Our starting point in our exploration of Space is to study the motion of particles. The language you will learn can be used to describe the motion of galaxies, stars, planets, comets, satellites, rockets, golf balls, microscopic objects, and so on. Using this language, principles, laws, theories and models will be given to help us understand the physical world surrounding us and to make predictions using mathematical models.

We will often develop simple models that are not necessarily good approximation to the real world but none the less can be quite useful. A physicist is a person who can take a complex situation and begin to understand its working by first creating a simple model which ignores many aspects of the real situation, but by doing this, they can start to develop an understanding of the situation. To improve your physics ability throughout this course it is essential that you learn how to visualise and simplify a physical situation. Often a starting point to answering an examination question is to draw a scientific but simple diagram of the physical situation. People who are good at physics do this automatically. Those that struggle with physics and think it is a “hard” subject do so because they can’t visualise and then draw an appropriate annotated diagram of the situation as part of their answer.

Our goal is to develop a set of models to describe, understand and predict the motion of objects. In developing our simplest models, any object is considered to be a particle with a mass. Therefore, any object we are going to study in this Unit should be simply visualised as a dot • presenting a particle. The mass of the object is always represented by the symbol $M$ or $m$ and if there is more than one object, subscripts are used to identify each object. For example, $m_1, m_2, m_3$ or $M_A, M_B, M_C$ could be used to represent the mass of three cars. The mass of the three cars together is $M_A + M_B + M_C$. The S.I. Unit for mass is the kilogram [kg]. Physical quantities like mass where a single number gives a measurement of the quantity and add together as simple numbers are called scalar quantities. Any quantity that has a magnitude but no direction is called a scalar and examples include time, temperature, volume, density, energy and electric charge.

One of the driving forces on developing the scientific method was the study of astronomy starting in the 17th century. This lead to a model in which our Universe is composed entirely of particles such as electrons, protons, and neutrons. If at some time one new the position each particle and the forces between those particles then you could know everything about the Universe as it would be
theoretically possible to predict the future. This was the viewpoint of nearly all scientists until about 1927 when the ideas of classical physics (ideas dating until the beginning of the 20th century) were strongly held. At the beginning of the 20th century numerous phenomena could not be explained in terms of the ideas of classical physics and this has led to completely new and strange ideas in what we call modern physics. Modern physics includes the theories of special relativity (e.g. different observers measure different time intervals of an event); general relativity (e.g. clocks run slower in stronger gravitational fields – the time synchronisation of mobile phones uses the principles of general relativity); and quantum theory (no clear distinction between what is a particle and wave; Heisenberg Uncertainly Principle, e.g. can’t know the position and momentum of a particle simultaneously).

FRAMES OF REFERENCE

Our starting point to understanding the Universe was that we had to know the position of each particle in space. Therefore, we need to set up a method of specifying the position of particles which is precise and unambiguous. To simplify this we will consider a two-dimensional universe. The methods we will develop can easily be extended give the position of particles in our real three-dimensional world (in terms of modern physics time and space are interwoven and a better model is to consider a four-dimensional world \([x, y, z, t]\)).

To clearly specify the position of a collection of particles we need to have a frame of reference. We take any point in space as an origin O. Through the origin O, we construct two lines at right angles to each other called the coordinate axis. These lines could be labelled \([x\text{-axis} \; y\text{-axis}]\) or \([\text{N} \; \text{S} \; \text{E} \; \text{W}]\) or \([\text{horizontal} \; \text{vertical}]\) as shown in figure (1).

![Frames of references: origin O and coordinate axes](image)

The most useful frame of reference is the one which has the axes labelled \([x\text{-axis} \; y\text{-axis}]\) and are called Cartesian axes. Figure (2) shows the position of three particles located at the points P, Q and R. The location of the particles in uniquely given in terms of their x and y coordinates. For example, the location of point P in terms of the given frame of reference has an x coordinate of 3 and a y coordinate of 4. The coordinates of the points are P(3,4), Q(6,-8) and R(-5 -5). The position can also be given in terms of the straight line distance between the origin and the point and an angle usually measured with respect to the +x-axis.
The location of a point in space can be uniquely specified by its x and y coordinates. The position of the three points can be written as P(3,4), Q(6,-8) and R(-5,5).

The distances $d$ between the origin O(0,0) and a point at $(x, y)$ is given by equation (1)

$$d = \sqrt{x^2 + y^2} \quad \text{Pythagoras' Theorem}$$

The distances from the origin O to the three points P, Q and R are given by $d_P$, $d_Q$ and $d_R$

$$d_P = \sqrt{3^2 + 4^2} = 5.00$$
$$d_Q = \sqrt{6^2 + (-8)^2} = 10.00$$
$$d_R = \sqrt{(-5)^2 + (-5)^2} = 7.07$$

The direction of a point at $(x, y)$ makes with the x-axis is be given by the angle $\theta$ (Greek letter theta) as expressed by equation (2)

$$\tan \theta = \frac{y}{x} \quad \theta = \arctan \left( \frac{y}{x} \right) \equiv \tan^{-1} \left( \frac{y}{x} \right)$$

and is shown in figure (3).
Fig. 3. The position of the three points expressed in terms of a straight line distance and an angle measured with respect to the x-axis.

The angles for the three points are (orientation of the angles is shown in figure (3))

\[
\theta_P = \tan^{-1} \left( \frac{4}{3} \right) = 51.3^\circ
\]

\[
\theta_Q = \tan^{-1} \left( \frac{-8}{6} \right) = -51.3^\circ
\]

\[
\theta_R = \tan^{-1} \left( \frac{-5}{-5} \right) = 45.0^\circ
\]

The location of the points can thus be written in terms of polar coordinates \((d, \theta)\) and the angle can also be given in terms of the points of the compass (NSEW)

\[
P \left( 5.00, +53.1^\circ \right) \quad 51.3^\circ \text{ NofE}
\]

\[
Q \left( 10.0, -53.1^\circ \right) \quad 51.3^\circ \text{ SofE}
\]

\[
R \left( 7.07, -45.0^\circ \right) \quad 45.0^\circ \text{ SofW}
\]
DISTANCE AND DISPLACEMENT

To proceed with our description of the Universe, three Cartesian coordinates \((x,y,z)\) are all that are need to determine the position of a particle at any time. In our Universe all particles are in motion so we need to develop a way to describe this motion. We are going to consider a single particle in a two-dimensional world. Our starting point is to define a frame of reference and at a time \(t_1\) we can choose the particle to be at the origin \(0\). The particle moves along a path or trajectory \(#1\) as shown in figure (4) to reach the point \(P\) at time \(t_2\). But there is no unique path from the origin \(O\) to the point \(P\). Another possible trajectory \(#2\) is also shown in figure (4).

The distance or distance travelled for trajectory \(#1\) is 22.0 m and for trajectory \(#2\) it is much more than 22.0 m. The distance is a scalar quantity. A much more useful quantity to describe the journey is the displacement where you only consider the initial position \(O\) at time \(t_1\) and the final position \(P\) at time \(t_2\). The displacement by definition is the straight line distance between the initial and final positions and the direction of the final position with respect to the initial position. Therefore, to specify the displacement one needs to state its magnitude (size) and direction. Such physical quantities are called vectors. You will come across many vector quantities such as velocity, acceleration, force, momentum, electric field and magnetic field.
Fig. 4. Two possible trajectories for the motion of the particle which starts at the origin $O$ at time $t_1$ and finishes at the point $P$ at time $t_2$.

A vector is represented graphically as an arrow whose length is proportional to the magnitude of the vector and points in the same direction as the vector quantity. The vector for the displacement for the journey from the origin $O$ to the point $P$ is shown in figure (5). The magnitude of the displacement is given by Pythagoras’ theorem and the angle to give the direction is determined from the right angle as shown in figure(5). Therefore, the displacement $\vec{s}$ of the point $P$ with respect to the origin is

$$s = \sqrt{6^2 + 8^2} \text{ m} = 10.0 \text{ m}$$

magnitude

$$\tan \theta = \frac{8}{6} \Rightarrow \theta = \tan^{-1} \left( \frac{8}{6} \right) = 53.1^\circ \text{ direction given by angle w.r.t x-axis}$$

There are many symbols to represent a displacement, e.g., $\vec{s}, \vec{r}, \vec{h}, \vec{x}, \vec{y}, \ldots$. The arrow over the top of the symbol indicates a vector quantity. In many books a vector is represent by a character in bold type, e.g. $\mathbf{s}$.

To indicate the displacement as a vector it is shown as $\vec{s}$ and to indicate only the magnitude it is written as $s$ (no arrow on top). To write a vector on paper you can place an arrow on top of the symbol or a curved line underneath

$$\vec{S} \quad \vec{s}$$
Fig. 5. Displacement is a vector represented by an arrow whose length is proportional to the magnitude of the vector quantity. The direction in which the arrow points gives the direction of the vector quantity.

**Resolving a vector into its components**
A vector quantity can be *resolved* into *components* along each of the coordinate axes. For the displacement shown in figure (5) it can be resolved into an $x$-component $s_x \equiv x$ and its $y$-component $s_y \equiv y$ (note: you could use $x$ and $y$ for the symbols or use $x$ and $y$ as subscripts).

To find the components of a vector draw a box around the vector. Then draw the two rectangular components as shown in figure (6). Note carefully that the two rectangular components replace the original vector. Avoid the mistake of many students who add the two components to the original vector, thus counting it twice.
From figure (6) we can conclude that

\[ \vec{S} = \vec{S}_x + \vec{S}_y \]

addition of vectors

\[ s = \sqrt{s_x^2 + s_y^2} \]

Pythagoras’ theorem – magnitude of the vector

\[ \tan \theta = \frac{y}{x} \quad \theta = \tan^{-1} \left( \frac{y}{x} \right) \]

angle to give direction of vector

\[ \sin \theta = \frac{y}{s} \quad y = s \sin \theta \]

\( y \)-component of the vector

\[ \cos \theta = \frac{x}{s} \quad x = s \cos \theta \]

\( x \)-component of the vector
Given the components of a vector we can find its magnitude and an angle to give its direction.

Given the magnitude of a vector and an angle w.r.t. a coordinate axis we can find its components.

For the displacement vector shown in figure (6)

\[
s = \sqrt{6^2 + 8^2} \text{ m} = 10.0 \text{ m}
\]

magnitude

\[
\tan \theta = \frac{8}{6} \quad \theta = \arctan \left( \frac{8}{6} \right) = 53.1^\circ
\]

direction given by angle w.r.t x-axis

\[
s_x = 10\cos(53.1^\circ) \text{ m} = 6.0 \text{ m}
\]

x-component

\[
s_y = 10\sin(53.1^\circ) \text{ m} = 8.0 \text{ m}
\]

y-component

**Addition of vectors**

Vectors do not add as simple numbers. We have to develop a set of rules for vector addition. Consider the addition of three displacements \( \vec{s}_1, \vec{s}_2, \) and \( \vec{s}_3 \) as shown in figure (7). The journey consist of:

- the displacement \( \vec{s}_1 \) from the origin \( O(0,0) \) to the point \( P(6,8) \)
- the displacement \( \vec{s}_2 \) from the point \( P(6,8) \) to the point \( Q(-8,6) \)
- the displacement \( \vec{s}_3 \) from the point \( Q(-8,6) \) to the point \( R(2,8) \)

The **resultant vector** \( \vec{s} \) is the sum of the three vectors

\[
\vec{s} = \vec{s}_1 + \vec{s}_2 + \vec{s}_3
\]

There are a number of methods in which vectors can be added. We will only consider the addition of components to give the resultant vector. This is not the necessarily the easiest method but it is the most useful. For vector addition, the vectors are drawn head-to-tail after each other as in figure (7). The resultant vector then points from the starting position (the origin in this example) to the final position (the point \( R \)). The resultant vector corresponds to the displacement of the particle at the end of the journey from our origin.
Fig. 7. The addition of three vectors to give the resultant vector. In a diagram for vector addition show the vectors connected head-to-tail and then the resultant is drawn from the start to the finish.

To find the components of each of the three vectors we are going to add, it is best to draw a diagram of each vector with a box around it. It is then an easy task to add the components for each vector as shown in figure (8).

Fig. 8. Each vector is shown separately with a box around it. It is then an easy task to determine the magnitude, direction and components of each vector.
Since all the x-components point in the same direction we can add them as numbers, and the same goes for the y-components. The resultant displacement is found as follows:

components of each vector are

\[ s_{x1} = (6.0 - 0) \text{ m} = 6.0 \text{ m} \]
\[ s_{y1} = (8.0 - 0) \text{ m} = 8.0 \text{ m} \]
\[ s_{x2} = (-8.0 - 6) \text{ m} = -14.0 \text{ m} \]
\[ s_{y2} = (6.0 - 8.0) \text{ m} = -2.0 \text{ m} \]
\[ s_{x3} = (-2.0 - (-8.0)) \text{ m} = 6.0 \text{ m} \]
\[ s_{y3} = (-8.0 - 6.0) \text{ m} = -14.0 \text{ m} \]

adding the components

\[ s_x = s_{x1} + s_{x2} + s_{x3} = (+6 - 14 + 6) \text{ m} = -2 \text{ m} \]
\[ s_y = s_{y1} + s_{y2} + s_{y3} = (+8 - 2 - 14) \text{ m} = -8 \text{ m} \]

From figure (7) it is obvious that the components of the resultant vector \( \vec{s} \) are \( s_x = -2 \text{ m}, s_y = -8 \text{ m} \) and these values are in agreement given by adding the components of each individual vector.

The magnitudes of the vector are (using equation (1))

\[ s_1 = \sqrt{6^2 + 8^2} \text{ m} = 10.0 \text{ m} \]
\[ s_2 = \sqrt{(-14)^2 + (-2)^2} \text{ m} = 14.1 \text{ m} \]
\[ s_3 = \sqrt{6^2 + (-14)^2} \text{ m} = 15.2 \text{ m} \]
\[ s = \sqrt{(-2)^2 + (-8)^2} \text{ m} = 8.2 \text{ m} \]

The total distance travelled from O to P to Q to R is

\[ (10.0 + 14.1 + 15.2) \text{ m} = 39.3 \text{ m} \]

whereas the magnitude of the displacement of the point R w.r.t. the origin 0 is 8.2 m. Distance travelled is a scalar quantity while displacement is a vector. Hence, must use different rules in adding scalars and vectors. Let us emphasize this again, the addition of vectors is a very special procedure clearly distinct from the addition of the scalar magnitude of the quantities.
The directions of the vector are found by using equation (2)

Best way to avoid ambiguity for direction / angle is to use a diagram to show the orientation. When using the function atan(y/x) to find the angle ignore the sign of the numbers for x and y. The angles are given w.r.t. the x-axis

\[
\theta_1 = \tan\left(\frac{s_{y1}}{|s_{x1}|}\right) = \tan\left(\frac{8}{6}\right) = 53.1^\circ
\]

\[
\theta_2 = \tan\left(\frac{s_{y2}}{|s_{x2}|}\right) = \tan\left(\frac{2}{14}\right) = 8.1^\circ
\]

\[
\theta_3 = \tan\left(\frac{s_{y3}}{|s_{x3}|}\right) = \tan\left(\frac{14}{6}\right) = 66.8^\circ
\]

\[
\theta = \tan\left(\frac{s_z}{|s_z|}\right) = \tan\left(\frac{8}{2}\right) = 80.0^\circ
\]