methods ignore flare statistics
  - GOES prediction: $\epsilon_{\mathrm{M-X}}$, ϵ_{X} probabilities of ≥ 1 M-X, X class events within a given time
  - For a power-law size distribution it follows that

$$R = \frac{\ln(1 - \epsilon_{\mathrm{M-X}})}{\ln(1 - \epsilon_{\mathrm{X}})}$$

    should be constant


NOAA PREDICTIONS FOR 2005
40
30
20
10
R
50
100
150
200
250
300
Day number

Event statistics method

(Wheatland 2004, ApJ 609, 1134-1139; Wheatland 2005, Space Weather 3, No. 7, S07003)

- S_1=reference size ('small event'), S_2=size of 'big' event
- λ_1=observed rate above S_1; PL size distribution $\Rightarrow$

$$\lambda_2 = \lambda_1 \left(S_1/S_2\right)^{\gamma-1}$$

  (even if no big events are observed)

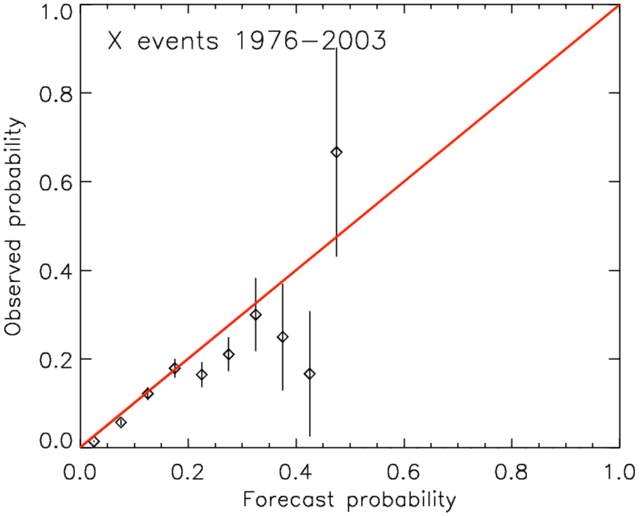
- Poisson probability of ≥ 1 big event in time τ is

$$\epsilon = 1 - \exp(-\lambda_2 \tau)$$

- If M events are involved in the rate estimation then

$$\sigma_\epsilon/\epsilon \approx M^{-1/2}$$

- Bayesian version: determine $P(\epsilon)$ given a sequence of events with sizes $s_1, s_2, \ldots, s_M$ at times $t_1, t_2, \ldots, t_M$
- Application to GOES flares: Wheatland (2005)
  - tested on GOES record 1976-2003



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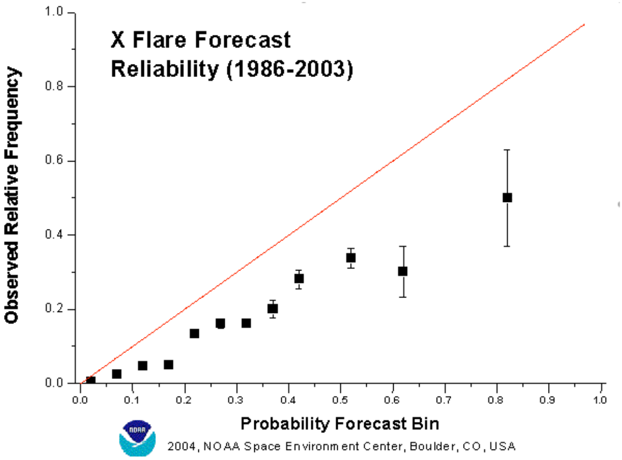

From: `http://www.sec.noaa.gov/forecast_verification`

Summary

- Power-law size distribution $\sim A(t)S^{-\gamma}$
  - appears to be fundamental
- Waiting-time distribution harder to characterize
  - information on inter-dependence, rate variation
  - likely to be less fundamental
- Physical models for the size distribution are difficult to reconcile with models for individual flares
  - one approach presented (Craig & Wheatland 2002; Wheatland & Craig 2003)
- Existing methods of flare prediction ignore the statistical rules
  - approach based on statistics presented (Wheatland 2004; Wheatland 2005)