

**ELECTRON DYNAMICS**

**PREPARATION**

You will need: to read and understand section 26-8 of SZY.

**INTRODUCTION**

These experiments are concerned with the dynamics of sub-microscopic particles - electrons. Although individual electrons cannot be seen, the trajectory of a beam of electrons in a cathode ray tube can be studied. The influence of electric and magnetic forces on the electrons can be observed as changes in the position at which the beam strikes a screen. The weight of an electron is minute in comparison to the electric and magnetic forces used in this experiment so that gravity produces no detectable effects on the electron trajectories.

The theoretical description of the electron motion is based on Newton's laws of motion together with formulae for the electric and magnetic forces. The conclusions derived from this theoretical description will not necessarily agree with experimental results because they are based on inadequate descriptions of the geometry of the electric and magnetic fields produced in the apparatus.

**THEORY**

**Electrostatic Deflection of Electrons**

Electrons, with mass \( m \) and charge \( e \), emerge from an electron gun with a kinetic energy determined by the accelerating voltage \( V_1 \) (see fig. A3.1). The velocity \( v \) of the beam is (see eqn (26-31) in SZY):

\[
v = \sqrt{\frac{2eV_1}{m}}.
\]  

(A3.1)

![Electrostatic Deflection Diagram](image)

**Fig. A3.1 Deflection of an electron beam by a uniform electric field**

Now the beam of electrons, with velocity \( v \), enters a region of uniform electric field. This region has potential difference \( V_2 \) and is in between two parallel plates which are a distance \( l \) apart. If the length of the parallel plates is \( L \) and the electron beam travels an additional distance \( D \) after emerging from the plates, as shown in fig. A3.1, then, the deflection \( y \) is given by (see SZY § 26-8):

\[
y = \left[ \frac{L}{2l} \left( D + \frac{L}{2} \right) \right] \frac{V_2}{V_1}.
\]  

(A3.2)
Motion of Electrons Normal to a Uniform Magnetic Field

An electron moving with velocity $v$ in a magnetic field $B$ experiences a force:

$$ F = ev \times B $$

This is a vector equation and the magnitude of the force is:

$$ F = evB \sin \theta $$

where $\theta$ is the angle between $v$ and $B$ and that the direction of the force is perpendicular to both $v$ and $B$ (in a sense given by the right hand rule).

An electron injected into a uniform magnetic field and moving perpendicular to the field experiences a force:

$$ F = e v B $$

Now the force remains perpendicular to the velocity and the electron moves in a circular path of radius $R$. The radial (centripetal) acceleration is:

$$ a = \frac{v^2}{R} $$

Applying Newton's second law of motion:

$$ F = ma $$

$$ e v B = m \frac{v^2}{R} $$

i.e. $\frac{v}{R} = \frac{e}{m} B$.

Now the speed of an electron travelling in a circle is the circumference of the circle divided by the period (time taken for one revolution):

$$ v = \frac{2\pi R}{\text{period}} = 2\pi Rf = \omega R $$

where $\omega$ is the angular velocity or angular frequency of the circular motion.

Thus:

$$ \omega = \frac{v}{R} = \frac{e}{m} B \quad \text{(A3.3)} $$

Note that $\omega$ is independent of $v$, the electron's speed. This angular frequency is often called the angular cyclotron frequency for the particle in the field, $B$.

Magnetic Deflection of Electrons

Now consider an electron which is fired along the axis of a cathode ray tube with speed $v$ and then enters a region, of length $b$, containing a uniform magnetic field, $B$, which is perpendicular to the axis of the tube, see fig. A3.2.

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**Fig. A3.2** Deflection of an electron beam by a uniform magnetic field normal to the velocity.
The electron moves in a circle of radius $R$ and is deflected through an angle:
\[ \theta = \omega T \]  
(A3.4)

and a linear distance:
\[ y_0 = b \tan \frac{\theta}{2} \]

before it leaves the region of the field. $T$ is the time spent in the field.

If $\theta$ is small, i.e. if the transverse velocity acquired is small compared with $v$, the following approximations can be made:
\[ T = \frac{b}{v} \]  
\[ \tan \frac{\theta}{2} = \frac{\theta}{2} = \frac{\omega T}{2} = \frac{\omega b}{2v} \]

Hence
\[ y_0 = \frac{\omega b^2}{2v} \]

The electron leaves the field region with a transverse velocity:
\[ v' = v \sin \theta \]

which, by the same approximations, can be written:
\[ v' = b \omega \]

If the electron now travels a further longitudinal distance, $c$, in a field free region it acquires an additional transverse displacement $y_1$. Since the longitudinal velocity is still approximately $v$,
\[ \frac{y_1}{c} = \frac{v'}{v} \]
\[ y_1 = \frac{\omega}{v} bc \]

Hence the total transverse displacement is:
\[ y = y_0 + y_1 = \frac{\omega}{v} (b^2 + bc) \]

Substituting for the electron cyclotron frequency $\omega$ from eqn (A3.3) shows that $y$ is proportional to $B$ and hence, also to the current $I$ which produces $B$.
\[ y = \frac{eB}{mv} (b^2 + bc) \]

Since the force on the electron is always perpendicular to its velocity, the magnetic field does no work on the electron. Hence its kinetic energy is unchanged and its speed remains constant. Substituting for $v$ from eqn (A3.1):
\[ y = \sqrt{\frac{eB}{2m \sqrt{V_1}}} (b^2 + bc) \]  
(A3.5)

The situation considered above is an approximation to the experimental arrangement; $B$ is not uniform and does not extend over a sharply defined region. However this does not affect the two conclusions just stated, provided the assumption of a small deflection angle is adhered to. To see this, note that the non-uniform field can be divided into many small regions, each one small enough for $B$ to be regarded as constant in that region, and in each case the electron undergoes a small deflection according to eqn (A3.4). In each case:
\[ \Delta y_0 \propto \frac{B}{\sqrt{V_1}} \]
\[ \Delta y_0 \propto \frac{1}{\sqrt{V_1}} \]

Note that eqn (A3.2) shows that the deflection is inversely proportional to the accelerating potential difference. The difference is, of course, that here the magnetic deflecting force is proportional to the electron velocity whereas the electrostatic deflecting force is independent of velocity.
APPARATUS AND TECHNIQUES

Design of Apparatus

The central piece of apparatus is a cathode ray tube (CRT). It is an evacuated glass tube whose basic structure is shown schematically in fig. A3.3. An electron gun, consisting of several electrodes, accelerates and focuses a beam of electrons. Electrons leaving the gun travel along the axis of the tube until they encounter regions of transverse electric field produced by applying potential differences between two sets of deflection plates (X and Y). After deflection by these fields the electrons again travel with uniform velocity until they strike a phosphorescent screen at the end of the tube.

The CRT is permanently connected to the voltage supply which produces all necessary accelerating, focusing, intensity control and deflection voltages. Controls and meters are labelled and should be self-explanatory.

**Fig. A3.3  Schematic diagram of the cathode ray tube, CRT**

Safety Precautions

**DO NOT, UNDER ANY CIRCUMSTANCES, TRY TO REMOVE OR POKE AROUND UNDER THE FIXED COVERS. THE VOLTAGES USED IN THIS APPARATUS ARE LETHAL.**

HANDLE THE CATHODE RAY TUBE WITH GREAT CARE.

The tube has been highly evacuated and if the envelope is broken the resulting implosion could spray shattered glass with possible serious consequences. Avoid handling the tube while wearing diamond rings which might scratch the glass. Do not subject the tube to any impact or shock.

Make sure that the milliammeter is set on a safe range. Start by using the highest available range and switch to a lower range only if the reading is low enough. Check the polarity (+,-) of the connections before switching on.
Construction of X and Y Deflection Plates

In most CRTs the deflection plates are constructed in tapered sections as shown in fig. A3.4 thus the electric field between the plates is not uniform. However, provided that the geometry of the apparatus remains fixed, eqn (A3.2) can be rearranged into the following format:

\[ y = k_1 \frac{V_p^2}{V_i^2} \]

where

\[ k_1 = \left[ \frac{L}{2} \right] \left( D + \frac{L}{2} \right) \]

(A3.6)

and \( l' \) is an effective average value of \( l \) over the whole length of the plates.

Fig. A3.4 Tapered deflection plates

For the CRT type D7-36 used in this experiment, the effective values of \( L, D \) and \( l' \) are as follows:

<table>
<thead>
<tr>
<th></th>
<th>( D ) / mm</th>
<th>( L ) / mm</th>
<th>( l' ) / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>X plates (horizontal deflection)</td>
<td>159.0 ± 0.5</td>
<td>27.0 ± 0.2</td>
<td>3.45 ± 0.15</td>
</tr>
<tr>
<td>Y plates (vertical deflection)</td>
<td>191.6 ± 0.4</td>
<td>27.8 ± 0.1</td>
<td>3.0 ± 0.1</td>
</tr>
</tbody>
</table>

Table A3.1 Effective dimensions of CRT type D7-36. Uncertainties quoted are limits of inaccuracy based on the variations in actual dimensions amongst different tubes.

Providing a Magnetic Field Normal (Transverse) to the Electron Path

Magnetic fields are produced by passing electric current through solenoids placed outside the CRT. The arrangement required to produce a transverse magnetic field is shown in fig. A3.5.

Fig. A3.5 Connection of solenoids for transverse magnetic field
Provided the geometry of the apparatus remains fixed, eqn (A3.5) can be rearranged into the following format:

\[ y = k_2 \frac{B}{\sqrt{V_1}}. \]

However

\[ B \propto I. \]

Therefore

\[ y = k_3 \frac{I}{\sqrt{V_1}}. \]

where

\[ k_3 = \frac{1}{\sqrt{2m}} \left( \frac{r^2 + b^2}{a^2} \right) \left( \frac{4}{a^2} \right)^{3/2} \frac{\mu_0 N}{r}. \]  \hspace{1cm} (A3.7)

If spot deflection \( y \) is plotted as a function of current \( I \) for a fixed value of accelerating voltage \( V_1 \), then the slope \( s' \) is:

\[ s' = k_3 \frac{1}{\sqrt{V_1}}. \]

And if this is repeated for another fixed value of accelerating voltage \( V_1^{''} \) with slope \( s'' \), then the ratio of the slopes is:

\[ \frac{s'}{s''} = \sqrt{\frac{V_1^{''}}{V_1}}. \]  \hspace{1cm} (A3.8)

**Magnetic Field of the Helmholtz Coils**

The field produced on the axis of a single circular Helmholtz coil (solenoid) is:

\[ B = \frac{\mu_0}{2} \frac{NIr^2}{(r^2 + z^2)^{3/2}}; \]

where \( r \) = radius of the coil,

\( z \) = distance along the axis from the centre of the coil,

\( N \) = number of turns in the coil,

\( \mu_0 \) = permittivity of free space = \( 4\pi \times 10^{-7} \) Weber Amp\(^{-1}\) m\(^{-1}\),

\( I \) = current in the wire.

If two such coils are separated by a distance equal to their radius, \( r \), then the resultant field at a point on their common axis, mid-way between the coils is:

\[ B = \left( \frac{4}{a} \right)^{3/2} \frac{\mu_0 NI}{r}. \]  \hspace{1cm} (A3.9)

This field changes very little in the region of space between the coils. For points off the axis the calculation is nasty. Actual measurements show however, that at a distance \( \frac{r}{3} \) from the axis in the mid-plane \( B \) is down by less than 2%, and at \( \frac{r}{2} \) it is down by less than 5%. 

EXPERIMENT AND ANALYSIS

Part I Investigating the Deflection of a Beam of Electrons by an Electric Field

From eqns (A3.2) and (A3.6) the deflection of the spot on the CRT screen depends on the accelerating voltage \( V_1 \) and the deflecting voltage \( V_2 \). We study these relationships by controlling the parameters \( V_1 \) and \( V_2 \). While examining these relationships we determine the value of \( I' \).

1. Set up the CRT. Ensure that the spot is well focused and the intensity setting is suitable.
2. Choose either X or Y deflection but stick to whichever axis you choose. Set the accelerating voltage to some non-zero value; record this value and its limits of resolution.
3. With the deflecting voltage at zero (switched off) record zero spot position.
4. For 10 or more values of deflecting voltage measure as large a range of spot positions as possible. Subtract zero spot position from spot positions to get spot deflection. Record the limits of resolution of your measurements.
5. Plot a graph of spot deflection against deflecting voltage. Measure the slope of the graph and hence find a value \( I' \) with its uncertainty.
6. Switch off the deflecting voltage. The spot position is now at zero. Qualitatively observe if the zero spot position varies with accelerating voltage. If it does, then you have a shifting zero.
7. Set the deflecting voltage to some non-zero value. Take at least 10 readings of spot deflection (difference between spot positions with deflecting voltage on and off) as a function of accelerating voltage.
8. Plot a graph of spot deflection against (accelerating voltage)\(^{-1}\). Estimate another value for \( I' \) with its uncertainty.
9. Compare \( I' \) obtained from 8 and 5 with the value quoted in table A3.1.

Part II Investigating the Deflection of a Beam of Electrons by a Magnetic Field

The aim here is to study the relationship between spot deflection and an applied transverse magnetic field, eqn (A3.5) and (A3.7).

1. Set up the apparatus as shown in fig. (A3.5) with the deflecting voltages at zero.
2. For a non-zero accelerating voltage take at least 10 measurements of spot deflection (difference between spot positions with deflecting voltage on and off) as a function of current (using both positive and negative values for the current).
3. Repeat step 1 for another non-zero value of accelerating voltage.
4. Plot spot deflection as a function of current for both sets of data.
5. What relationship do your results indicate between magnetic field and spot deflection? Do the ratio of the slopes behave as predicted by eqn (A3.8)?
6. Why is the spot not at the centre of the screen in the absence of applied electrostatic and magnetic fields? This suggests that even when there are no deliberately applied fields some electric or magnetic field is affecting the beam.

With the meter function switch set to zero and the magnetic field coils removed lift up the CRT and vary its orientation. Can you find an orientation for which the spot does fall at the centre of the screen? What is the significance of this orientation? Can you now suggest why the spot does not appear at the centre when the CRT is in its usual orientation?

Part III Determining the e/m ratio

1. Evaluate the constant \( k_3 \) (see eqns (A3.7) and (A3.9)) for your set up in Part II.
2. Use results from Part II and eqn (A3.7) to estimate the e/m ratio.
3. How does this compare with the expected value.

CHALLENGE

Theoretical
How is this problem (part II step 5) avoided in oscilloscopes and television sets?

Experimental
Experimentally examine the relationship between accelerating voltage and deflection in the presence of a constant transverse magnetic field.