The magnetic field and its consequences in solar eruptive regions

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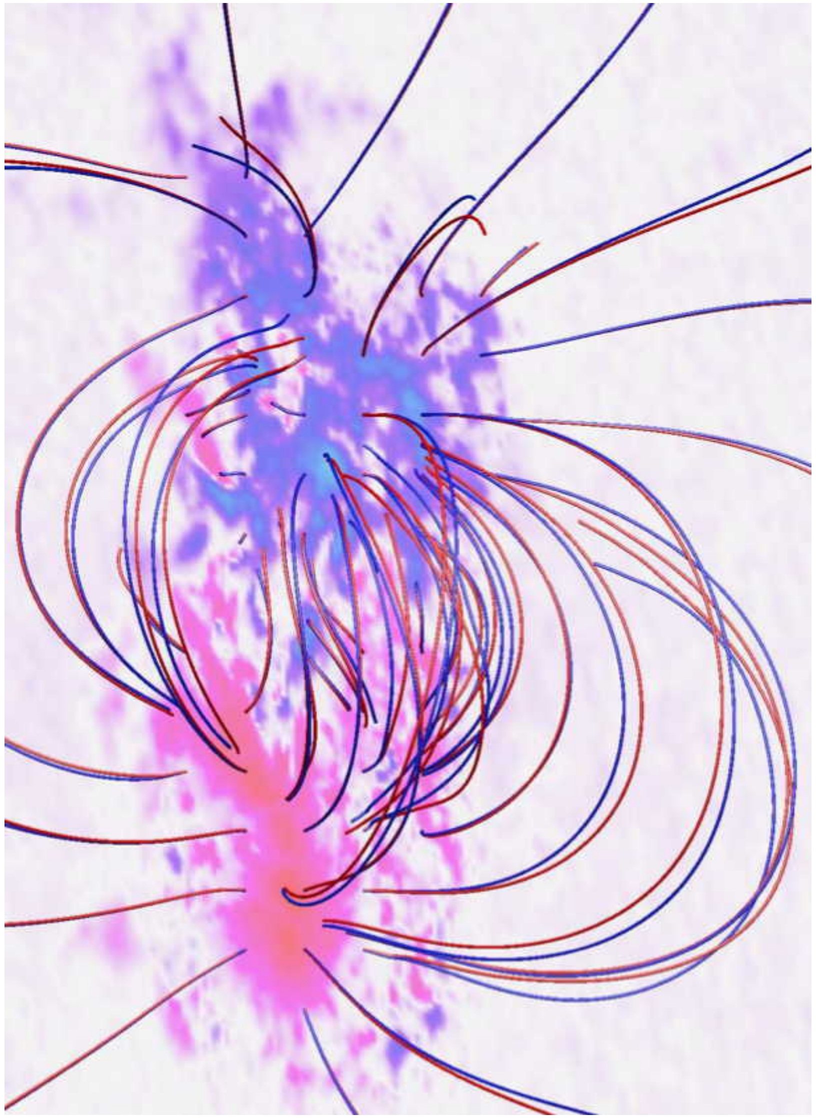


Nonlinear force-free model for AR 11029
(Gilchrist, Wheatland & Leka 2011)

Overview

- *Background*
  - *Flares, eruptions, and space weather*
  - *The data – vector magnetograms*
  - *Nonlinear force-free modeling*
  - *The inconsistency problem*
  - *Self-consistency recipe*
- *Modeling AR 11029*
  - *A dynamic region at deep minimum*
  - *Data*
  - *Results*
- *Modeling eruptive regions*
- *Summary*

Background: Flares, eruptions, and space weather

- Sunspot magnetic fields power large-scale solar activity
  - solar flares, large eruptive events (CMEs)
- Space weather effects motivate modeling
  (US National Research Council workshop report, Baker et al. 2008)
  - potential for large economic losses (Odenwald, Green & Taylor 2006)

SDO 171Å image of AR 11164 (Feb 2011) which produced a number of eruptions (`http://sdo.gsfc.nasa.gov/`)

Background: The data – vector magnetograms

Nobody can measure physical quantities of the solar atmosphere
(Del Toro Iniesta & Ruiz Cobo (1996), Sol. Phys. 164, 169)

- Zeeman effect imprints **B** on photospheric lines (del Toro Iniesta 2003)
  - Stokes polarisation profiles $I(\lambda)$, $Q(\lambda)$, $U(\lambda)$, $V(\lambda)$ measured
  - 'Stokes inversion' is the process of inferring magnetic field
  - an inference rather than a direct measurement/observation
- 180° ambiguity in $B_\perp$ must be resolved
  (Metcalf 1994; Metcalf et al. 2006; Leka et al. 2009)
- Vector magnetogram: photospheric map of $\mathbf{B} = (B_x, B_y, B_z)$
  - local heliocentric co-ordinates (z radially out)
  - common to neglect curvature on active region scale
- Vector magnetograms are not direct measurements/observations
  - inversion results are very method and model dependent

- In principle, VMs give BCs for coronal field modeling
  - referred to as coronal magnetic field reconstruction
- Vertical current density J_z may be estimated at photosphere:

$$\mu_0 J_z\big|_{z=0} = \frac{\partial B_y}{\partial x}\bigg|_{z=0} - \frac{\partial B_x}{\partial y}\bigg|_{z=0} \tag{1}$$

- New generation of instruments
  - US NSO Synoptic Long-term Investigations of the Sun
    - Vector Spectro-magnetograph (SOLIS/VSM)
      (Jones et al. 2002)
  - Hinode satellite
    - Solar Optical Telescope Spectro-Polarimeter (SOT/SP)
      (Tsuneta et al. 2008)
  - Solar Dynamics Observatory satellite
    - Helioseismic & Magnetic Imager (SDO/HMI)
      (Scherrer et al. 2006)

Background: Nonlinear force-free modeling

- Force-free model for coronal magnetic field **B**:

$$\mathbf{J} \times \mathbf{B} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{2}$$

  - $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is electric currrent density
  - physics: static model in which Lorentz force dominates
  - coupled nonlinear PDEs
- Writing $\mathbf{J} = \alpha \mathbf{B}/\mu_0$ (**J** is parallel to **B**):

$$\mathbf{B} \cdot \nabla \alpha = 0 \quad \text{and} \quad \nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{3}$$

  - α is the force-free parameter

Mini glossary
Model: a solution to the force-free model
Solution: a solution to the model

- Boundary conditions: (Grad & Rubin 1958)
  - B_z over $z = 0$
  - α over $z = 0$ where $B_z > 0$ *or* where $B_z < 0$
    - α is prescribed over one polarity
    - we refer to the polarities as P and N respectively
- Vector magnetograms give two sets of boundary conditions
  - values of $\alpha = \mu_0 J_z / B_z$ over both P and N are available
- Methods of solution of Eqs. (3) are iterative (e.g. Wiegelmann 2008)
- Current-field iteration/Grad-Rubin iteration (Grad & Rubin 1958)
  - at iteration k solve the linear system

$$\mathbf{B}^{[k-1]} \cdot \nabla \alpha^{[k]} = 0 \quad \text{and} \quad \nabla \times \mathbf{B}^{[k]} = \alpha^{[k]} \mathbf{B}^{[k-1]} \qquad (4)$$

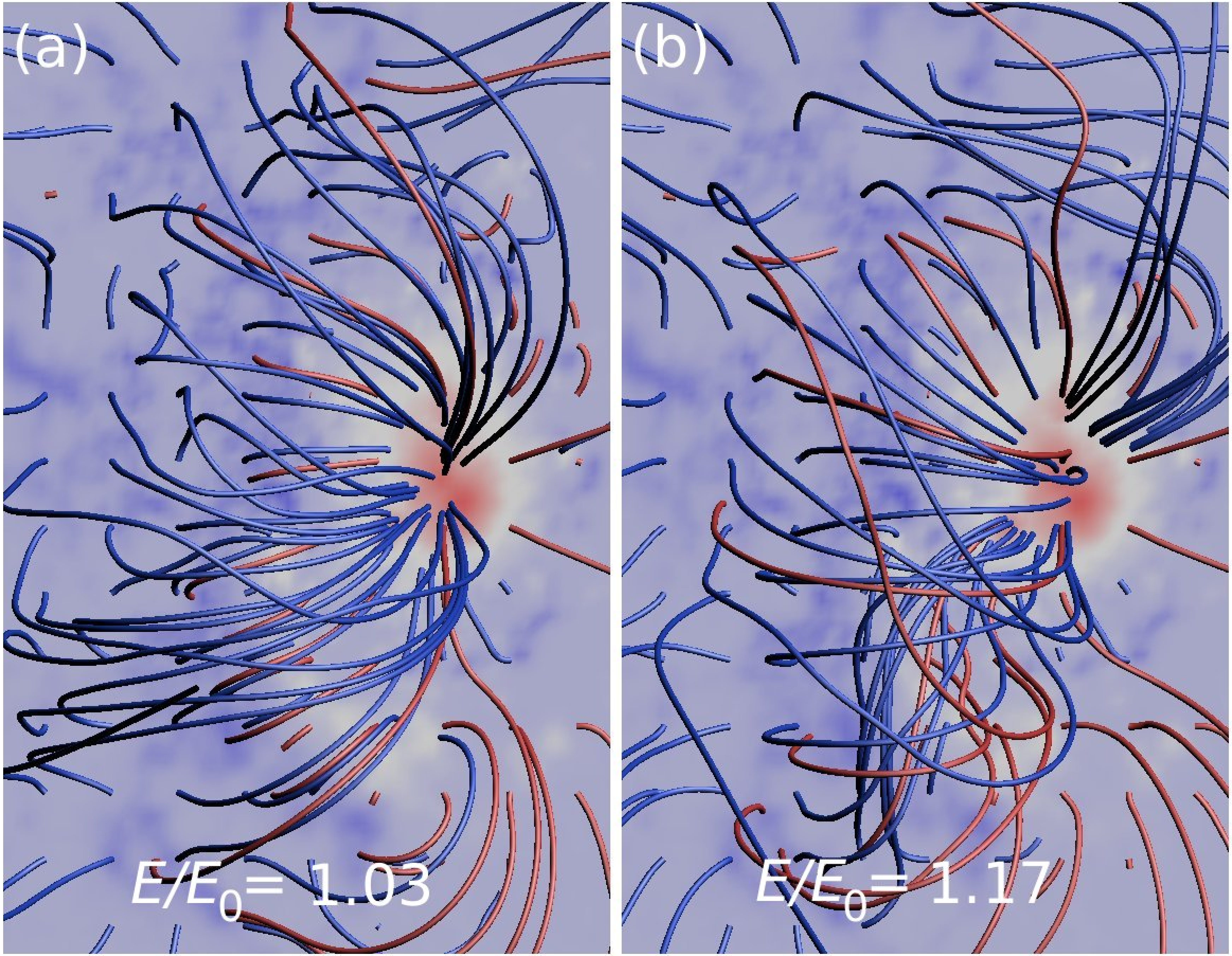
  - BCs imposed on $B_z^{[k]}$ and on $\alpha^{[k]}$ over P or N

Mini glossary
P solution: a solution using α values over $z = 0$ where $B_z > 0$
N solution: a solution using α values over $z = 0$ where $B_z < 0$

Background: The inconsistency problem

- Force-free methods work for test cases but fail for solar data (Schrijver et al. 2006; Metcalf et al 2008; Schrijver et al. 2008; DeRosa et al. 2009)
  - e.g. P and N solutions do not agree for a Grad-Rubin method
  - some force-free methods use $\mathbf{B}|_{z=0}$ as BCs (Wheatland, Sturrock & Roumeliotis 2000; Wiegelman 2000)
    - the 'solutions' have $\mathbf{J} \times \mathbf{B} \neq 0$ and/or $\nabla \cdot \mathbf{B} \neq 0$ somewhere
- Vector magnetogram BCs inconsistent with force-free model
  - errors in measurements and field inference
  - field at photospheric level is not force free (Metcalf et al. 1995)
  - necessary conditions for a force-free field are not met (Molodenskii 1969)
- 'Preprocessing' does not solve this problem
  - 'preprocess': modify BCs to meet necessary model conditions (Wiegelmann et al. 2006)
  - preprocessed BCs remain inconsistent with the model (DeRosa et al. 2009)
- In general different energies for P and N solutions

- Illustration of the problem: AR 10953 on 30 June 2007



Inconsistent solutions from vector magnetogram BCs: (a) *P* solution; (b) *N* solution (Wheatland & Leka 2011)

Background: Self-consistency recipe

(Wheatland & Régnier 2009; Wheatland & Leka 2011)

1. Calculate P and N solutions using Grad-Rubin (Wheatland 2006; 2007)
   - BCs: unpreprocessed vector magnetogram data
2. Adjust boundary values using solutions and uncertainties
   - Each solution has α constant along **B**...
   - ...so they define two sets of α values at $z = 0$:

$$\alpha_P \pm \sigma_P \quad \text{and} \quad \alpha_N \pm \sigma_N \tag{5}$$

   - Each is consistent with the force-free model
   - Bayesian probability is used to estimate 'true' values:

$$\alpha_{\text{est}} = \frac{\alpha_P/\sigma_P^2 + \alpha_N/\sigma_N^2}{1/\sigma_P^2 + 1/\sigma_N^2} \quad \sigma_{\text{est}} = \left(1/\sigma_P^2 + 1/\sigma_N^2\right)^{-\frac{1}{2}} \tag{6}$$

   - Still inconsistent but closer to consistency

3. Iterate 1. & 2. until P and N solutions agree ($\alpha_{\rm est}$ consistent)

- Step 1. uses $\alpha_{\rm est}$ for BCs at subsequent iterations

Mini glossary
Iteration: one step in a procedure, e.g. a Grad-Rubin step from $k \rightarrow k+1$
Self-consistency cycle: sequence of G-R iterations to produce P and N solutions

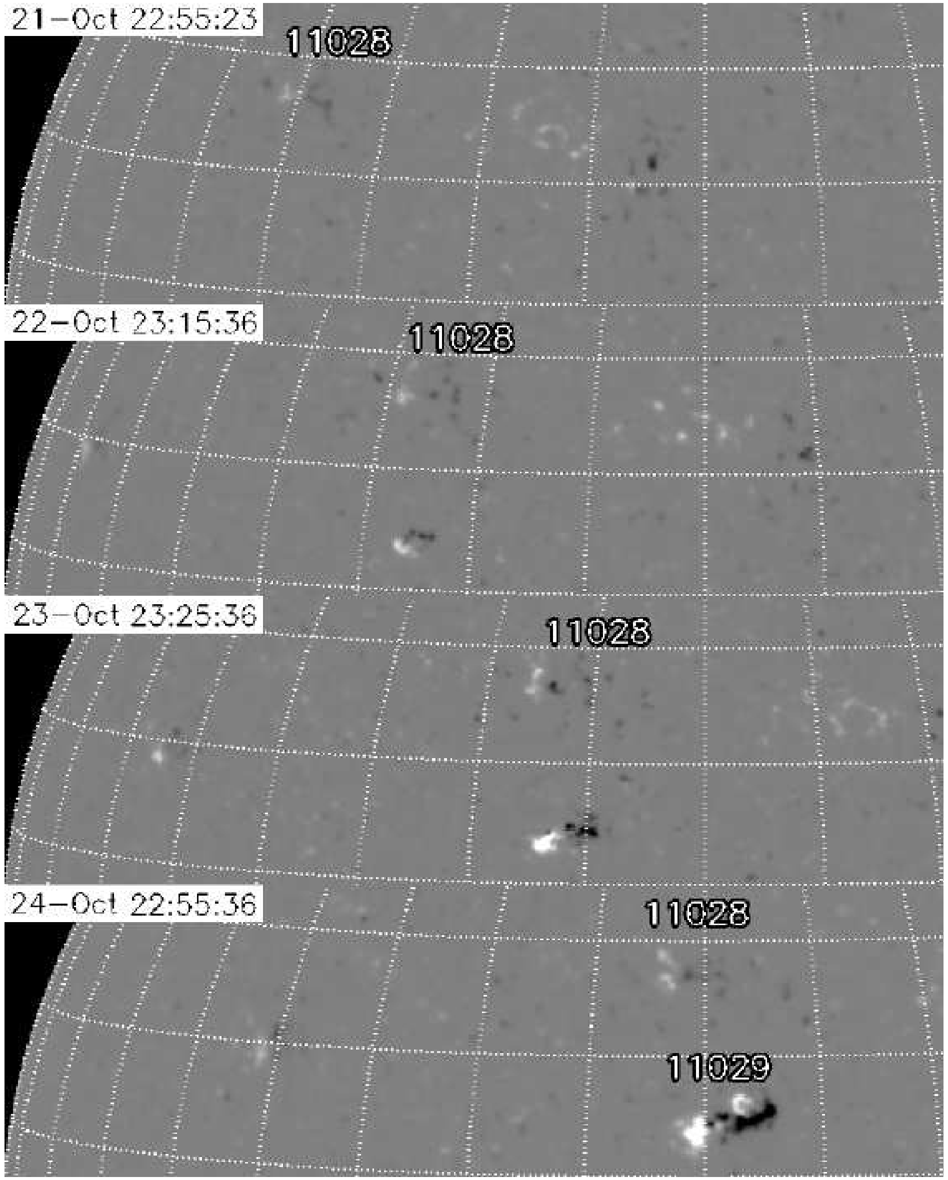
- Self consistency provides a single energy value
- Method previously applied to AR 10953

  (Wheatland & Régnier 2009; Wheatland & Leka 2011)

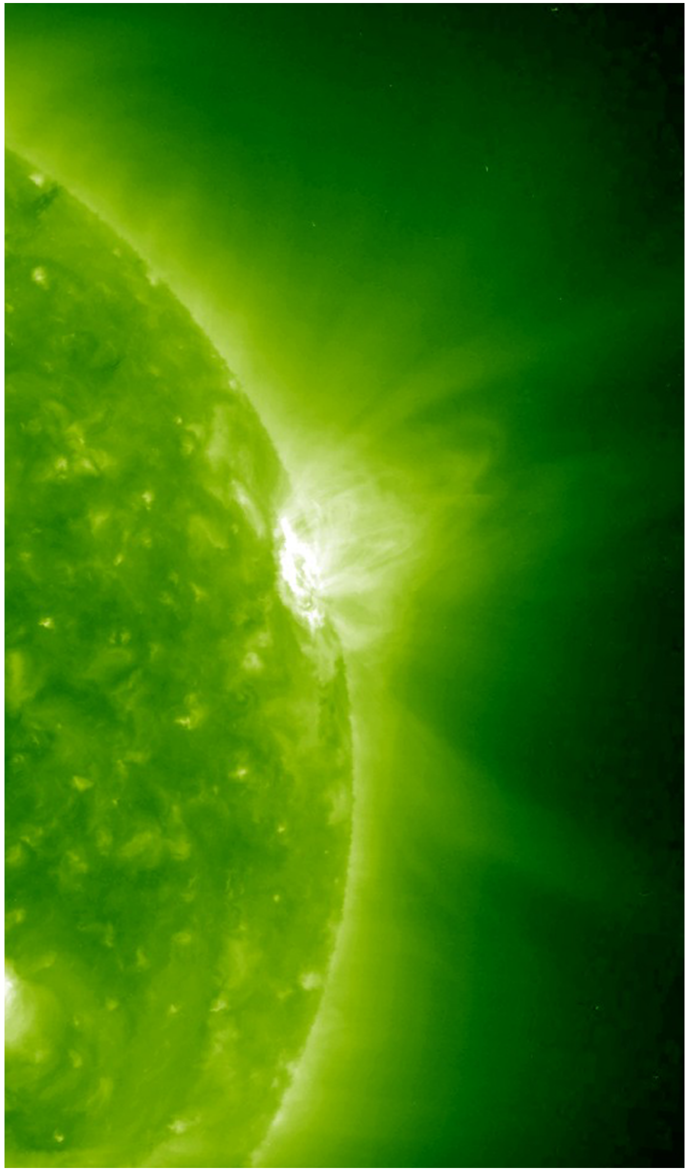
Modeling AR 11029: A dynamic region at deep minimum

(Wheatland 2011)

- Active region 11029 emerged on the disk on 21-22 Oct 2009

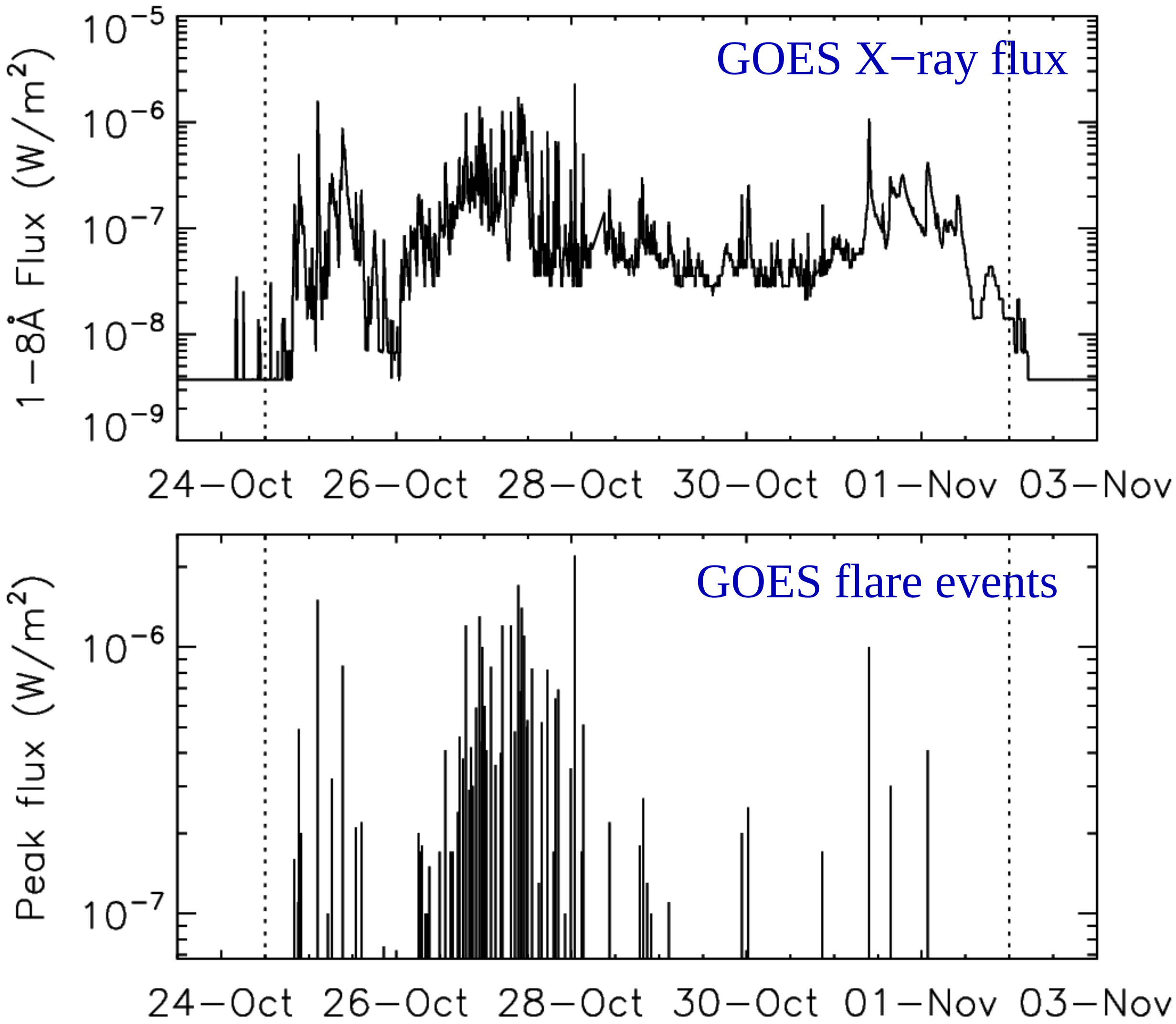


Line-of-sight magnetic field 21-24 Oct (`www.solarmonitor.org`)



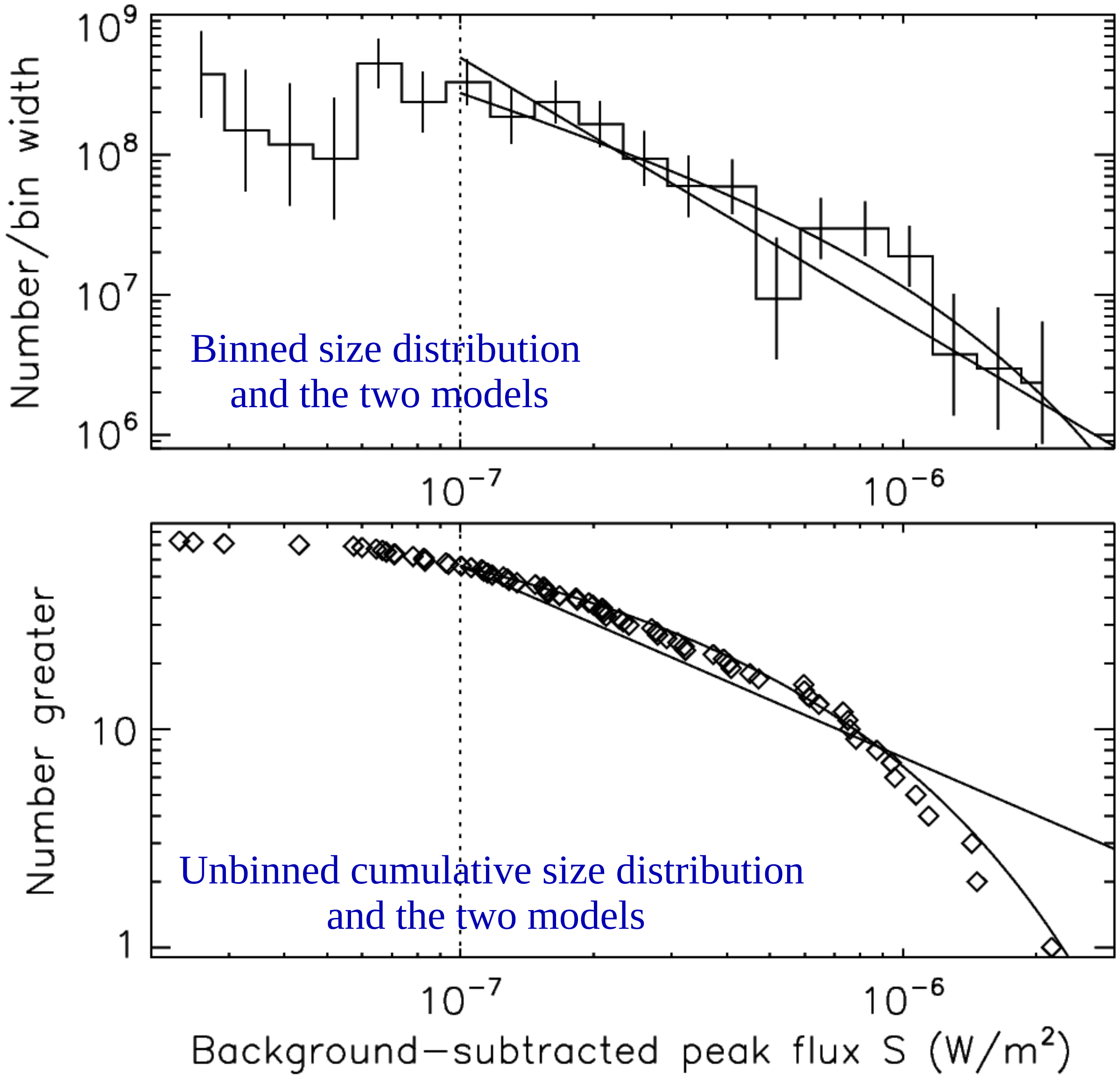
STEREO A on (`sohowww.nascom.nasa.gov`)

- Highly flare-productive but small ($< 400\,\mu$-hemispheres)
  - observed at a time with very low soft X-ray background
  - 73 small GOES events: one A-class, 60 B-class, and 11 C-class
  - produced many eruptions (SOHO LASCO CME catalog)



Time history of X-rays from AR 11029, and the 73 flare events for the region (Wheatland 2011)

- Largest flare was C2.2
  - a departure from the power-law flare size[1] distribution?



Peak-flux distributions for GOES events and power-law/power-law plus rollover models (Wheatland 2011)

[1] Size S: a measure of the magnitude, e.g. peak GOES flux, which is a proxy for energy.

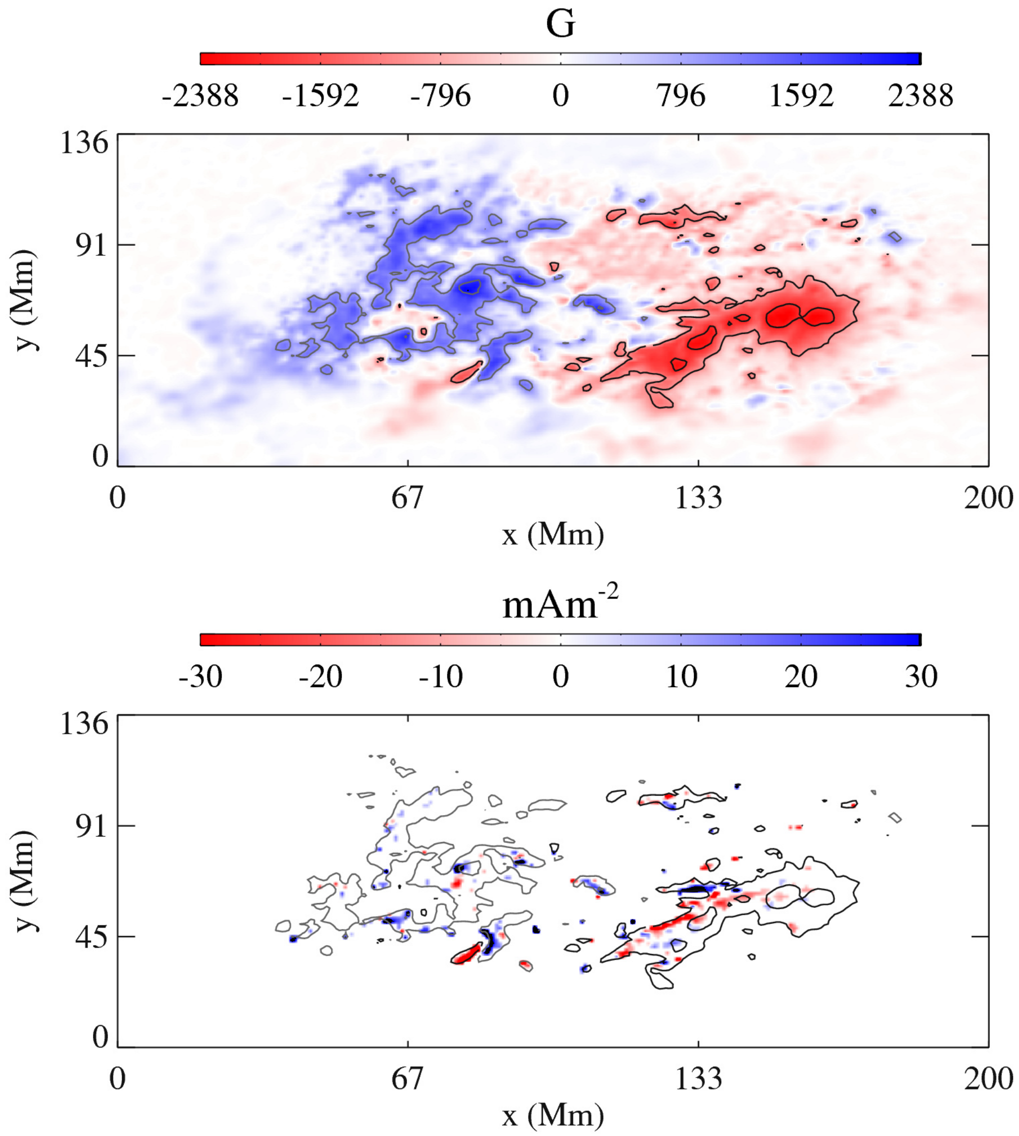
- Flares obey a power-law size distribution: (e.g. Akabane 1956)

$$f(S) = AS^{-\gamma} \tag{7}$$

  - $f(S)$ is number of flares per unit time, per unit S
  - power-law index $\gamma \approx 1.5$–2
  - universal: same index at different times, in different regions
- An upper limit to the power law must exist
  - there is a finite amount of energy available for flaring
  - however it has proven very hard to identify this
  - some evidence based on *many* small regions (e.g. Kucera et al. 1997)
- Is the AR 11029 distribution revealing a limit on the energy?
- Idea: estimate the ‘free’ magnetic energy of the region...
  - ...from self-consistent nonlinear force-free modeling
  - this provides an upper limit to the energy of the largest flare
  - how does it compare with the largest observed flare?

Modeling AR 11029: Data (Gilchrist, Wheatland & Leka 2011)

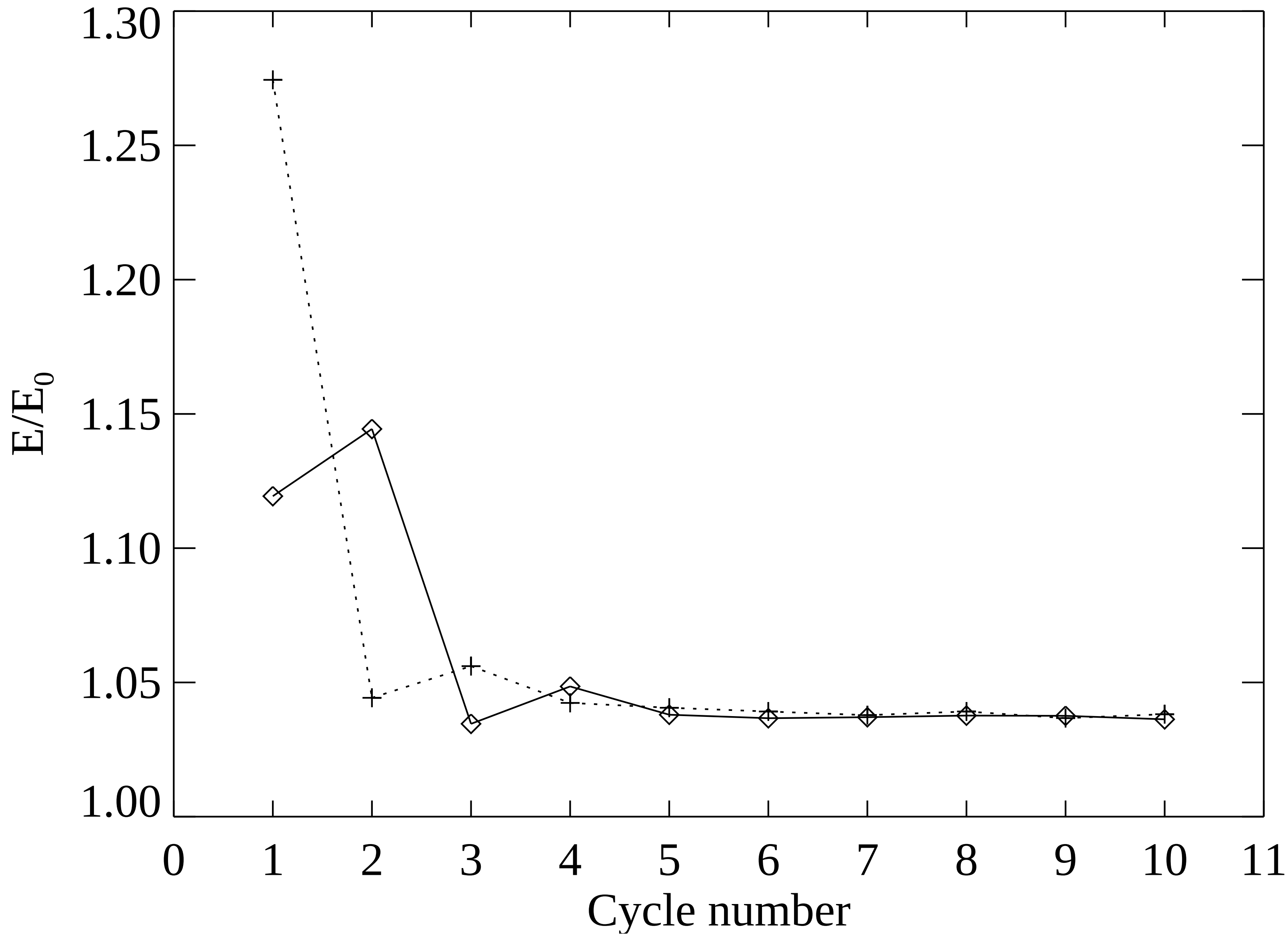
- Magnetogram based on Hinode SP and MDI data (27 Oct)
  - uncertainties from Stokes inversion



Boundary conditions on B_z (upper) and J_z (lower) (Gilchrist, Wheatland & Leka 2011)

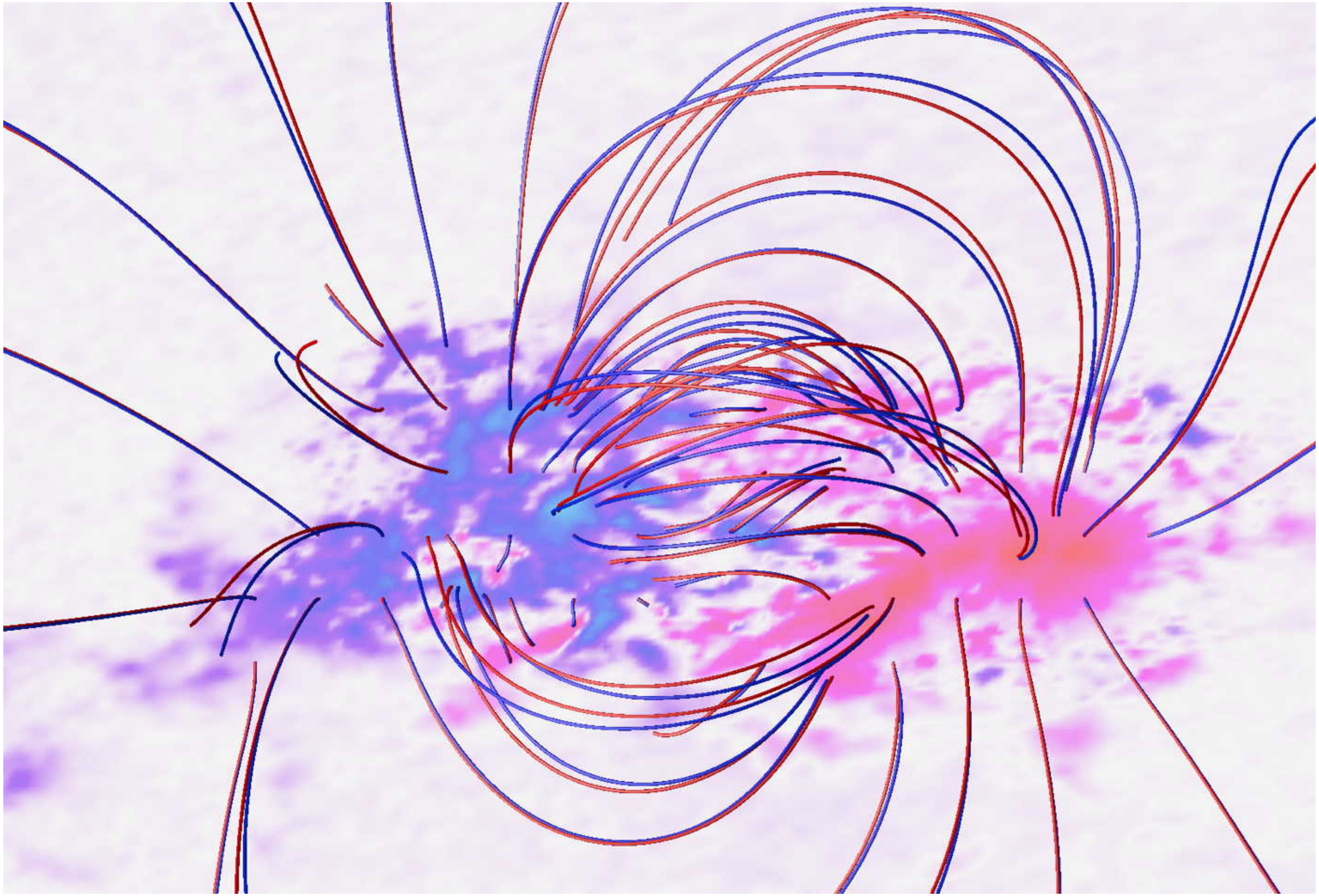
Modeling AR 11029: Results (Gilchrist, Wheatland & Leka 2011)

- Convergence in energy of self-consistency procedure



Energy of P solution ($+$) and N solution ($\diamond$) versus self-consistency cycle (Gilchrist, Wheatland & Leka 2011)

- Self-consistent solution from Hinode/MDI data
  - calculation on a $440 \times 300 \times 200$ grid
  - 20 Grad-Rubin iterations per cycle

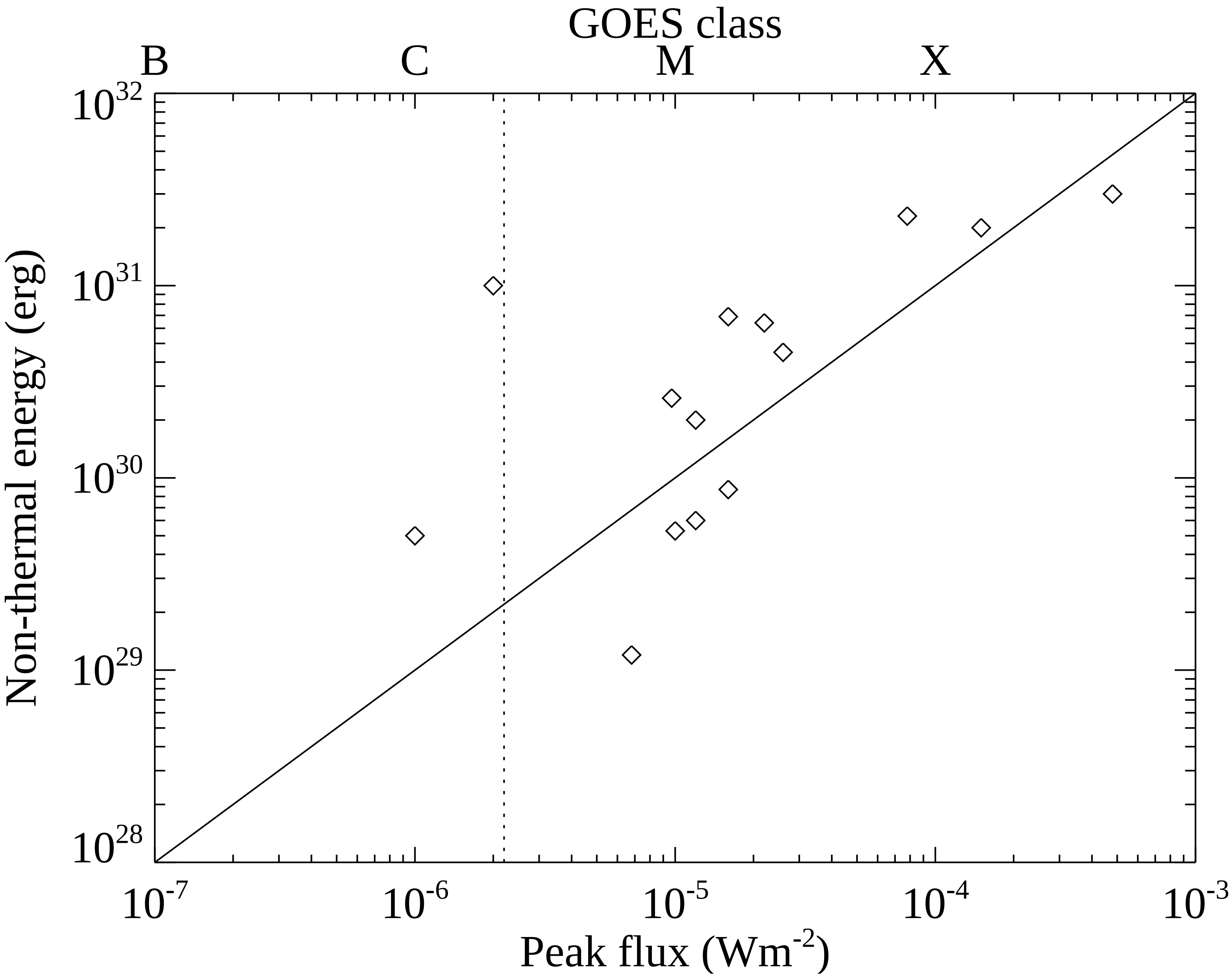


Self-consistent *P* solution (blue curves) and *N* solution (red curves) (Gilchrist, Wheatland & Leka 2011)

- Energy of self-consistent solution $E/E_0 = 1.04$
  - large potential field energy: $E_0 = 1.7 \times 10^{33}$ erg
  - free energy $E_f = E - E_0 = 6 \times 10^{31}$ erg
- Early self-consistency cycles do not converge strictly
  - oscillations in energy (a symptom of inconsistency)
  - introduces some arbitrariness in the modeling
  - results depend on the number N_{GR} of GR iterations
- Modeling repeated with $N_{GR} = 30$
  - results very similar which suggests the process is robust
  - order of magnitude free energy estimate: $E_f \sim 10^{32}$ erg

G-R iterations	Sol.	E (10^{33} erg)	E_0 (10^{33} erg)	$E_f = E - E_0$ (10^{31} erg)
20	P	1.769	1.707	6.16
	N	1.772	1.707	6.50
30	P	1.787	1.707	7.94
	N	1.791	1.707	8.35

- Energy-GOES peak flux scaling from the literature

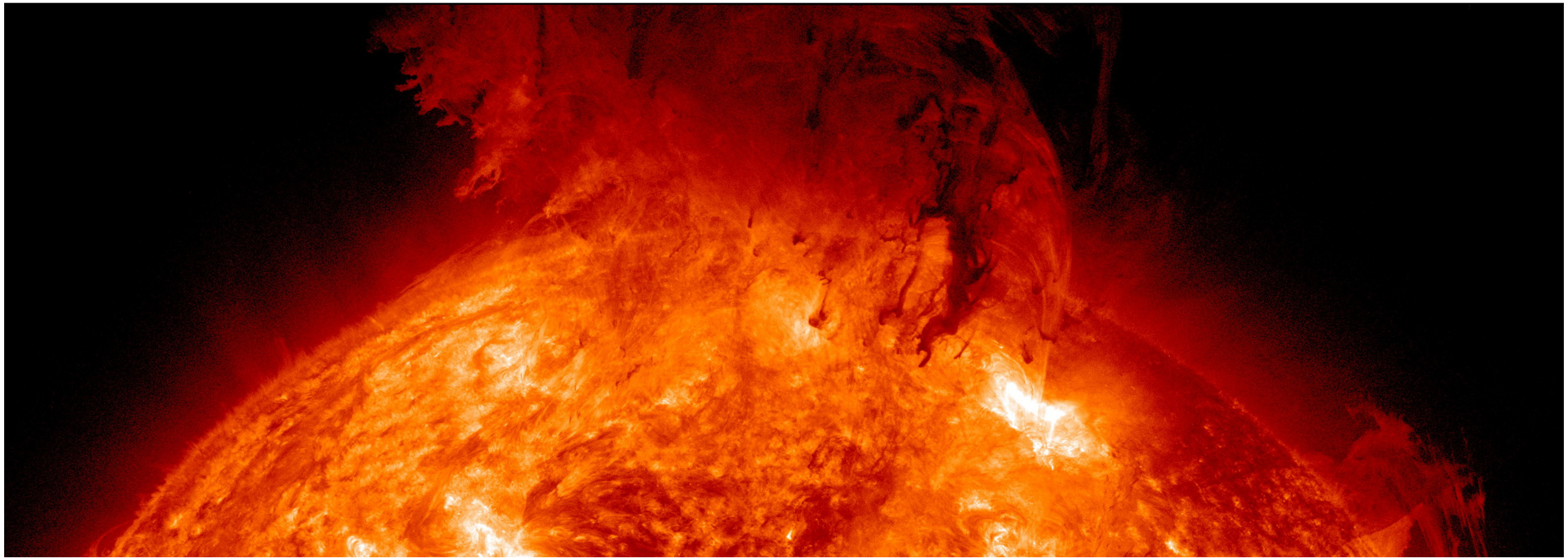


RHESSI nonthermal electron energy estimates versus GOES peak flux for 14 flares (Gilchrist, Wheatland & Leka 2011)

- Recall the hypothesis:
  - absence of large GOES events due to limited energy of region?
- But $E_f \sim 10^{32}$ erg is consistent with an X-class flare
  - the largest observed flare was C2.2
  - hence the results do not support the hypothesis
- SOLIS/VSM vector magnetogram data for 24 Oct available
  - the region was newly emerged and smaller at this time
  - the flaring rate was much smaller
- Self-consistent solution energy for 24 Oct: $E \sim 10^{29}$ erg
  - consistent with C- or M-class flare energy

Modeling eruptive regions

- Force-free model is static so eruption is not described
- However – for magnetograms before and after eruptions:
  - construct self-consistent solutions
  - investigate e.g. changes in connectivity, energy
- Energy estimates may assist in forecasting eruptions...
  - ...or constraining 'largest possible' event
- Global nature of many eruptions a difficulty for modeling
  - SDO shows separate regions on disk often involved
  - full disk modeling based on data is needed



SDO 304Å image of June 7 2011 eruptive event (http://sdo.gsfc.nasa.gov/)

Summary

- Vector magnetograms give BCs for coronal field modeling
  - but the modeling is difficult
- The nonlinear force-free model is popular
  - but vector magnetogram data are inconsistent with the model
  - the model gives unreliable results for solar data
  - the self-consistency procedure provides one solution...
  - ...with a unique energy
- Self-consistency modeling for AR 11029
  - motivated by non power-law flare size distribution
  - hypothesis: evidence for an upper limit to region energy?
- Self-consistent magnetic free energy on 27 Oct: $E_f \sim 10^{32}$ erg
  - based on Hinode SOT/SP magnetogram
  - consistent with X-class event
  - does not support hypothesis
- Application of self-consistency modeling to eruptions discussed