Computational modelling of stellar magnetic fields from observational boundary values

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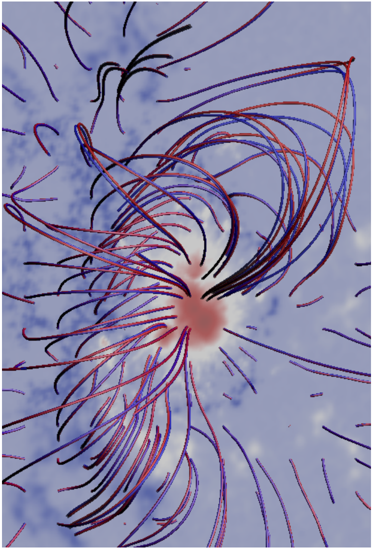
Cosmic Magnetism
Kiama 8 June 2010



Model for solar AR 10953 from Hinode/SOT data

Overview

Background

- *Zeeman effect on spectral lines*

The Sun

- *Modelling active regions*
- *The data – vector magnetograms*
- *Nonlinear force-free modelling*
- *The inconsistency problem*
- *Self-consistent nonlinear force-free modelling*

Other cool stars

- *Zeeman Doppler Imaging (ZDI)*
- *Modelling*

Summary

Nobody can measure physical quantities of the solar atmosphere

– Del Toro Iniesta & Ruiz Cobo (1996), Sol. Phys. 164, 169

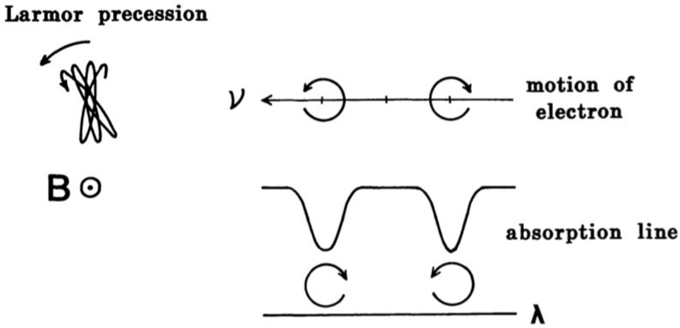
Background: Zeeman effect on spectral lines

- Classical model: dipole-oscillator atom (Sakurai 1989; Jefferies et al. 1989)
  - absorbs light near e^- oscillation frequency ν_0
- Introduce field **B**: motion of e^- parallel to field unaffected
  - in plane perpendicular to **B** the e^- precesses
  - frequency of precession is Larmor frequency $\Delta\nu_B = eB/4\pi m_e$
  - motion described in terms of frequencies $\nu_0 \pm \Delta\nu_B$
  - superposed CCW and CW motions at $\nu_0 + \Delta\nu_B$ and $\nu_0 - \Delta\nu_B$
- Wavelength shift (Landé factor g_L is quantum correction)

$$\Delta\lambda_B = \nu_B \lambda_0^2/c = 11.7 g_L \frac{B[\mathrm{G}]}{1000\,\mathrm{G}} \left(\frac{\lambda_0[\text{Å}]}{5000\,\text{Å}}\right)^2 \ [\text{mÅ}] \qquad (1)$$

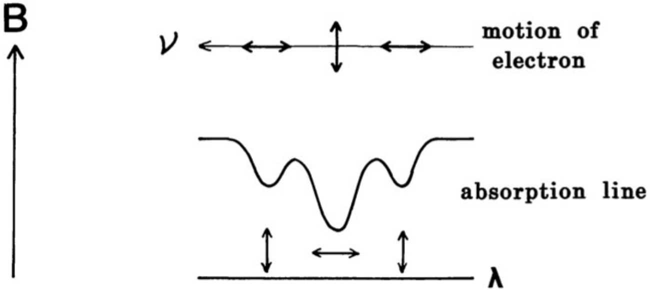
  - small effect except for large fields

- Viewed along **B**: observe circular motions
  - line replaced by two shifted circularly-polarized lines
  - σ-components



Classical explanation of longitudinal Zeeman effect (Sakurai 1989)

- Viewed transverse to **B**: observed two linear motions of e^-
  - central unshifted linearly-polarized π component
  - two shifted linearly-polarized σ-components



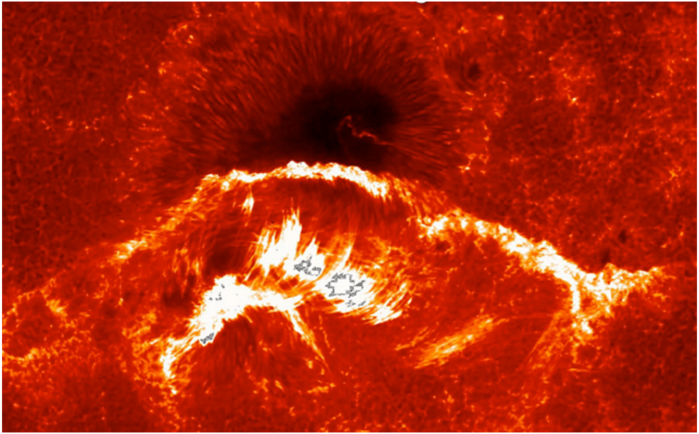
Classical explanation of transverse Zeeman effect (Sakurai 1989)

- For oblique **B** interpretation requires radiative transport [1]
  - for stellar case specification of an atmospheric model
  - result is not a measurement of **B** but an inference
    - "Nobody can measure..."
- Unno & Rachovsky analytic solution (Unno 1956; Rachkovsky 1962; 1967)
  - radiative transfer with uniform **B** and simple atmosphere
  - often the basis for interpreting spectro-polarimetric data
  - simpler weak-field approximation also used (e.g. Ronan et al. 1987)

[1] For more details see e.g. Landi degl'Innocenti & Landolfi (2004).

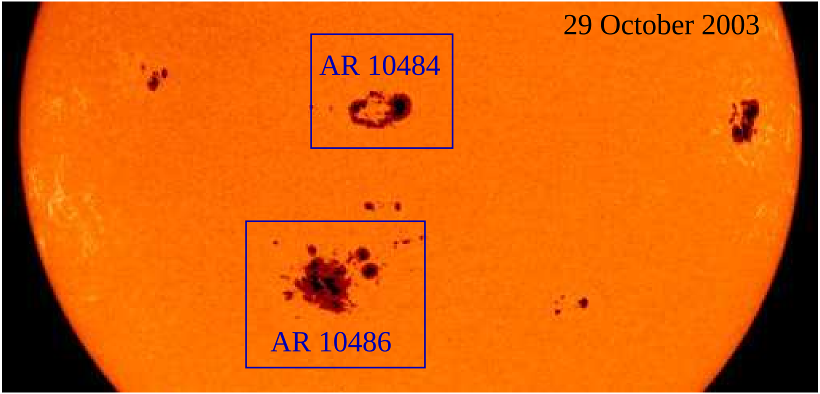
The Sun: Modelling active regions

- Sunspot magnetic fields power solar activity:
  - solar flares – magnetic explosions in the atmosphere (corona)
  - Coronal Mass Ejections (CMEs) – expulsions of material
- *Space weather:* CMEs influence local conditions
  - storms of energetic particles (*Solar Proton Events*)



A flare and a sunspot: 12 Dec 2006 (Hinode/SOT)

- Large active regions flare repeatedly
  - e.g. ARs 10484 and 10486 in Oct-Nov 2003[2]
- Problem: model the coronal magnetic fields of these regions



ARs 10484 and 10486 produced a sequence of huge flares in October-November 2003 [MDI]

[2] A good read: Stuart Clark 2007, "The Sun Kings," Princeton University Press

The Sun: The data – vector magnetograms

- Stokes profiles $I(\lambda)$, $Q(\lambda)$, $U(\lambda)$, $V(\lambda)$ measured
- *Stokes inversion:* vector magnetic field inferred[3]
  - nonlinear least-squares fitting to Unno-Rachovsky solution

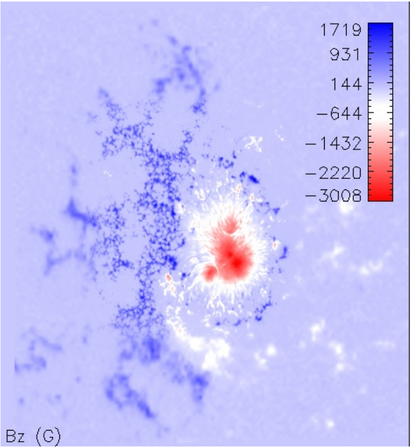
    (Auer et al. 1977; Skumanich et al. 1987; Skumanich & Lites 1987; Lites & Skumanich 1990)
  - line-of-sight and transverse field are parameters of fit
  - transverse field subject to a 180° ambiguity
- 180° ambiguity must be resolved (Metcalf 1994; Metcalf et al. 2006)
- *Vector magnetogram:* photospheric map of $\mathbf{B} = (B_x, B_y, B_z)$
- Vertical current density J_z may be calculated at photosphere:

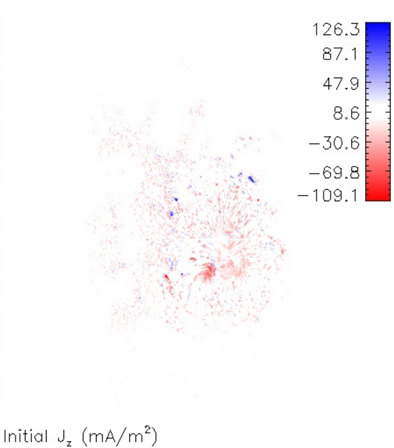
$$J_z = \frac{1}{\mu_0}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) \quad \text{at} \quad z = 0 \tag{2}$$

  - locally planar approximation to photosphere

[3] For more details see e.g. Landi degl'Innocenti & Landolfi (2004).

- New generation of instruments
  - Hinode Solar Optical Telescope (SOT) Spectro-Polarimeter (Tsuneta et al. 2008)
  - Solar Dynamics Observatory Helioseismic & Magnetic Imager (Borrero et al. 2007)
- Hinode-derived vector magnetogram for active region 10953





Active region AR 10953 on 30 April 2007. Left: B_z. Right: J_z (Wheatland & Leka in preparation; Hinode/SOT).

The Sun: Nonlinear force-free modelling

- Vector magnetograms provide boundary conditions for models
  - coronal magnetic field *reconstruction*
- *Force-free model* for coronal magnetic field:

$$\mathbf{J} \times \mathbf{B} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0 \tag{3}$$

  - $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$ is electric currrent density
  - physics: Lorentz force dominates over other forces
  - coupled nonlinear PDEs
- Writing $\mathbf{J} = \alpha \mathbf{B}/\mu_0$ ($\mathbf{J}$ is parallel to $\mathbf{B}$):

$$\mathbf{B} \cdot \nabla \alpha = 0 \quad \text{and} \quad \nabla \times \mathbf{B} = \alpha \mathbf{B} \tag{4}$$

  - α is the force-free parameter
  - $\alpha = \mu_0 J_z / B_z$ at $z = 0$ defines values over vector magnetogram

- Boundary conditions (Grad & Rubin 1958):
  - B_n in boundary
  - α in boundary over region where $B_n > 0$ *or* where $B_n < 0$
    - over *one polarity*
    - we label the polarities P and N respectively
- Vector magnetograms give two sets of boundary conditions
  - values of $\alpha = \mu_0 J_z / B_z$ over both P and N are available
- Eqs. (4): methods of solution are iterative (e.g. Wiegelmann 2008)
- Current-field iteration (Grad & Rubin 1958)
  - at iteration k solve the linear system

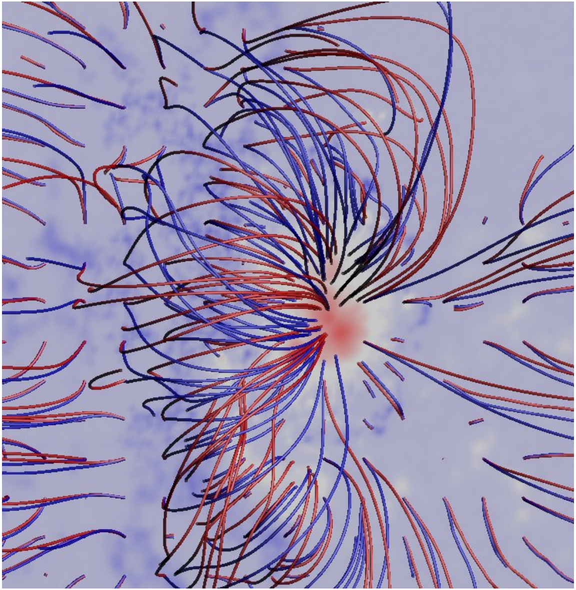
$$\mathbf{B}^{[k-1]} \cdot \nabla \alpha^{[k]} = 0 \quad \text{and} \quad \nabla \times \mathbf{B}^{[k]} = \alpha^{[k]} \mathbf{B}^{[k-1]} \qquad (5)$$

  - BCs imposed on $\alpha^{[k]}$ and $B_z^{[k]}$
  - Wheatland (2007): a fast implementation

The Sun: The inconsistency problem

- Force-free methods work for test cases but fail for solar data
  (Schrijver et al. 2006; Metcalf et al 2008; Schrijver et al. 2008; DeRosa et al. 2009)
  - different methods give different solutions
  - P and N solutions do not agree for the same method
- Vector magnetogram data inconsistent with force-free model
  - errors in field determination
  - field at photospheric level is not force free (Metcalf et al. 1995)
  - necessary conditions for a force-free field not met (Molodenskii 1969)

- AR 10953 on 30 April 2007
  - P (blue) and N (red) solutions from vector magnetogram

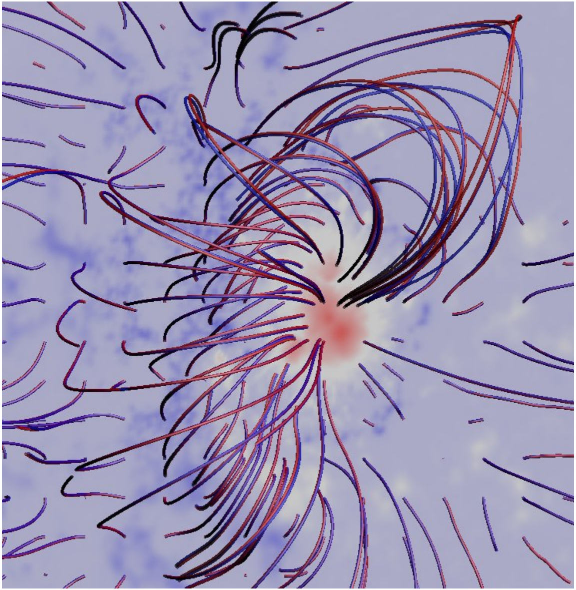


Force-free solutions from K. D. Leka's vector magnetogram data for AR 10953

The Sun: Self-consistent nonlinear force-free modelling

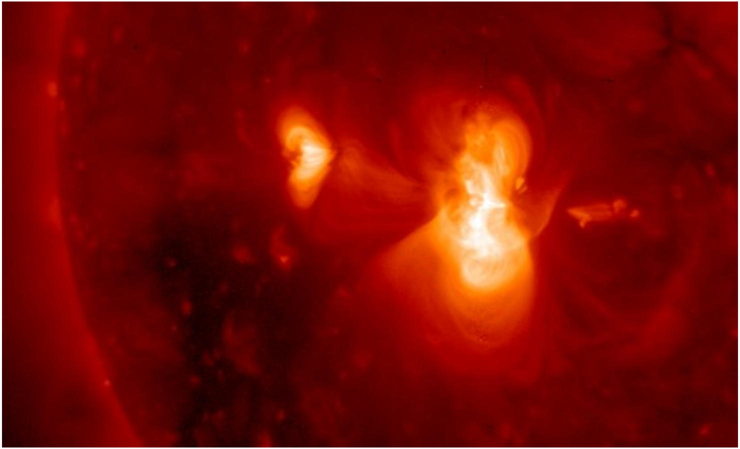
- Find the *closest* force-free solution to the observed data
- Self-consistency procedure (Wheatland & Régnier 2009)
  - P and N solutions constructed (current-field iteration)
  - Bayesian probability plus solutions used to modify BCs on α
    - taking into account relative uncertainties in boundary values
  - procedure iterated until the P and N solutions agree
- Wheatland & Régnier (2009): demonstrated on AR 10953
  - method shown to work
  - but uncertainties were not available for the boundary data
  - self-consistent solution was close to potential (current-free)
  - result was considered a proof of concept
- Problem re-visited with data including uncertainties
  - solution with large currents obtained (Wheatland & Leka in preparation)

- AR 10953 on 30 April 2007
  - New self-consistent solution(s): P (blue) and N (red)



Self-consistent nonlinear force-free solutions for AR 10953

- Soft X-ray image of AR 10953 on 30 April 2007



Hinode/XRT broadband soft X-ray image (Hinode/XRT)

Other cool stars: Zeeman Doppler Imaging (ZDI)

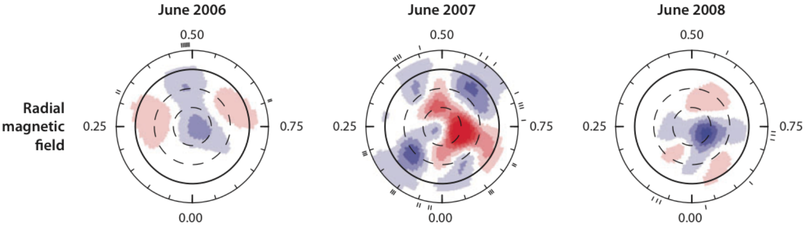
- Permits determination of surface field over cool stars
- Proposed by Semel (1989)[4]
  - applicable to rapidly rotating stars
  - assumes field evolves on a time scale longer than a period
- Basic technique:
  - combine Stokes $V(\lambda, t)$ profiles for many lines to improve SNR
  - fit composite profiles to profiles for a surface field model
  - Unno-Rachovsky solution or weak-field approximation used
- Donati et al. (2006) model:

$$\mathbf{B} = [B_r(\theta,\phi), B_\theta(\theta,\phi), B_\phi(\theta,\phi)] \tag{6}$$

  - components expanded in spherical harmonics
  - fitting determines coefficients in the expansion

[4] Further developmeants e.g. Brown et al. (1991); Donati & Brown (1997); Donati (2001).

- Evidence for stellar global polarity switches (Donati et al. 2008)
  - planet-hosting F8 star τ Boo
  - successive polarity switches of field components over two years



Surface distribution of B_r inferred by ZDI for τ Boo (Donati & Landstreet 2009)

Other cool stars: Modelling

- Source surface modelling (e.g. Jardine et al. 1999; Jardine et al. 2002)
  - a potential (current-free) model for global field
  - developed for the Sun (Altschuler & Newkirk 1969; Schatten et al. 1969)
  - mimics radial stretching of field at height due to stellar wind
- Source surface model field (which satisfies $\nabla \times \mathbf{B} = 0$):

$$\mathbf{B}(r, \theta, \phi) = -\nabla \Psi = (B_r, B_\theta, B_\phi) \tag{7}$$

  - boundary conditions:

$$B_r(R_*, \theta, \phi) = B_r^{\mathrm{ZDI}}(\theta, \phi) \tag{8}$$

$$B_\phi(R_s, \theta, \phi) = B_\theta(R_s, \theta, \phi) = 0 \tag{9}$$

  - field is purely radial at source surface $R_s \approx 3R_* - 5R_*$
  - field components may be expanded in spherical harmonics
    - coefficients determined by imposing boundary conditions
- ZDI values of B_ϕ, B_θ inconsistent with potential model
  - non-potential models also tried (e.g. Hussain et al. 2002)

Summary

- Stellar magnetic fields are inferred not measured
  - inferred surface values permit coronal field modelling
- The Sun
  - active region modelling motivated by activity/space weather
  - photospheric vector magnetogram data is available
  - nonlinear force-free modelling has been developed
  - boundary data is inconsistent with the model
  - self-consistency solution presented
- Other cool stars
  - inference of surface fields using Zeeman Doppler Imaging
  - coronal field modelling e.g. source surface solutions
- List of solar sites including pictures and movies:
  http://sydney.edu.au/science/physics/~wheat/[5]

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[5] Easier: search for Mike Wheatland on google.