Modelling magnetic fields in the corona using nonlinear force-free fields

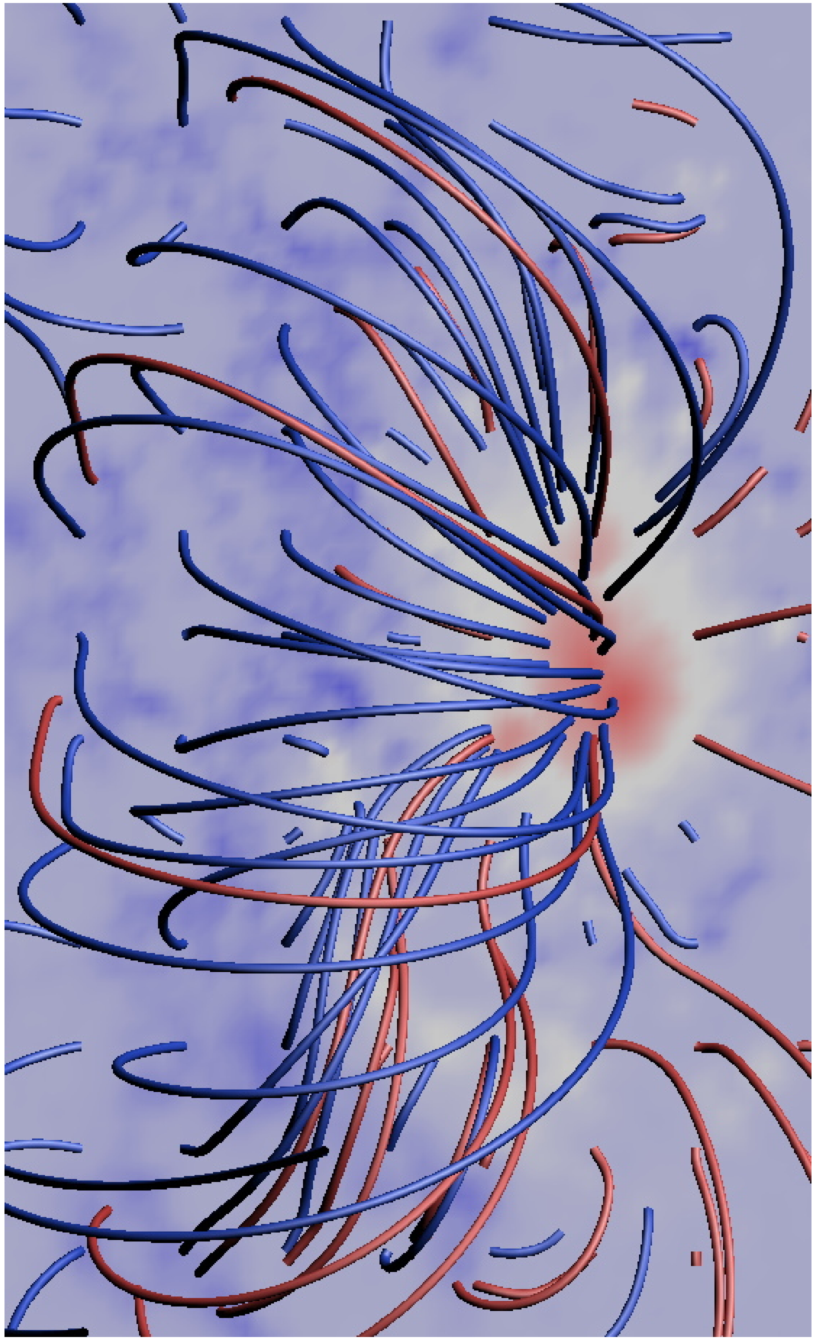
M. S. Wheatland[1] and K. D. Leka[2]

[1]School of Physics
Sydney Institute for Astronomy
The University of Sydney

[2]North West Research Associates
Colorado Research Associates Division

1st APSPM, Indian Institute of Astrophysics
Bangalore, March 2011

THE UNIVERSITY OF SYDNEY



Model for AR 10953 field (Wheatland & Leka 2011)

Overview

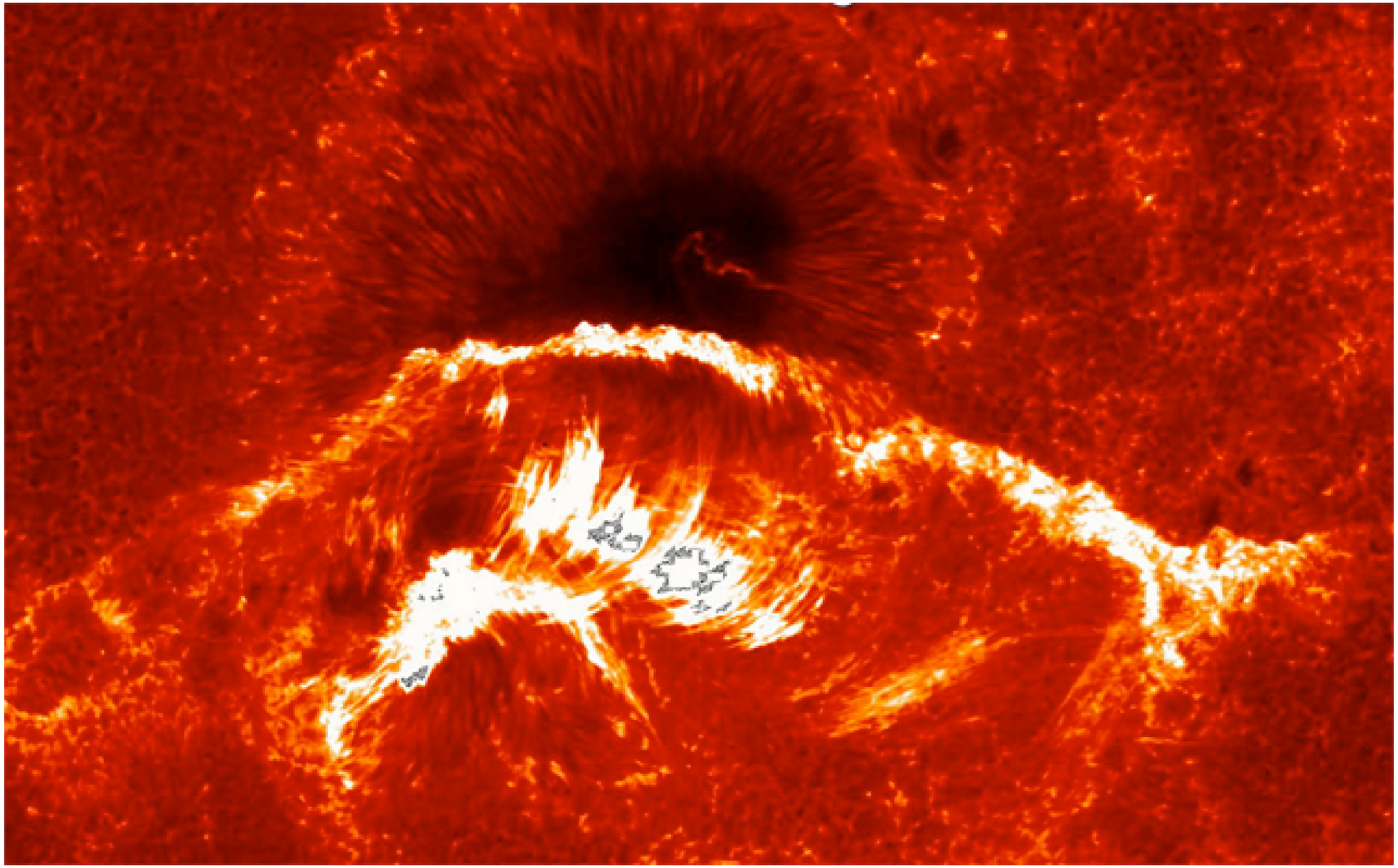
Background
- *Flares, CMEs and space weather*
- *The data – vector magnetograms*
- *Nonlinear force-free modelling*
- *The inconsistency problem*
- *Self-consistency recipe*

Modelling AR 10953 with uncertainties
- *Data*
- *Results*

Summary

Background: Flares, CMEs and space weather

- Sunspot magnetic fields power large-scale solar activity
  - solar flares, Coronal Mass Ejections
- Space weather effects motivate modelling
  (US National Research Council workshop report, Baker et al. 2008)
  - potential for large economic losses (Odenwald, Green & Taylor 2006)

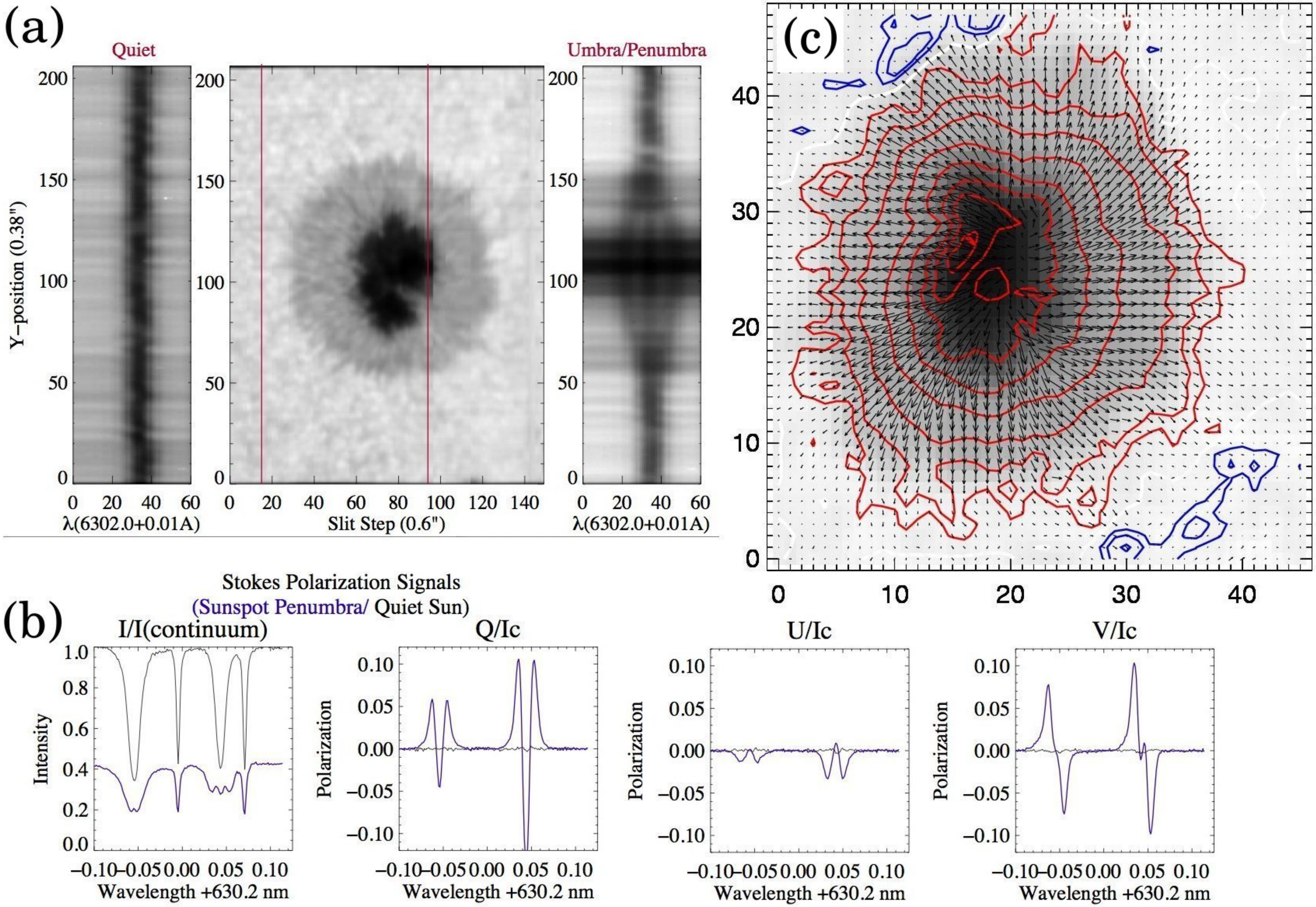


12 Dec 2006 X-class flare (Hinode/SOT)

Background: The data – vector magnetograms

Nobody can measure physical quantities of the solar atmosphere
(Del Toro Iniesta & Ruiz Cobo (1996), Sol. Phys. 164, 169)

- Zeeman effect imprints **B** on photospheric lines (del Toro Iniesta 2003)
    - Stokes polarisation profiles $I(\lambda)$, $Q(\lambda)$, $U(\lambda)$, $V(\lambda)$ measured
    - 'Stokes inversion' is the process of inferring magnetic field
    - an inference rather than a direct measurement/observation
- 180° ambiguity in $B_\perp$ must be resolved
  (Metcalf 1994; Metcalf et al. 2006; Leka et al. 2009)
- Vector magnetogram: photospheric map of $\mathbf{B} = (B_x, B_y, B_z)$
    - local heliocentric co-ordinates (z radially out)
    - common to neglect curvature on active region scale
- Vector magnetograms are not direct measurements/observations
    - inversion results are very method and model dependent



The data: (a) Sunspot image and line observations; (b) Stokes profiles for sunspot and quiet Sun observations; (c) vector magnetogram field values (Advanced Stokes Polarimeter/Imaging Vector Magnetograph)

- In principle, VMs give BCs for coronal field modelling
  - referred to as coronal magnetic field reconstruction
- Vertical current density J_z may be estimated at photosphere:

$$\mu_0 J_z\big|_{z=0} = \left.\frac{\partial B_y}{\partial x}\right|_{z=0} - \left.\frac{\partial B_x}{\partial y}\right|_{z=0} \qquad (1)$$

- New generation of instruments
  - US NSO Synoptic Long-term Investigations of the Sun Vector Spectro-magnetograph (SOLIS/VSM) (Jones et al. 2002)
  - Hinode Solar Optical Telescope Spectro-Polarimeter (SOT/SP) (Tsuneta et al. 2008)
  - Solar Dynamics Observatory Helioseismic & Magnetic Imager (SDO/HMI) (Scherrer et al. 2006)

Background: Nonlinear force-free modelling

- Force-free model for coronal magnetic field:

$$\mathbf{J} \times \mathbf{B} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{2}$$

  - $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is electric currrent density
  - physics: static model in which Lorentz force dominates
  - coupled nonlinear PDEs
- Writing $\mathbf{J} = \alpha \mathbf{B}/\mu_0$ ($\mathbf{J}$ is parallel to $\mathbf{B}$):

$$\mathbf{B} \cdot \nabla \alpha = 0 \quad \text{and} \quad \nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{3}$$

  - α is the force-free parameter

- Boundary conditions: (Grad & Rubin 1958)
  - B_z over $z = 0$
  - α over $z = 0$ where $B_z > 0$ *or* where $B_z < 0$
    - over one polarity
    - we refer to the polarities as P and N respectively
- Vector magnetograms give two sets of boundary conditions
  - values of $\alpha = \mu_0 J_z / B_z$ over both P and N are available
- Methods of solution of Eqs. (3) are iterative (e.g. Wiegelmann 2008)
- Current-field iteration/Grad-Rubin iteration (Grad & Rubin 1958)
  - at iteration k solve the linear system

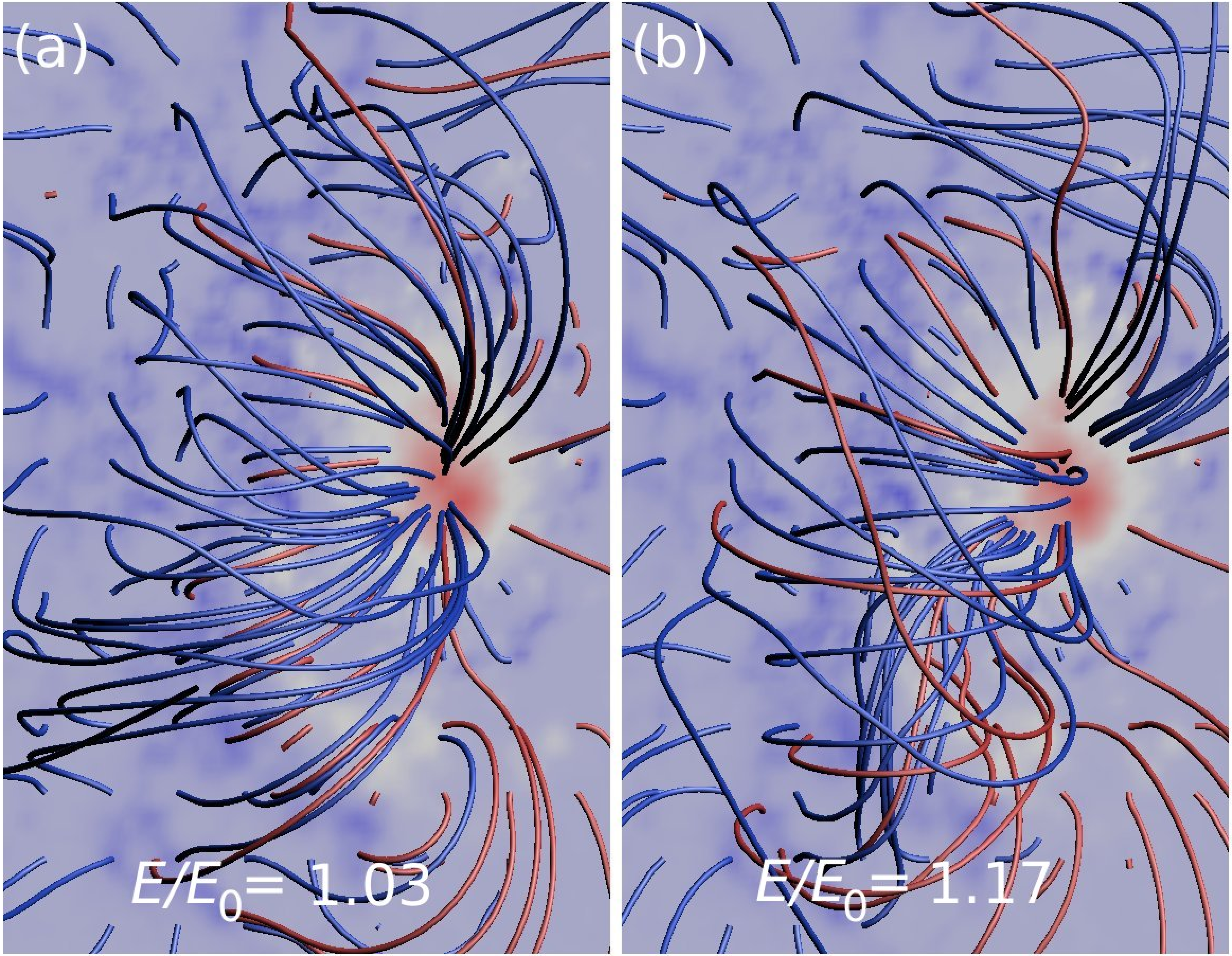
$$\mathbf{B}^{[k-1]} \cdot \nabla \alpha^{[k]} = 0 \quad \text{and} \quad \nabla \times \mathbf{B}^{[k]} = \alpha^{[k]} \mathbf{B}^{[k-1]} \tag{4}$$

  - BCs imposed on $B_z^{[k]}$ and on $\alpha^{[k]}$ over P or N

Background: The inconsistency problem

- Force-free methods work for test cases but fail for solar data (Schrijver et al. 2006; Metcalf et al 2008; Schrijver et al. 2008; DeRosa et al. 2009)
    - different methods give different fields
    - *P* and *N* solutions do not agree for a Grad-Rubin method
    - some force-free methods use $\mathbf{B}|_{z=0}$ as BCs (Wheatland, Sturrock & Roumeliotis 2000; Wiegelman 2000)
        - the 'solutions' have $\mathbf{J} \times \mathbf{B} \neq 0$ and/or $\nabla \cdot \mathbf{B} \neq 0$
        - they fail to solve the model
- Vector magnetogram BCs inconsistent with force-free model
    - errors in measurements and field inference
    - field at photospheric level is not force free (Metcalf et al. 1995)
    - necessary conditions for a force-free field are not met (Molodenskii 1969)
- Force-free models from vector magnetograms are unreliable

- Illustration of the problem: AR 10953 on 30 June 2007



Inconsistent solutions from vector magnetogram BCs: (a) *P* solution; (b) *N* solution (Wheatland & Leka 2011)

- One approach to the problem is ‘preprocessing’(Wiegelmann et al. 2006)
  - BCs modified to satisfy necessary force-free conditions...
  - ...but they are necessary, not sufficient
  - preprocessed BCs are inconsistent with the force-free model (DeRosa et al. 2009)
  - this procedure typically also smooths, which is undesirable
- Alternative approach:
  - find the ‘closest’ force-free solution to the observed data

Background: Self-consistency recipe (Wheatland & Régnier 2009)

1. Calculate P and N solutions using Grad-Rubin (Wheatland 2006; 2007)
   - BCs: unpreprocessed vector magnetogram data
2. Adjust boundary values using solutions and uncertainties
   - Each solution has α constant along **B**...
   - ...so they define two sets of α values at $z = 0$:

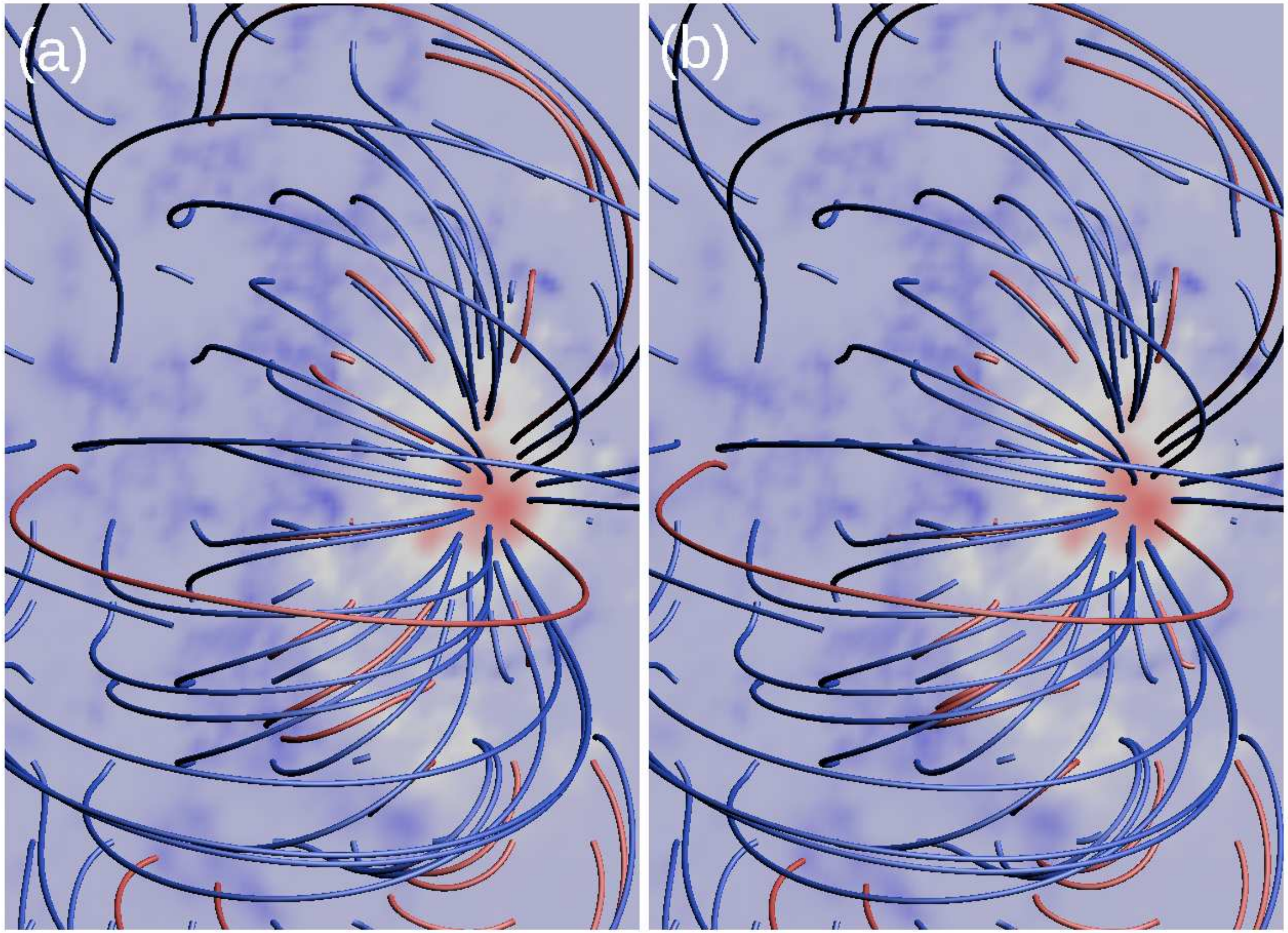
$$\alpha_P \pm \sigma_P \quad \text{and} \quad \alpha_N \pm \sigma_N \tag{5}$$

   - Each is consistent with the force-free model
   - Bayesian probability is used to estimate 'true' values:

$$\alpha_{\text{est}} = \frac{\alpha_P/\sigma_P^2 + \alpha_N/\sigma_N^2}{1/\sigma_P^2 + 1/\sigma_N^2} \quad \sigma_{\text{est}} = \left(1/\sigma_P^2 + 1/\sigma_N^2\right)^{-\frac{1}{2}} \tag{6}$$

   - Still inconsistent but closer to consistency
3. Iterate 1. & 2. until P and N solutions agree (α_{est} consistent)
   - Step 1. uses α_{est} for BCs at subsequent iterations

- Initial test on AR 10953 (Wheatland & Régnier 2009)
  - method shown to work: a 'proof of concept'
  - but uncertainties not included
  - self-consistent solution near to potential
    - energy $E/E_0 = 1.02$ (potential field energy is E_0)

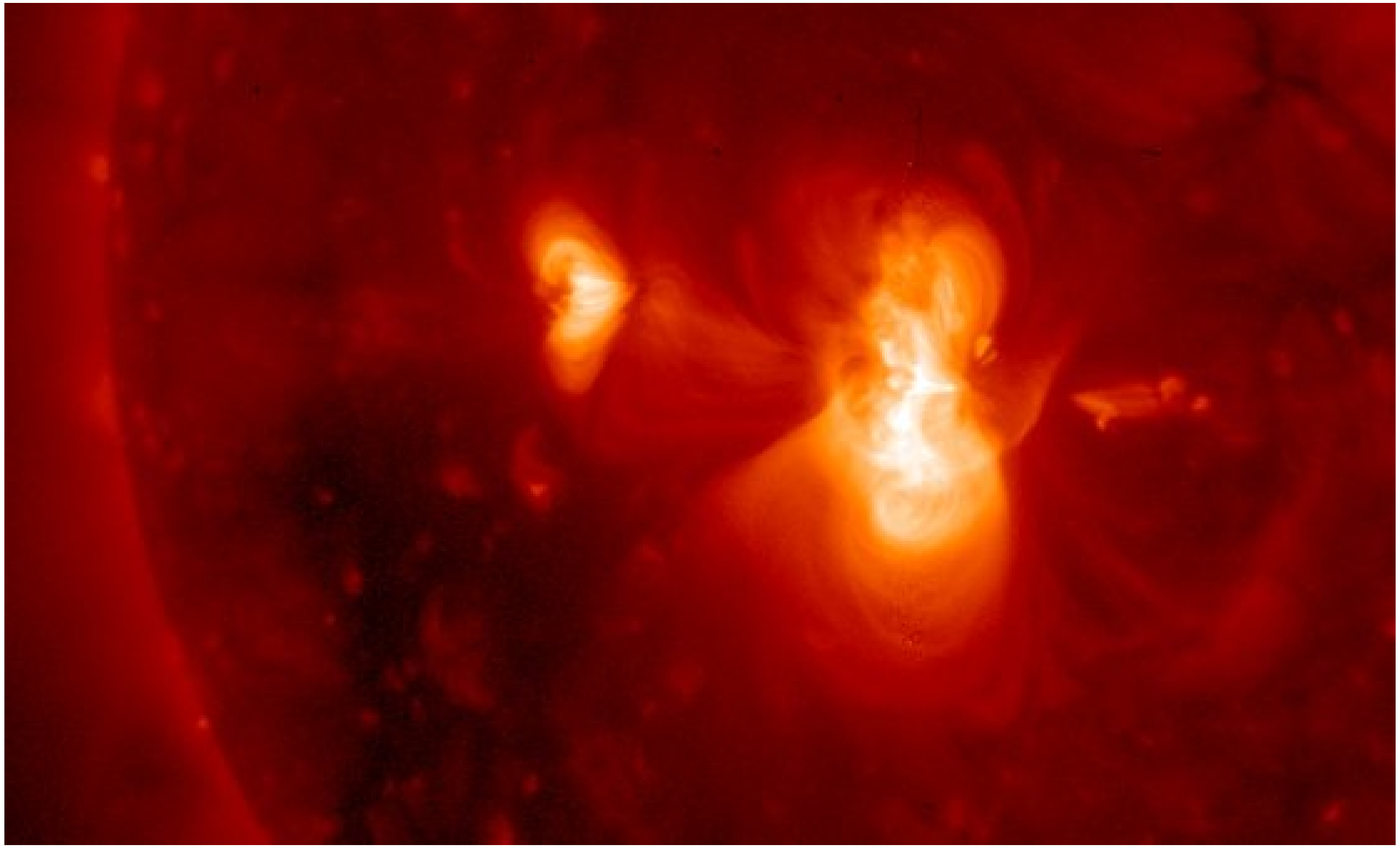


Self-consistent P (left) and N (right) solutions for AR 10953 (Wheatland & Régnier 2009)

Modelling AR 10953 with uncertainties: Data

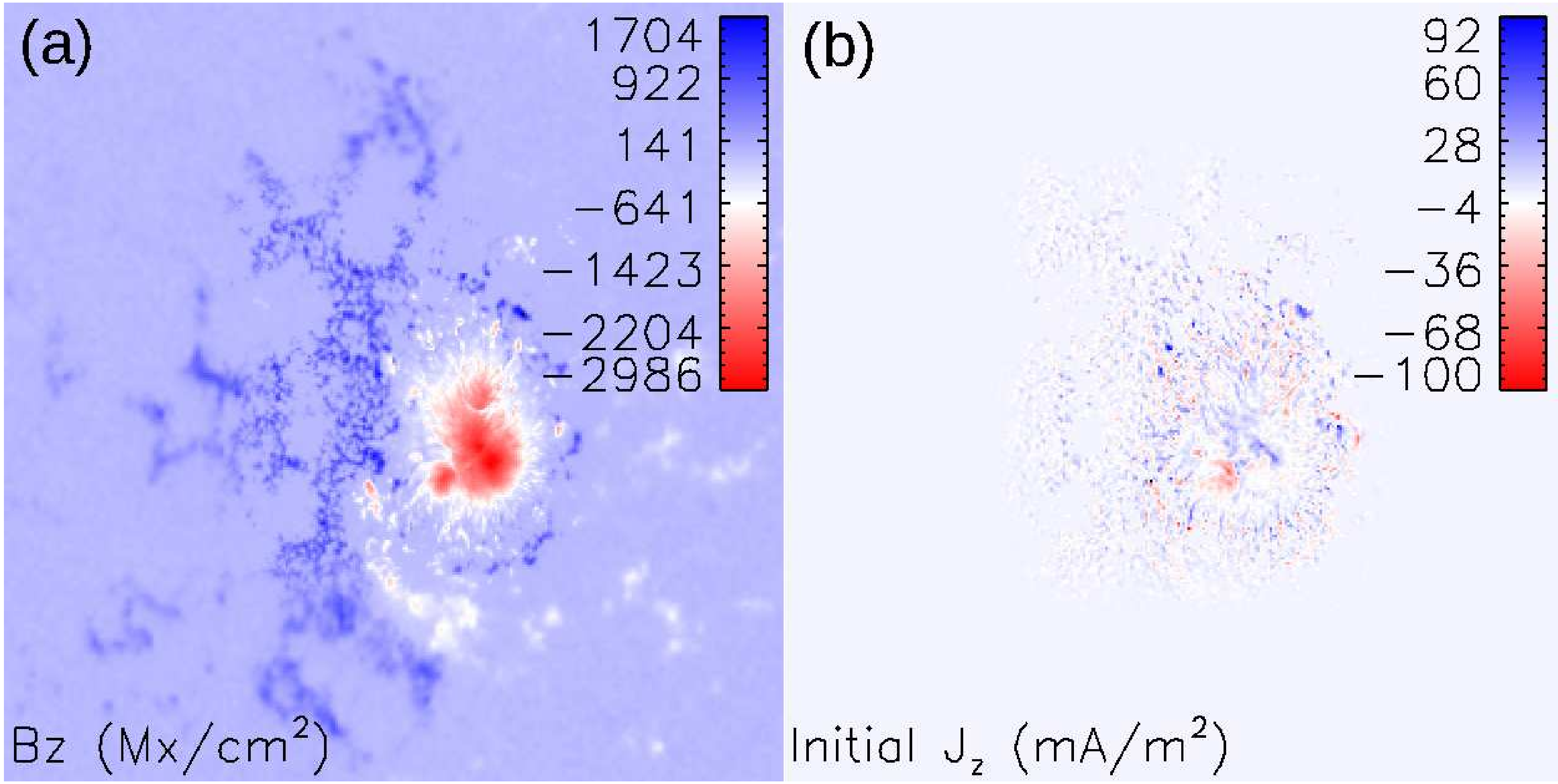
(Wheatland & Leka 2011)

- AR 10953 on 30 April 2007 is again the region of study
  - many force-free methods applied before (De Rosa et al. 2009)
  - self-consistent modelling test case (Wheatland & Régnier 2009)
- Hinode SOT/SP and MDI data used
  - new treatment: improved data merging and uncertainties

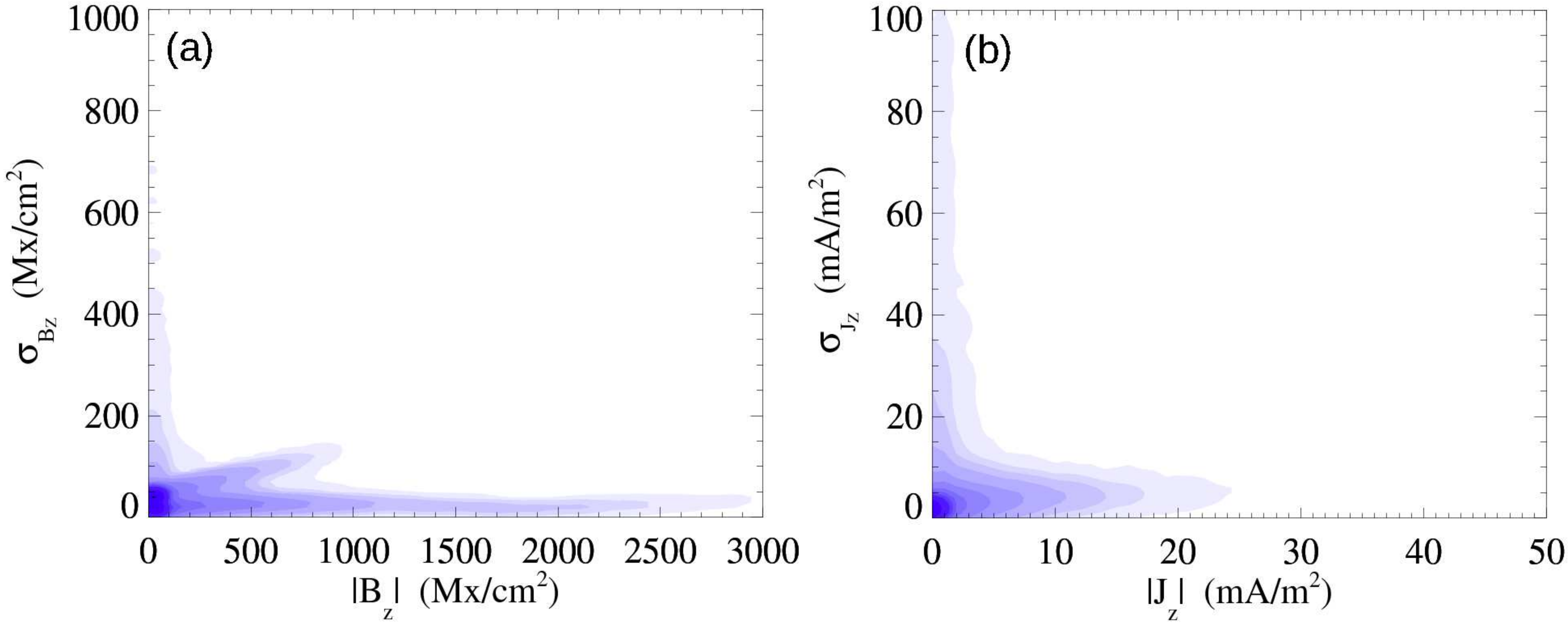


Hinode/XRT broadband soft X-ray image (Hinode/XRT)

- MDI data used to provide a wider FOV
- Uncertainties derived from Stokes inversion fit: 'lower limits'
- Boundary values $\alpha_0 \pm \sigma_0$ calculated from $B_i \pm \sigma_{B_i}$
  - points in MDI region assigned maximal uncertainties



Vector magnetogram B_z values (left) and J_z values (right) (Wheatland & Leka 2011)

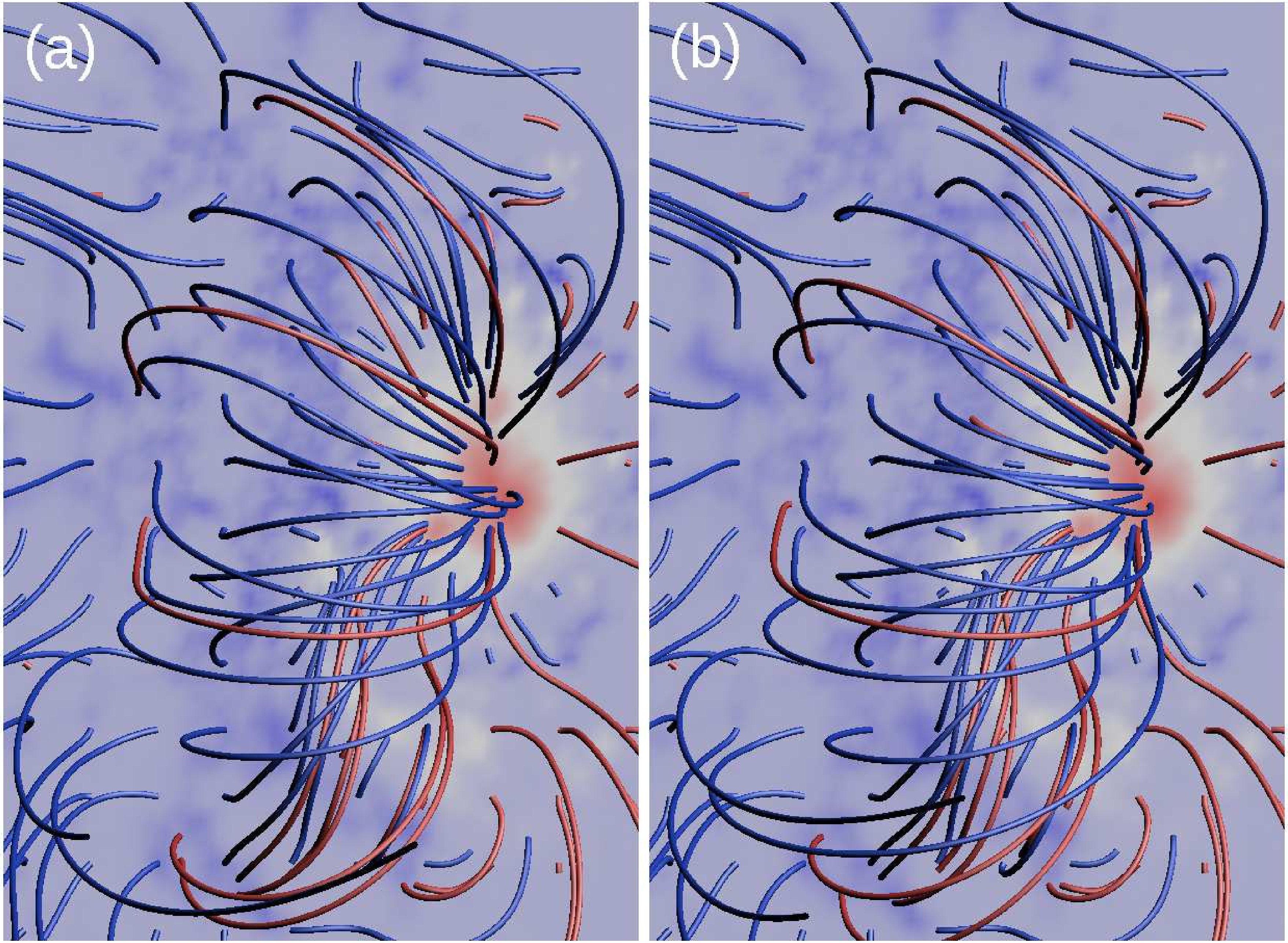


Vector magnetogram B_z values (left) and J_z values (right) (Wheatland & Leka 2011)

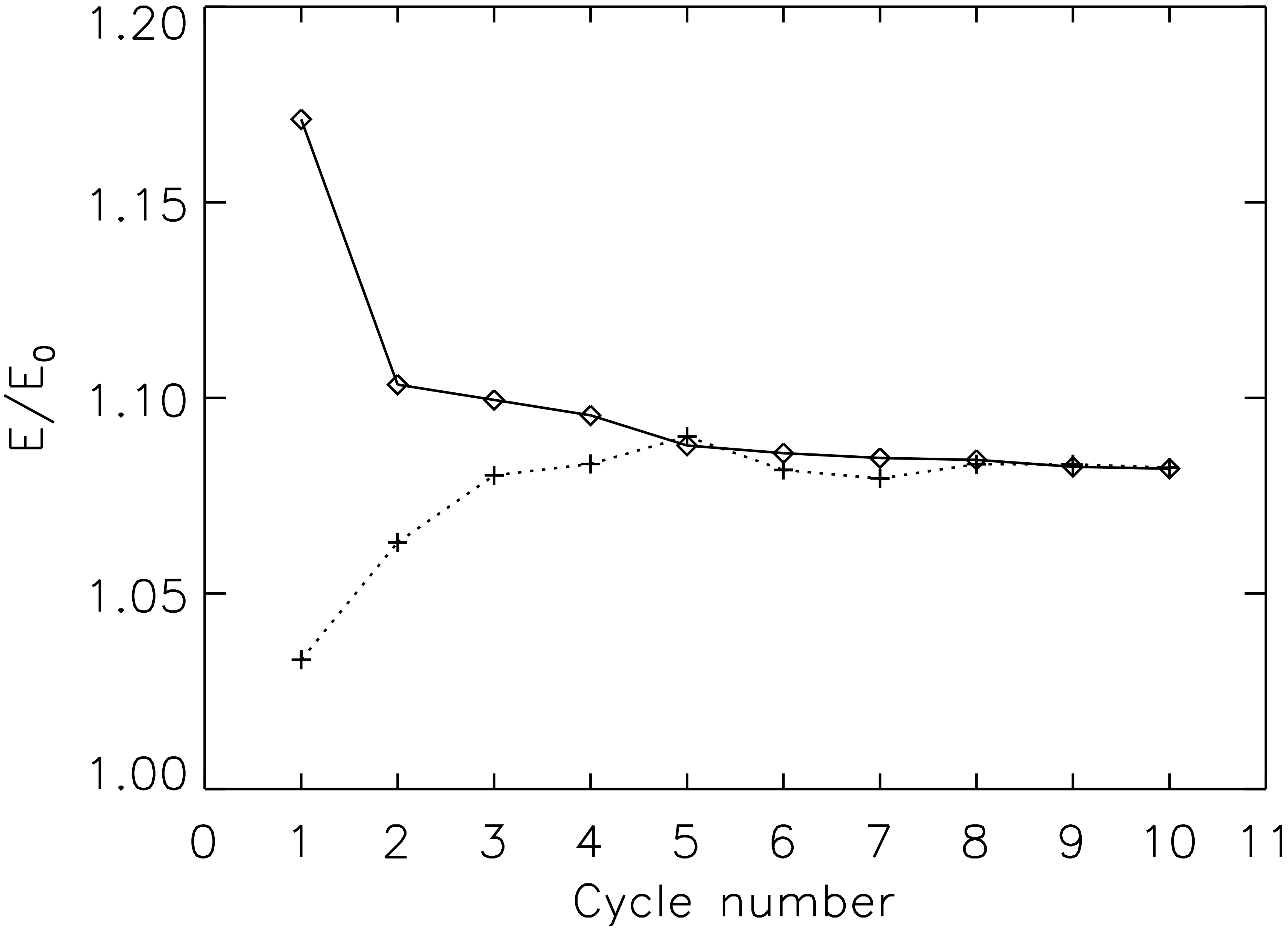
Modelling AR 10953 with uncertainties: Results

(Wheatland & Leka 2011)

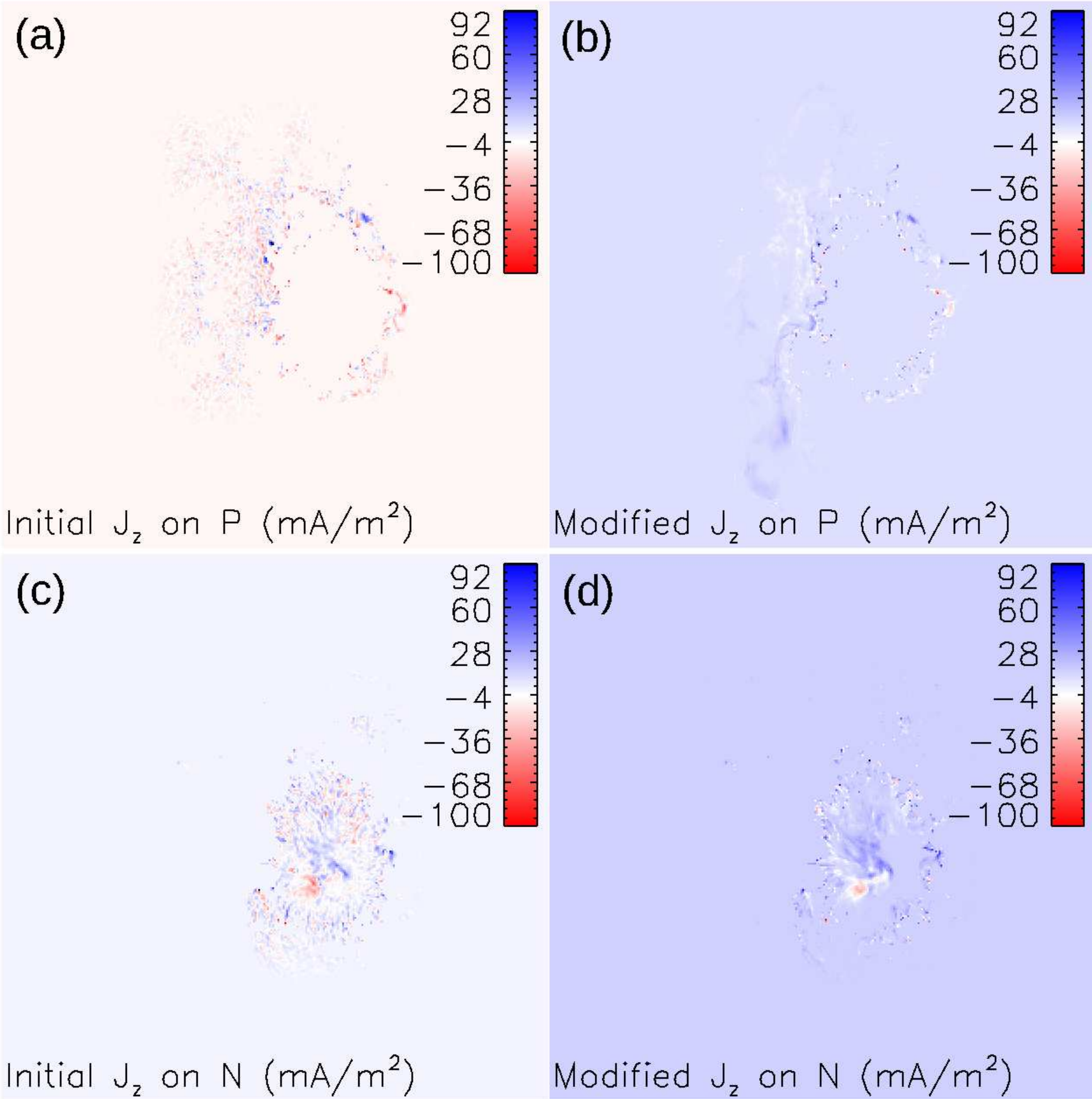
- 10 self consistency cycles used
  - grid size is $313 \times 313 \times 300$ (spacing is 0.8 arcsec)
  - $N_{GR} = 30$ Grad-Rubin iterations per cycle
  - currents crossing side and top boundaries omitted (Wheatland 2007)
- Procedure converges in < 10 cycles
  - energy of final solution(s) is $E/E_0 = 1.08$
    - significantly non-potential
  - energies of P and N solutions differ by $< 0.03\%$
    - self-consistency is achieved
- B_x and B_y are modified by self-consistency procedure
  - the changes exceed the nominal uncertainties
  - but are similar to those imposed by preprocessing
  - this implies the initial data are quite inconsistent
- BCs on α are preserved at locations with small σ
  - attention is paid to the most believable inference



Self-consistent solutions: (a) *P* solution; (b) *N* solution (Wheatland & Leka 2011)

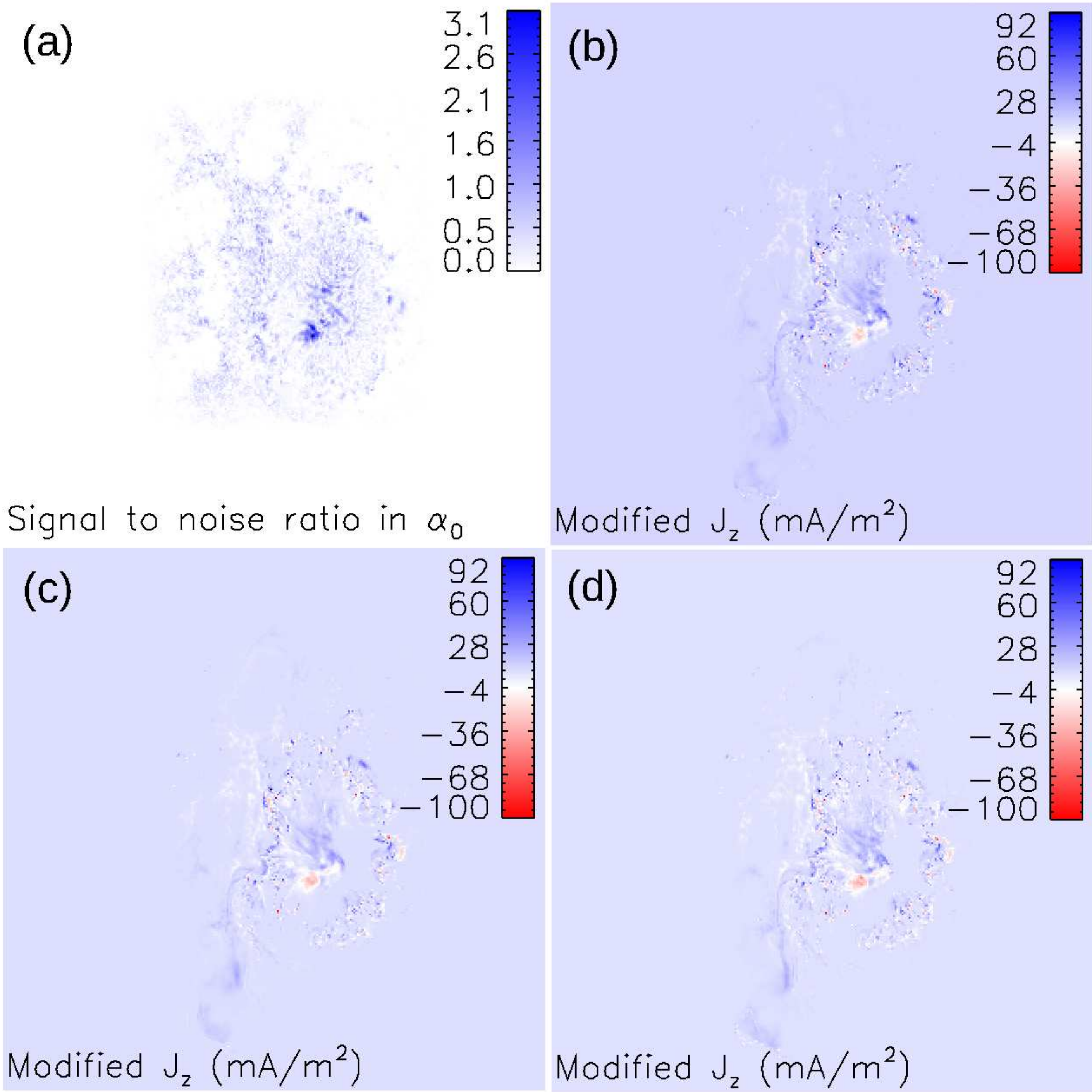


Energy of P solution (+) and N solution (◇) versus self-consistency cycle (Wheatland & Leka 2011)



Initial BCs on J_z (left column) and self-consistent values (right column) (Wheatland & Leka 2011)

- Grad-Rubin iteration does not converge strictly initially
  - but oscillates in energy (another symptom of inconsistency)
  - which introduces some arbitrariness in the modelling
    - a dependence on the number N_{GR} of GR iterations
- Modelling repeated with $N_{GR} = 20$ and $N_{GR} = 40$
  - results very similar which suggests the process is robust
  - energies of two new solutions are $E/E_0 = 1.08$ to 1 s.f.
  - minor differences in final BCs and field structure
- Energy of solution is higher when uncertainties are included
  - large J_z values in strong field regions have small σ
  - these values are preserved giving higher E
- Energy $E/E_0 = 1.08$ is between initial N and P energies
  - in the range of energies found in other studies (de Rosa et al. 2009; Canou & Amari 2010)
  - higher GR-method energies in other studies are the N solution
    - the P solution is ignored (e.g. Canou & Amari 2010)



Signal to noise ratio in α_0 and the BCs on J_z for the solutions with $N_{GR} = 20, 30, 40$ (Wheatland & Leka 2011)

Summary

- Vector magnetograms give BCs for coronal field modelling
  - the field values are inferences not measurements
  - the modelling is difficult
- The nonlinear force-free model is popular
  - but vector magnetogram data are inconsistent with the model
  - nonlinear force-modelling gives unreliable results
  - the self-consistency procedure provides one solution
- Self-consistency modelling for AR 10953
  - relative uncertainties in boundary data accounted for
  - significantly non-potential force-free model field obtained
  - results robust against different choices in the method
- Self-consistency is a promising method
  - however more physical modelling should be pursued