VIBRATION OF A WIRE

This experiment investigates the modes of vibration of a stretched wire. These are standing waves and the first five modes are shown in Fig. 1. The observed phenomena are common to all vibrating systems including springs, musical instruments, vibrating windows, cars, buildings, electrical circuits, atoms, molecules, planets, etc. A vibrating wire is chosen because it is one of the simplest systems to study.

![Fig. 1. The first five modes of vibration of a stretched wire. The nodes are marked N.](image)

In this experiment a steel wire is stretched between two bridges (supports) and passes between the poles of a strong magnet (see Fig. 2). An AC current is made to flow in the wire and so a periodic force perpendicular to both the wire and the field is produced. The wire vibrates at the applied frequency. The amplitude of the vibration becomes large when the applied force is at a frequency close to one of the natural modes of vibration of the wire and provided that the magnet is not located at one of the nodes.

![Fig. 2. Stretched wire and magnet for the vibrating wire experiment.](image)
Apparatus and Techniques

Design of Apparatus

The aim of this experiment is to test eqn (3) by varying $\tau$, $L$, $\mu$ or $n$ and measuring $f'$. The apparatus is arranged so that each of the modes can be excited separately. Instead of bowing or plucking the wire it can be driven continually at a fixed frequency. The set up is as shown below.

A steel wire is stretched between two fixed bridges. One end of the wire passes over a roller and is attached to a load carrier which hangs over the end. If the friction forces at the roller bearing and the contact between the wire and the bridge are small then the tension $\tau$ in the wire should be equal to the weight of the load, which can be varied using a set of slotted brass discs whose masses are nominally 1.0 kg each. The length of the vibrating part of the wire can be altered by placing an additional bridge between the two fixed bridges.

Safety Precautions

- The load carrier should be attached to the wire using a chain, so that it hangs fairly close to the floor. Avoid standing or sitting where your foot could be underneath the load.

There is a perspex cover over the top of the apparatus. This is to prevent the wire from flying out and hitting somebody if it should break. Don't put head or hands between the wire and this cover.

Keep watches away from the magnet.
Setting up the Apparatus
Place the base board so that the roller is over the end of the bench.

Select the thinner of the steel wires (the brown phosphor bronze wire is not suitable).

Hook the wire onto the end of the board, and place it over the two fixed bridges and the roller. Attach the chain and the load carrier so the carrier hangs just above the floor.

Check that the wire rests in the V notches on the bridges.

Add one or two brass discs to the carrier to stretch the wire. If there are kinks or bends in the wire it may be necessary to increase the tension to get a satisfactory vibration.

Gently pluck the wire - you should be able to hear a low-pitched tone. This tone should be clear, without rattles or buzzes. If that is not so check the alignment of the wire to make sure that it does not touch anything other than the bridges and the roller.

If the vibrations on the wire are to be driven electrically, put the magnet at a place where an antinode is expected and position it so that the wire is in the middle of the gap between the pole faces. If the wire will not stay in the middle, try increasing the tension.

Electrically Driving the Wire
The wire can be set into free vibration by plucking it. It can also be forced to vibrate at a particular frequency using an alternating current at frequency \( f \) is passed through the wire. This current is generated by passing the signal from an audio signal generator (oscillator) to a power amplifier. Part of the wire lies between the poles of a magnet where the magnetic field \( B \) is perpendicular to the wire. The magnetic force on the wire acts in a direction perpendicular to both \( B \) and the current in the wire and varies with time at frequency \( f \). The wire is thus forced to vibrate at the same frequency as the oscillator. Fig. 1 shows a possible location of the magnet for some modes; ideally it should be located at an antinode.

The EM Detector Probe
Whether plucked or driven electrically, the vibration of the wire can be observed by eye and usually heard. It is also possible to detect the vibrations using an electromagnetic detector which is mounted on a rail above the wire. The detector consists of a coil of wire (a solenoid) and an amplifier connected to the input of an oscilloscope. When the steel wire vibrates, the component of the magnetic field along the axis of the coil changes and produces an induced EMF in the coil. This changing voltage is amplified and displayed on the oscilloscope. Although the amplitude of the signal increases with amplitude of the mechanical vibration of the wire, it is not safe to assume that these two amplitudes are proportional.

Measuring the Driving Frequency
- Connect the signal generator output to the EXT COUNTER input terminal as well as the amplifier's input terminals. When the instrument is set to the EXT COUNTER function (press in the push button switch), the frequency meter counts the number of cycles of oscillation in a fixed time interval (there are seven ranges, 10 µs to 10 s) and displays the result as a frequency reading. Consult apparatus note AN97 for more details about the
KENWOOD FUNCTION GENERATOR FG-273; this note is available from the note racks in each laboratory.

Searching for Resonances
In order to find a resonance frequency experimentally, set the oscillator amplitude to maximum, then adjust the oscillator frequency slowly and wait for the resonance to build up as described below. It is a good idea to make a rough calculation of the frequency to be expected, then start the search a small amount above or below that value.

The EM detector unit can also be used to measure the frequency of free vibrations (ie with no current through the wire) to get an idea of the approximate resonant frequency. If you pluck the wire near the centre then the probe output should be close to the fundamental frequency.

To find a resonance proceed carefully and patiently. The range of driving frequencies over which the wire responds noticeably is quite small. Thus a rapid twiddling of knobs may miss a resonance altogether. When near a resonance, the wire will start to vibrate very slightly. Adjust the frequency very slowly until the wire vibrates with maximum amplitude. See Fig. 4.

Even when the driving frequency exactly matches the resonance frequency, it may take a few seconds to establish this resonating vibration. This time interval is about equal to the time taken for free vibrations to die out after the wire has been plucked.

When resonance has been found, reduce the amplitude of the driving signal and carefully repeat the search near the same frequency. Repeat this process until the maximum amplitude of vibration of the wire is less than 2 mm (4 mm peak to peak) at the antinode. This is the resonant frequency, $f'$. (This is necessary because when the amplitude of vibration is large, the resonant frequency is amplitude dependent. See Appendix 2.)

It is possible to have the wire resonating at frequencies $\frac{f'}{2}$, $\frac{f'}{3}$, \ldots $\frac{f'}{n}$. To guard against this, check that the driving frequency and the driven frequency are similar.

![Fig. 4. Dependence of amplitude on frequency, for a steady vibration near resonance.](image)

Auxiliary Measurements
Although the discs are supposed to have equal masses, measure the mass of each disc. Use the double beam balances - not the electronic (Mettler) balance - for this.

To find a value for the linear density $\mu$ of the wire either measure its diameter and use a given value for the density (see Data section below) or measure the length and the mass of a sample of the wire.
Hint

\[ \mu = \frac{m}{L} \quad \text{and} \quad \rho = \frac{m}{V}. \]

Thus \( \mu = \rho A \)

Data

- Density of steel: \( \rho = (7.8 \pm 0.1) \times 10^3 \text{ kg.m}^{-3} \)
- Local gravitational field: \( g = (9.797 \pm 0.0005) \text{ m.s}^{-2} \)
- Young's modulus for steel: \( E = (2.10 \pm 0.01) \times 10^{11} \text{ N.m}^{-2} \)

Experiment and Analysis

- Fundamental Frequency: Tension and Length (thin wire)

- Modes of Vibration (thick wire) and resonant frequency of a stiff wire (see appendix)

- Chaotic Behaviour of a Non-magnetic Wire

In the above experiment the steel wires vibrate in a vertical plane rather than in the horizontal plane since the force on the wire is in a vertical direction. Steel is attracted by magnets and this together with the shape of the magnetic field between the poles of the magnet keep its vibrations vertical.

A surprising result is obtained if the steel wire is replaced with a non-magnetic wire such as phosphor-bronze. In this case, the wire tends to rotate in an elliptical or circular orbit, see Fig. 5, even though the applied force is in the vertical direction. Any slight bend in the wire or any slight motion of the wire away from the vertical direction will start it moving in an elliptical orbit. The same thing happens in a pendulum. Provided the pendulum is free to move in any direction, it will eventually move in an elliptical orbit even if it starts swinging in a straight line.

Whenever the frequency is changed, the motion becomes chaotic. It is impossible to repeat the experiment and get the same sequence of chaotic vibrations. The theory of chaotic phenomena is one of the interesting developments in modern physics. A useful introduction may be found in an article "Chaos" by J.P. Crutchfield et al in Scientific American, pp 38 - 49, Dec 1986.

T (ms) = 13.35 ................................. 13.13
The aim of this exercise is to observe the chaotic behaviour of a phosphor-bronze wire.

Change the steel wire for a phosphor-bronze one and observe the elliptical motion by eye, especially near resonance where the amplitude is large (>5 mm peak-to-peak). It is necessary to observe the vibration from both vertical and horizontal directions to see the elliptical circular motion.

Sketch the orbit while increasing or decreasing the frequency through resonance, see Fig. 4.

**Theoretical exercise**

This exercise illustrates chaotic behaviour.

If you have access to a computer, try plotting $x_{n+1}$ against $y_n$ where

$$x_{n+1} = 1 - ax_n^2 + y_n; \quad y_n = bx_n$$

starting with $x_1 = 0, y_1 = 0$ for $b = 1$ or -1 and a value of $a$ in the range $0.1 < a < 1.5$. A catastrophe sets in after about 1000 iterations, depending on the value of $a$. Try, for example $a = 0.5, 0.3007, 0.225, 0.205, 0.2$, and 0.14.

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**APPENDIX 1**

**Resonant Frequency of a Stiff Wire**

As the wire bends to accommodate standing waves in its various modes of vibration, some portions of the wire elongate, some portions compress and some portions are unchanged. The ability of a wire to execute these standing waves is therefore related to its radius and stiffness (Young's modulus). It is more difficult to bend a wire into standing waves:

1. as the thickness of the wire increases,
2. as the stiffness of the wire increases,
3. and as the mode number increases (it has to form tighter loops).
The simple theory does not contain a reference to any of these effects. When they are included eqn (3) becomes:

\[ f' = \frac{n}{2L} \sqrt{\frac{\tau}{\mu}} \sqrt{1 + Bn^2} \]  

(4)

where

\[ B = \frac{\pi^3 E r^4}{4 L^2 \tau} \]

and

\[ E = \text{Young's modulus of the wire} \]

\[ r = \text{radius of the wire} \]

Note that \((1 + Bn^2)\) is dimensionless and that the correction varies as the mode number. Thus the correction becomes more important at the higher modes.

For some of your points which deviate systematically from a straight line, check the effect of this correction.

**APPENDIX 2**

**Dependence of Resonant Frequency on Amplitude for Large Amplitudes.**

![Graph showing dependence of amplitude on frequency](image)

Fig. 5. Dependence of amplitude on frequency, when increasing and decreasing frequency near resonance.

The fundamental resonant frequency is obtained when the frequency is steadily increased from below, but not when decreased from above - i.e. there is hysteresis present, see Fig. 5. If the maximum amplitude is steadily decreased by reducing the signal strength, the difference between the two curves shown in Fig. 5 decreases.

Large amplitudes cause an increase in tension. The weights producing the tension vibrate upwards from their rest position twice per cycle. The tension must increase over the equilibrium value to move the masses upwards. In addition any stretching of the wire that takes place requires an increase in tension. Thus during oscillation the tension reaches some higher average value which causes an increase in the resonant frequency. The small amplitude resonant frequency is lower in value and provides a better estimate.