Light scattering by surface tension waves

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Abstract: Measurements of the ratio of surface tension to density for tap water, mercury and methanol are reported in an assessment of an experiment first described by Weisbuch and Garbay and involving the diffraction of laser light by ripples on the surface of a liquid.

1. Introduction

Weisbuch and Garbay\textsuperscript{1} have described a novel undergraduate teaching experiment in which laser light is diffracted by a regular pattern of ripples generated on the surface of a liquid by a simple vibrator. We have set up this experiment as shown in Fig. 1 in our second year undergraduate teaching laboratory, and used it to measure the ratio of surface tension to density for tap water, mercury and methanol. As an indication of the success of the experiment as a teaching aid we report the results of our measurements together with some effects and precautions not mentioned by Weisbuch and Garbay\textsuperscript{1} as well as some comments on the theory.

![Diagram of equipment](image)

Fig. 1. Diagram of equipment.

2. Experimental details

It simplifies the theoretical description if the ripples have straight wave fronts rather than circular as in the set-up of Weisbuch and Garbay\textsuperscript{1}. Accordingly, in addition to the "needle" probe of Weisbuch and Garbay,\textsuperscript{1}
we constructed a vibrator in the shape of an inverted stiff wire "goal-post" glued to the diaphragm of a loudspeaker as illustrated in Fig. 2. A further advantage of the "goal-post" probe is improved vibrational energy transfer to the liquid. We also built a custom-made electronics unit housing a sine wave voltage generator with frequency variable between 40 Hz and 2 kHz and variable output amplitude up to 3 V peak-to-peak with a low source impedance to drive the loudspeaker. The unit incorporates a digital frequency meter.

The experiment was situated on a concrete flagstone isolated from vibrations of the laboratory bench by partially inflated motor-car inner tubes. This latter precaution is essential as is the avoidance of draughts. The "lazy-tongs" platform for supporting the saucer containing the liquid is very useful for adjusting the probe depth and changing liquids.

The depth of the water is of the order of a centimetre and the frequencies available ensure that the deep-water limit and the neglect of gravity and viscous terms as described by Weisbuch and Garbay all apply. It is important to clean and degrease the surfaces in contact with the liquids and to use clean liquids.

3. Theory

With straight ripples, one has a type of reflection grating. Weisbuch and Garbay suggest that the scattered amplitude is merely the Fourier transform of the sinusoidal surface-wave displacement, and hence they predict only two diffraction peaks, as in Brillouin scattering. This is oversimplified. The appropriate aperture function of the grating is determined mainly by the phase shifting of the reflected wave due to variations of surface height. There may also be blocking of reflections by the wave crests. Thus the "envelope" function of the diffraction pattern is subtle and interesting, and depends on the amplitude of the ripples as well as their spacing. By adjusting the ripple amplitude some of the spots may even be partially suppressed with the appearance of dark bands through their centres. We found that a useful model was to think of the troughs
and peaks of the surface waves as two separate reflection gratings.

However, the angular positions of the diffraction spots are given by the usual simple considerations. Suppose the light beam (wavelength \( \lambda \)) is directed at right angles to the ripples (wave vector \( q \)), and at glancing angle \( \theta \) to the liquid surface. The \( n \)th reflection occurs at angle \( \theta_n \) given by

\[
(2\pi/q)(\cos \theta - \cos \theta_n) = n\lambda
\]  

We write \( r_n = (\theta_{n+1} - \theta_n) \). Then, subtracting the \( n \)th equation from the \( (n+1) \)th,

\[
q = (4\pi/\lambda) \sin (r_n/2) \sin [(\theta_n + \theta_{n+1})/2] \tag{2}
\]

If \( r_n < \theta_n \) (e.g. \( \theta = 70^\circ \) and \( \pi/\theta = 0.02 \) in our case),

\[
q = (2\pi/\lambda) r_n \sin \theta_n \tag{3}
\]

We note that the spacing of the spots \( r_n \) varies with \( n \). With circular ripples the same values of \( \theta_n \) are to be expected. This is because, firstly, at glancing incidence the laser beam samples a long, narrow swathe of surface so that the curvature of the ripples is immaterial. Secondly, although the surface displacement is given by a zero-order Bessel function with unequally spaced ripples, this reduces to the form \( r^{-1} \exp(\text{i}qr) \) after several wavelengths from the centre. The laser must not be reflected from the central region in any case, for either probe, because the ripples spread apart here. In effect the beam would sample two separate gratings in relative motion, producing a fringe pattern which varies on a time scale the order of the wave period.

Although Weisbuch and Garbay\(^1\) appear to suggest that the diffraction pattern for circular ripples differs from that for straight ripples, nevertheless their equation for circular ripples is the same as our equation (2) so long as \( r + l = r - 1 \).

4. Observations

Figure 3 shows typical spot patterns produced by the "goal-post". Progressively more vibration amplitude is necessary to produce a visible pattern as the frequency is increased. It is important to avoid over-driving the loudspeaker since this generates unwanted harmonics. Best results were obtained with the probe just below the liquid surface. If the probe is too high, splashing and other non-linear effects can occur.

When the angular spread of the pattern is large, it becomes quite clear that the angular spacing of the spots decreases towards the top, as predicted by equation (3). As the loudspeaker drive amplitude is increased the pattern extends showing more spots, as in Figs 3 (a) and (b), until a
stage is reached where the central spot is diminished in intensity through the appearance of a horizontal line through its centre. This is shown in Fig. 3 (c). With further increase in amplitude the central spot regains its brightness but the first two side spots \((n = \pm 1)\) become partially suppressed in the same way. Even more exotic patterns appear as the amplitude is increased still further. We also observed the disappearance of the fringes when the beam is reflected from the area of the probe. However, none of these three phenomena were investigated quantitatively.

![Fig. 3](image)

**Fig. 3.** Spot patterns on a plane vertical screen obtained for tap water with vibrator frequencies of (a) 253 Hz, (b) 456 Hz, (c) 393 Hz. The increased spot spacing and reduction in visibility caused by increasing the frequency is evident by comparing (b) with (a). The reduction in the spacing of the spots towards the top described by equation (3) is apparent in (a) and overcompensates for the opposing effect arising from the inclination of the flat screen to the light beam. Partial extinction of the central spot is demonstrated in (c).

5. Results

As Wiesbuch and Garbay\(^1\) point out, the dispersion relation for straight ripples under the experimental condition is

\[
\omega = (A/\rho)^{1/2} q^{1/2}
\]

where \(\omega\) and \(q\) are the angular frequency and wave vector of the ripples, \(A\) is the surface tension and \(\rho\) the density. We obtained \(q\) by measuring the distances of the first intense spots from the central spot with a ruler and averaging, and \(\omega\) from the frequency meter reading.

We checked the 1.5 exponent by drawing log–log plots of our results as did Wiesbuch and Garbay\(^1\) but measuring intercepts on these plots to
obtain $A/p$ produces results with unnecessarily large random errors. More reliable $A/p$ results are obtained by assuming the 1.5 exponent and measuring the gradients of plots of $q^3$ against $\omega^2$. These are shown in Fig. 4.

Fig. 4. Plot of $q^3$ against $\omega^2$ for three liquids: tap-water ◦, mercury □ and methanol ×. The representative error bars are estimates based on the precision with which the spot positions could be measured.

It is worth mentioning that the measurement of the height of the central spot above the liquid surface which is necessary in determining \( \theta \), must be done carefully. Errors here are systematic in the determination of \( q \) and then become tripled in the $q^3$, $\omega^2$ plots as well as affecting the exponent determinations. We carried out this measurement by setting the laser beam horizontal and grazing the liquid surface to produce a spot on the screen corresponding to the height of the liquid.

Our results on the three liquids are summarised in Table 1 where the experimental errors include both random errors through the measurement of the spot spacing and an estimate of the systematic error mentioned above. It can be seen that the 1.5 exponent is verified within the uncertainty and that the $A/p$ measurements for mercury and methanol agree with accepted values obtained from Kaye and Laby. However our $A/p$ value for tap water is lower than the accepted value and outside our
error estimates. We suspect this is a result of the use of tap water which may have had some surface contamination.

6. Conclusions

We did not find any substantial difference between the results obtained with circular and straight ripples. In view of this it would probably be sensible to dispense altogether with the needle probe.

This experiment has the merit of containing elements of several branches of physics, namely, properties of matter, optics, mechanics of vibrating systems and waves. It can be completed in 2 to 3 days and is of first or second year degree level at a British university. It can be extended by investigating the interesting details mentioned in Sections 3 and 4. It was used for a 10-day project by students who wished to obtain intensity profiles of the diffraction pattern using a photo-diode detector.

REFERENCES