The objects that make up space are in motion, we move, soccer balls move, the Earth moves, electrons move, - - - . Motion implies change. The study of the motion of objects is called kinematics. We will construct simple models starting with the motion of a single object. The object will be modelled as a point particle in space and who dimensions are ignored. A particle is represented in a diagram as a dot •.

Since the beginning of “human time” the study of the motions of the heavens has been important. People who had knowledge of the motion of the heavens had power and control over others because they knew about the seasons, eclipses and could make predictions. In ancient cultures, people were very frightened during solar and moon eclipses.

Developing theories about the motion of the Sun and planets was very frustrating. Many religions had the Earth at the centre of the Universe. For presenting views opposing the Earth centred Universe one could be burnt alive at the stake.

Developing a consistent theory of motion was difficult. Physical theories are not just discovered, they are creations of the human mind and must be invented and not discovered. It is claimed that the study of physics is man’s greatest intellectual achievement. Physics is a creative process and physicists build on the best of the past to create new visions of the future.

A significant time for science was when Nicholas Copernicus (1473 – 1543) concluded from experimental data on the motion of the planets, that the Earth revolved around the Sun. Johannes Kepler (1571 – 1630) was able to decipher accurately and precisely the paths of the planets and produced mathematical laws accounting for the motion of the planets around the Sun.

The work of Galileo Galilei (1564 – 1642) produced a dramatic change in the way science was to be done. Galileo’s ideas overturned the Aristotelian view of the world that had persisted for more than 2000 years. Galileo’s genius was that he realized that the world can be described through mathematics and that experimentation and measurement were crucial elements in developing scientific theories where approximation were necessary to limit complexity.

Following on from Galileo was Isaac Newton (1642 – 1727), who developed the underlying theory of the causes of motion (dynamics). He published one of the most famous books of science in 1686, Principia.

WEB activity: Use the web to find out more about: Aristotle, Copernicus, Kepler, Galileo and Newton.

Group discussion: Why did people such as priests who had scientific knowledge of the seasons and astronomy have control over the bulk of the population in their societies?

Group discussion: In the period from the 15th century to the 19th century in western Europe there was a dramatic change in social, cultural and technological aspects for people but this did not occur in China, which at the beginning of the 15th century could be consider to be more “advanced” socially, culturally and technologically than Europe. Explain why the “no change” in China compared with the scientific and technological developments that occurred in Western Europe in this 400 year period.
DISTANCE AND DISPLACEMENT

In our study of kinematics will we only consider the motion of objects in one-dimension. This is called **rectilinear motion**. It is a relative straightforward task to extend the knowledge one gains in studying one-dimensional motion to two- and three-dimensions.

One advantage of studying one-dimensional motion is that we don’t have to use vector notation although displacement, velocity and acceleration are vectors. The sign of the physical quantity in this instance gives the direction of the vector. Consider the displacement vectors shown in figure (1). You can’t associate a positive or negative sign with a vector quantity unless it has a direction parallel to one of the coordinate axes. For example, a component of a vector which is parallel to x-axis in a direction pointing towards +x is assigned a positive number and a component pointing towards the –x is assigned a negative number.

For the components only:
Since the components of the vectors are directed along the coordinate axes we can assign a positive or negative value to the components of a vector. The sign then gives the direction.

\[
\begin{align*}
\hat{s}_{1x} & \equiv s_{1x} & s_{1x} > 0 \\
\hat{s}_{1y} & \equiv s_{1y} & s_{1y} > 0 \\
\hat{s}_{2x} & \equiv s_{2x} & s_{2x} > 0 \\
\hat{s}_{2y} & \equiv s_{2y} & s_{2y} < 0
\end{align*}
\]

Before reading further, view the animation of the rectilinear motion of a tram that travels to the right then back to the left. Think about the scientific language that you will need to describe the motion of the tram.

To describe the motion of a moving object you must first define a frame of reference (origin and x-axis) and the object becomes a particle. Consider the tram moving backward and forwards along a straight 2.00 km track. The x-axis is taken along the track with the origin at x = 0. The left end of the track is at x = -1.00 km and the right end of the track is at x = +1.00 km. The tram start at time \( t_1 \) at the left end of the track moves to the right where it stops at the right end and returns to the left end of the track arriving at a later time \( t_2 \). At any time \( t \), the position of the tram is given by its x-coordinate as shown in figure (2).
Fig. 2. A frame of reference for the rectilinear motion of the tram with the origin at $x = 0$. The tram travels from the left to the right end and travels back to the left end of the track.

At time $t_1 = 0$ the tram starts its journey at position $x_1 = -1.00$ km. At time $t_2 = 0.25$ hours the train completes one cycle to the right end of the track and back again where the final position of the train is $x_2 = -1.00$ km. The time interval $\Delta t$ for the journey is

$\Delta t = t_2 - t_1 = 0.25 \text{ h} = 900 \text{ s}$

$\Delta$ is the Greek letter Delta meaning change in or increment.

Note: Time and time interval are different physical quantities.

The **distance travelled** $\Delta d$ is

$\Delta d = 4.00$ km.

The magnitude of the **displacement** $\Delta s$ is the straight line distance between the initial position $x_1$ and the final position $x_2$

$\Delta s = x_2 - x_1 = -1.00 - (-1.00) \text{ km} = 0 \text{ km}$

Distance travelled (scalar) is not the same physical quantity as displacement (vector).
SPEED AND VELOCITY

Speed and velocity refer to how fast something is travelling but are different physical quantities. Also, we need to distinguish between average and instantaneous quantities.

The **average speed** $v_{avg}$ of a particle during a time interval $\Delta t$ in which the distance travelled is $\Delta d$ is defined by equation (1)

$$v_{avg} = \frac{\Delta d}{\Delta t} \quad \text{definition of average speed}$$

The average speed is a positive scalar quantity and not a vector.

The average velocity $\vec{v}_{avg}$ of a particle is defined as the ratio of the change in position vector $\Delta \vec{s}$ of the particle in the time interval $\Delta t$ during which the change occurred as given by equation (2)

$$\vec{v}_{avg} = \frac{\Delta \vec{s}}{\Delta t} \quad \text{definition of average velocity}$$

For one-dimensional motion directed along the x-axis we do not need the use the vector notation shown by the arrow above the symbol.

In the example of our tram:

$$\Delta t = t_2 - t_1 = 0.25 \text{ h} = 900 \text{ s} \quad \Delta d = 4.00 \text{ km} = 4.00 \times 10^3 \text{ m}.$$  
$$\Delta s = x_2 - x_1 = -1.00 - (-1.00) \text{ km} = 0 \text{ km} = 0 \text{ m}$$

**average speed**

$$v_{avg} = \frac{\Delta d}{\Delta t} = \left( \frac{4}{0.25} \right) \text{ km.h}^{-1} = 16.0 \text{ km.h}^{-1}$$

$$v_{avg} = \frac{\Delta d}{\Delta t} = \left( \frac{4 \times 10^3}{900} \right) \text{ m.s}^{-1} = 4.44 \text{ m.s}^{-1} \quad \text{S.I. units}$$

**average velocity**

$$v_{avg} = \frac{\Delta s}{\Delta t} = 0 \text{ km.h}^{-1} = 0 \text{ m.s}^{-1}$$

*Note: very different values for the average speed and average velocity. Often the same symbol is used for speed and velocity. You always need to state whether the symbol represents the speed or velocity.*

Our tram speeds up continually until it reaches the origin, it then slows down and stops at the right end. In the return journey, it continually speeds up until it reaches the origin it then slows down and stops at the left end of the track. The speed and velocity are always changing. Hence, it is more useful to talk around **instantaneous** values rather than average values. The instantaneous velocity $\vec{v}$ is a vector quantity whose magnitude is equal to the instantaneous speed $V$ and who instantaneous direction is in the same sense to that in which the particle is moving at that instant. The velocity vector is always tangential to the trajectory of the particle at that instant. Remember, speed and velocity are not the same physical quantity even though they have the same S.I. unit [m.s$^{-1}$].
The average velocity given by equation (2) is
\[ \bar{v}_{\text{avg}} = \frac{\Delta s}{\Delta t} \]

If we make the time interval \( \Delta t \) smaller and smaller, the average velocity approaches the instantaneous value at that instant. Mathematically it is written as
\[ \lim_{\Delta t \to 0} \left( \frac{\Delta s}{\Delta t} \right) \]

This limit is one way of defining the derivative of a function. The instantaneous velocity is the time rate of change of the displacement

\[ v = \frac{ds}{dt} \quad \text{definition of instantaneous velocity} \]

For rectilinear motion along the x-axis, we don’t need the vector notation and we can simply write the instantaneous velocity as

\[ v = \frac{dx}{dt} \quad \text{rectilinear motion} \]

You don’t need to know how to differentiate a function, but you have to be familiar with the notation for differentiation and be able to interpret the process of differentiation as finding the slope of the tangent to the displacement vs time graph at one instant of time figure (3).

Fig. 3. The average and instantaneous velocities can be found from a displacement vs time graph. The slope of the tangent gives the instantaneous velocity (derivative of a function).

The reverse process to differentiation is integration. Graphically, integration is a process of finding the area under a curve. The slope of the tangent to the displacement vs time graph is equal to the velocity. The area under a velocity vs time graph in a time interval \( \Delta t \) is equal to the change in displacement \( \Delta s \) in that time interval as shown in figure(5).

When you refer to the speed or velocity it means you are talking about the instantaneous values. Therefore, on most occasions you can omit the word instantaneous.
ACCELERATION

Velocity is related to how fast an object is travelling. Acceleration refers to changes in velocity. Since velocity is a vector quantity, an acceleration occurs when

- an object speeds up (magnitude of velocity increases)
- an object slows down (magnitude of velocity decreases)
- an object changes direction (direction of velocity changes)

Acceleration is a vector quantity. You don’t sense how fast you are travelling in a car, but you do notice changes in speed and direction of the car especially if the changes occur rapidly – you “feel” the effects of the acceleration. Some of the effects of acceleration we are familiar with include: the experience of sinking into the seat as a plane accelerates down the runway, the “flutter” in our stomach when a lift suddenly speeds up or slows down and “being thrown side-ways” in a car going around a corner too quickly.

The average acceleration of an object is defined in terms of the change in velocity and the interval for the change

\[
\bar{a}_{avg} = \frac{\Delta v}{\Delta t} \quad \text{definition of average velocity}
\]

The instantaneous acceleration is the time rate of change of the velocity, i.e., the derivative of the velocity gives the acceleration (equation 6). Again, you don’t need to differentiate a function but you need to know the notation and interpret it graphically as the acceleration is the slope of the tangent to a velocity vs time graph as shown in figure (4). The area under the acceleration vs time graph in the time interval is equal to the change in velocity in that time interval as shown in figure (5).

\[
\ddot{a} = \frac{dv}{dt} \quad \text{definition of instantaneous acceleration}
\]

\[
a = \frac{dv}{dt} \quad \text{rectilinear motion}\]
Fig. 4. Velocity vs time graphs for rectilinear motion. The slope of the tangent is equal to the acceleration. For the special case when the velocity is a linear function of time (straight line) the acceleration is constant.

\[ a = \frac{\Delta v}{\Delta t} \]

instantaneous acceleration at time \( t' \) is equal to the slope of the tangent at time \( t' \)

Fig. 5. The area under the velocity vs time graph in the time interval \( \Delta t \) is equal to the change in displacement \( \Delta s \) in that time interval. The area under the acceleration vs time graph in the time interval \( \Delta t \) is equal to the change in velocity \( \Delta v \) in that time interval.
VELOCITY AS A VECTOR

We will consider a number of problems (with solutions) to illustrate a how-to approach to solving problems involving vectors. It is important to consider the steps and strategies involved so that you will use similar techniques in answering your examination questions. Remember you want to strive for technical excellence.

For many vector problems you need to have mastered the mathematics of a right angle triangle.

For a right angle triangle with sides $a$, $b$ and $c$ (hypotenuse) and with angles $A$, $B$ and $C = 90^\circ$

\[
\sin A = \frac{a}{c} \quad \cos A = \frac{b}{c} \quad \tan A = \frac{a}{b} \\
\tan A = \frac{\sin A}{\cos A} \quad \sin^2 A + \cos^2 A = 1
\]

**Example 1**

An aircraft is climbing with a steady velocity of 200 m.$s^{-1}$ at an angle of $20^\circ$ to the horizontal. What are the speeds that the aircraft is moving horizontally and vertically?

**Solution 1**

How-to approach the problem

- Draw a frame of reference showing the x (horizontal) and y (vertical) coordinate axes.
- Represent the aircraft as a dot at the origin.
- Draw the velocity vector
- Show the components of the velocity vector on a separate diagram with a box around the velocity vector.
- Use the properties of a right angle triangle to calculate the values for the x and y components of the velocity.
- Check you answer – sensible, significant figures, units.

always identifying the unknowns
From the right angle triangle

horizontal component \( v_x = 200 \cos(20^\circ) \text{ m.s}^{-1} = 188 \text{ m.s}^{-1} \)

vertical component \( v_y = 200 \sin(20^\circ) \text{ m.s}^{-1} = 68.4 \text{ m.s}^{-1} \)

units included after all expressions, answers to 3 s.f.

check: \( v = \sqrt{v_x^2 + v_y^2} = \sqrt{188^2 + 68.4^2} \text{ m.s}^{-1} = 200 \text{ m.s}^{-1} \) ok

Example 2
An aircraft is trying to fly due north with a velocity of 200 m.s\(^{-1}\) but is subject to a cross wind blowing from the east at 50 m.s\(^{-1}\). What is the velocity of the plane with respect to the ground?

Solution 2
How to approach the problem

- Draw a frame of reference showing the x (WE) and y (SN) coordinate axes.
- Represent the aircraft as a dot.
- Draw the velocity vectors for the aircraft w.r.t. the wind and the wind w.r.t. the ground.
- Add the vectors using the head-to-tail method.
- Use the properties of a right angle triangle to calculate the resultant velocity.
- Check you answer – sensible, significant figures, units.

The resultant velocity of the plane w.r.t. the ground is \( \vec{v} = \vec{v}_1 + \vec{v}_2 = \text{ m.s}^{-1} \) m.

Adding the vectors head-to-tail method

\[ v = \sqrt{200^2 + 50^2} \text{ m.s}^{-1} = 206 \text{ m.s}^{-1} \]

\[ \theta = \tan^{-1} \left( \frac{50}{200} \right) = 14.0^\circ \]

The resultant velocity of the plane w.r.t. the ground is 206 m.s\(^{-1}\) and is flying in the direction 14.0\(^{\circ}\) W of N.
Example 3
A motor boat is travelling at 10.0 m.s\(^{-1}\) w.r.t the water. The boat wants to cross from one side to the other of a 400 m wide river in the shortest possible time but the river is flowing at a rate of 5.00 m.s\(^{-1}\)? In which direction must the boat point and how long will it take to cross the river?

Solution 3
How-to approach the problem

- Draw a frame of reference showing the x (perpendicular to the river) and y (parallel to the river) coordinate axes.
- Represent the boat as a dot.
- Draw the velocity vectors for the boat w.r.t the water and the water w.r.t. the ground.
- Add the vectors using the head-to-tail method so that the resultant points directly across the river.
- Use the properties of a right angle triangle to calculate the velocity of the boat w.r.t. the water.
- Calculate the time to cross the river.
- Check you answer – sensible, significant figures, units.

For the boat to cross the river in the shortest time it must travel directly across the river. So the boat must head up steam so that it will drift down with the flow of the water in the river.

resultant velocity must be directly across the stream (pointing in the +x direction)

$$\vec{v} = 8.66 \text{ m.s}^{-1}$$

$$\vec{v}_1 = 5.00 \text{ m.s}^{-1}$$

$$\vec{v}_2 = 10.0 \text{ m.s}^{-1}$$

$$\theta = 30^\circ$$

$$\Delta x = 400 \text{ m}$$

$$\Delta t = ? \text{ s}$$

$$\Delta t = \frac{\Delta x}{v} \quad \Delta t = \frac{400}{8.66} \text{ s} = 46.2 \text{ s}$$

From the right angle triangle

$$v = \sqrt{v_2^2 - v_1^2} = \sqrt{10^2 - 5^2} \text{ m.s}^{-1} = 8.66 \text{ m.s}^{-1}$$

$$\sin \theta = \frac{5}{10} \quad \theta = a \sin \left( \frac{1}{2} \right) = 30^\circ$$

The boat must head up-stream pointed at an angle of 30° as shown in the diagram. The boat crosses the stream at 8.66 m.s\(^{-1}\) and the shortest time to cross the river is 46.2 s.