INSTRUCTIONS:

There are four sections in this paper, each worth 45 marks:

Section A: BIOPHYSICS
Section B: PLASMA PHYSICS
Section C: HIGH ENERGY PHYSICS
Section D: THERMODYNAMICS

You should attempt the two (2) subjects in which you are enrolled.

You must answer each section in a separate booklet.

You are permitted to take two (2) A4 pages, handwritten on one side only, into the examination room.

Non-programmable calculators are permitted.

The following data may be useful:

- speed of light: $c = 3.00 \times 10^8 \text{ m s}^{-1}$
- fundamental charge: $e = 1.60 \times 10^{-19} \text{ C}$
- electron mass: $m_e = 9.11 \times 10^{-31} \text{ kg}$
- Boltzmann constant: $k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$
- electron volt: $\text{eV} = 1.60 \times 10^{-19} \text{ J}$
- gravitational constant: $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
- (Planck’s constant)/$2\pi$: $\hbar = 1.05 \times 10^{-34} \text{ J s}$
SECTION A: BIOPHYSICS
Use a separate booklet. Please write “Section A” on the cover.

1. **All students:** Give brief physical explanations for the following.

   (a) Using the fact that proteins do not fold in non-polar liquids, describe the role played by water in folding of proteins.

   (b) Describe in one sentence how enzymes catalyse reactions (just the key idea). Explain what causes the saturation of enzyme reactions and how it can be described using the Michaelis-Menten relationship.

   (c) Propagation of action potential requires two types of ions (Na$^+$ and K$^+$) diffusing in and out of axons. Give a qualitative description of this process and explain why it cannot be achieved with only one type of ion.

   (15 marks)

2. **All students:** Non-polar molecules attract each other in water due to hydrophobic forces, e.g., they are responsible for aggregation of Abeta proteins that causes Alzheimer’s disease. For simplicity, consider aggregation of cubic proteins of size $a = 5$ Å.

   (a) Using the van’t Hoff relation for osmotic pressure, $p_{\text{equil}} = ckT$, where $c$ is the number density of proteins, show that when two of these proteins stick together forming a dimer, the free energy changes by

   $$\Delta G_d = -ckT \times a^2 b,$$

   where $b = 3$ Å is the diameter of water. Estimate this energy for $c = 1$ mol/L solution of the proteins and comment on its strength by comparing with the average kinetic energy ($1$ mol/L = $1/1661$ Å$^{-3}$).

   (b) Using the above result, show that the free energy change when these proteins aggregate into larger cubes of size $na$ containing $N = n^3$ proteins is given by

   $$\Delta G_N = 3(n^3 - n^2)\Delta G_d.$$

   (c) Calculate $\Delta G_N$ for $n = 2, 3$ and compare with the average kinetic energy. Also find their formation probability relative to that of dimer. Use these results to discuss the role of cooperative effects on aggregation of weakly interacting proteins.

   (15 marks)
3. **Normal only:** The Goldman-Hodgkin-Katz solution of the Nernst-Planck equation for flux of charged particles across the membrane is given by

\[ j = \frac{DqV}{kTL} \left( \frac{c_{\text{out}} e^{qV/kT} - c_{\text{in}}}{e^{qV/kT} - 1} \right), \]  

where \( j \) is the flux of particles, \( D \) and \( q \) are their diffusion coefficient and charge, \( c_{\text{in}} \) and \( c_{\text{out}} \) are their concentration inside and outside the cell, \( L \) is the thickness of the membrane, and \( V \) is the membrane potential (i.e. \( V = V_{\text{out}} - V_{\text{in}} \)).

(a) Show that for \( q|V| \ll kT \), the above solution reduces to a superposition of Ohm’s and Fick’s laws.

(b) Find the approximate solutions of the flux when \( q|V| \gg kT \) for both positive and negative \( V \). Explain why the two results are different.

(c) Find the Nernst potential for the case \( c_{\text{in}} = 10 c_{\text{out}} \) and \( q = e \) (note, \( 1 kT = 1/40 \ eV \)). Sketch the behaviour of the flux as a function of \( V \) using the limiting values as a guide.

(15 marks)

4. **Advanced only:** The Nernst-Planck equation describes the transport of charged molecules. In 1-D, it is given by

\[ j = -D \left( \frac{qc}{kT} \frac{d\phi}{dx} + \frac{dc}{dx} \right), \]  

where \( j \) is the flux of molecules, \( D \) and \( q \) are their diffusion coefficient and charge, \( c \) is concentration and \( \phi \) is potential. For consistency, this equation should be solved together with the Poisson equation in 1-D. However, because this is difficult in practice, one often resorts to a constant field approximation.

(a) Solve the Nernst-Planck equation using \( d\phi/dx = E_0 \), constant, in Eq. (2). (Hint, first find the solution for the homogeneous equation, and then add to it the particular solution from the inhomogeneous equation.)

(b) Determine the constants in your solution using the boundary conditions: \( c(o) = c_0 \) and \( c(L) = c_L \), and find an expression for the flux \( j \).

(c) Examine the limits of your solution for \( j \) for \( qE_0 L/kT \ll 1 \) and \( qE_0 L/kT \gg 1 \). Show that they correspond to the Fick’s and Ohm’s laws, respectively.

(15 marks)
SECTION B: PLASMA PHYSICS
Use a separate booklet. Please write “Section B” on the cover.

5. **All students:** In no more than one or two sentences each, explain the following plasma physics concepts and say why they are physically relevant:

(a) The plasma frequency: estimate its value in the Earth’s ionosphere
(b) Debye screening
(c) The phase and group velocities
(d) Landau damping
(e) Dispersion relation: write down the dispersion relation for transverse waves in an isotropic plasma
(f) Runaway electrons
(g) Alfvén waves: write down their dispersion relation
(h) The frozen-in condition
(i) Magnetic pressure
(j) An adiabatic invariant: give an example

(15 marks)

6. **All students:**
A plasma consists of electrons and protons with equal number densities, \( n \), with a uniform magnetic field, \( \mathbf{B} \) along the \( x \)-axis. There is a uniform gravitational acceleration \( g \) along the negative \( z \)-axis.

(a) Show that a particle with mass \( m \) and charge \( q \) drifts with velocity \( mg \times \mathbf{B}/qB^2 \). (You may appeal to the electric drift as an analogy.)
(b) Calculate the current density, \( \mathbf{J} \), due to the gravitational drift of electrons and protons.
(c) Calculate the force per unit volume, \( \mathbf{J} \times \mathbf{B} \), due to this current density.
(d) Write down the total force per unit volume acting on the plasma, including the \( \mathbf{J} \times \mathbf{B} \) term and the gravitational force per unit volume.
(e) Give a physical interpretation of your answer to part (d).

(15 marks)
7. **Normal only:** Micro-particles or aerosols (1–10 μm) are usually negatively charged and in steady-state non-thermal plasma. Assume that micro-particles are spherical and located in a non-thermal plasma with electron temperature 1 eV and room temperature ions.

(a) Estimate the typical charge of such particles as a function of their radius.

(b) What happens to the charge when the particle moves from the plasma into the sheath?

(c) How will the charge change if the micro-particles are located in a plasma with electron and ion temperatures both equal to 1 eV?

(15 marks)

8. **Advanced only:** The plasma dispersion function is

$$Z(y) = -\frac{1}{y} \left\{ \begin{array}{ll} y^2 - \frac{1}{2}y^4 + \cdots & \text{for } |y^2| \ll 1, \\ 1 + (1/2y^2) + (3/4y^4) + \cdots & \text{for } |y^2| \gg 1, \end{array} \right.$$  \hspace{1cm} (3)

with \(y \rightarrow y_\alpha = \omega / 2^{1/2}kV_\alpha\) for thermal particles of species \(\alpha\). The longitudinal part of the dielectric tensor for a plasma composed of thermal particles is

$$K^L(\omega, k) = 1 + \sum_\alpha \frac{\omega_{\alpha\gamma}^2}{k^2 V_\alpha^2} \left[ 1 - \phi(y_\alpha) + i\pi^{1/2}y_\alpha \exp(-y_\alpha^2) \right].$$  \hspace{1cm} (4)

(a) An isotropic plasma consists of thermal electrons (\(\alpha = e\)) and thermal ions (\(\alpha = i\)). Show that for frequencies in the range \(2k^2V_i^2 \ll \omega^2 \ll 2k^2V_e^2\), (4) may be approximated by

$$K^L(\omega, k) = 1 + \frac{1}{k^2 \lambda_{De}^2} - \frac{\omega_{pi}^2}{\omega^2},$$  \hspace{1cm} (5)

(b) Solve the dispersion equation \(K^L(\omega, k) = 0\), with \(K^L(\omega, k)\) given by (5), for the dispersion equation for ion sound waves. Under what conditions does this reduce to \(\omega = kv_s\) with \(v_s = \omega_{pi} \lambda_{De}\)?

(c) Another plasma consists of thermal electrons and two cold (\(V_i \rightarrow 0\)) beams of oppositely directed ions. The beams have equal number densities, \(\frac{1}{2}n_i\), and opposite velocities, \(\pm v_b\). For \(\omega \ll \sqrt{2}kV_e\), justify approximating the response function by

$$K^L(\omega, k) = 1 + \frac{1}{k^2 \lambda_{De}^2} - \frac{\omega_{pi}^2 [\omega^2 + (k \cdot v_b)^2]}{[\omega^2 - (k \cdot v_b)^2]^2}. $$  \hspace{1cm} (6)

*Hint:* A frequency \(\omega'\) in the rest frame of particles moving with bulk velocity \(v\) is given by \(\omega' = \omega - k \cdot v\).

(d) Show that the dispersion equation \(K^L(\omega, k) = 0\) with \(K^L(\omega, k)\) given by (6), has two solutions, and show that one of these solutions can correspond to a reactive instability.

(e) Determine the maximum growth rate for this instability, and identify the value of \(k\) at which this maximum occurs.

(15 marks)
SECTION C: HIGH ENERGY PHYSICS
Use a separate booklet. Please write “Section C” on the cover.

FORMULAE & USEFUL DATA:

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark Content</th>
<th>Mass (GeV/c²)</th>
<th>Spin (h)</th>
<th>Strong Isospin (I, I₃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>uud</td>
<td>0.9383</td>
<td>1/2</td>
<td>(1/2, +1/2)</td>
</tr>
<tr>
<td>n</td>
<td>udd</td>
<td>0.9396</td>
<td>1/2</td>
<td>(1/2, -1/2)</td>
</tr>
<tr>
<td>Λ</td>
<td>uds</td>
<td>1.1157</td>
<td>1/2</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>Δ⁰</td>
<td>udd</td>
<td>1.2320</td>
<td>3/2</td>
<td>(3/2, -1/2)</td>
</tr>
<tr>
<td>Δ⁻</td>
<td>ddd</td>
<td>1.2320</td>
<td>3/2</td>
<td>(3/2, -3/2)</td>
</tr>
<tr>
<td>π⁺</td>
<td>uū or dū</td>
<td>0.1396</td>
<td>0</td>
<td>(1, +1)</td>
</tr>
<tr>
<td>π⁰</td>
<td></td>
<td>0.1350</td>
<td>0</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>K⁺</td>
<td>uū or dū</td>
<td>0.4937</td>
<td>0</td>
<td>(1/2, +1/2)</td>
</tr>
<tr>
<td>K⁰</td>
<td>dū or uū</td>
<td>0.4977</td>
<td>0</td>
<td>(1/2, -1/2)</td>
</tr>
<tr>
<td>e⁻</td>
<td></td>
<td>0.0005110</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>µ⁻</td>
<td></td>
<td>0.1057</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>τ⁻</td>
<td></td>
<td>1.7768</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>νₑ, νₘ, νₜ</td>
<td>≈ 0</td>
<td>1/2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W⁺</td>
<td></td>
<td>80.4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Z⁰</td>
<td></td>
<td>91.2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Cabibbo angle \( \theta_C = 12.7° \)

Relativistic Equations

\[
\begin{align*}
\beta &= v/c \\
E^2 &= (pc)^2 + (mc^2)^2 \\
E &= \gamma mc^2 \\
p &= \beta E/c \\
E' &= \gamma(E - \beta p_z c) \\
\not{p}_x &= \not{p}_x \\
\not{p}_y &= \not{p}_y \\
\not{p}_z &= \gamma(p_z - \beta E/c)
\end{align*}
\]

Selected Clebsch-Gordan Coefficients

Note: A \( \sqrt{\cdot} \) sign is to be understood over every coefficient, e.g. for \(-1/2\) read \(-\sqrt{1/2}\).
9. All students:

Consider the following scattering reactions of pions from protons

\[ \pi^- + p \rightarrow \Lambda + K^0 \]
\[ \pi^- + p \rightarrow \Delta^0 + n \]
\[ \pi^- + p \rightarrow \Delta^0 + \pi^0 \]
\[ \pi^- + p \rightarrow \Delta^- + \pi^+ \]

None of the final state particles in the above reactions are stable. Here is a list of example decays

\[ \Lambda \rightarrow p + \pi^- \]
\[ \Delta^0 \rightarrow p + \pi^- \]
\[ \Delta^0 \rightarrow n + \pi^0 \]
\[ \Delta^- \rightarrow n + \pi^- \]
\[ n \rightarrow p + e^- + \bar{\nu}_e \]
\[ \pi^0 \rightarrow \gamma + \gamma \]
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ K_S \rightarrow \pi^+ + \pi^- \]

(a) Draw a Feynman diagram to represent the first scattering reaction. Does this reaction proceed via the strong, electromagnetic, or weak interaction?

(b) One of the scattering reactions cannot proceed at all. Identify which it is, and give two reasons why it cannot occur.

(c) The \( \Lambda \) and \( \Delta^0 \) particles can decay to the same final state particles. Which of these will decay at the higher rate? Briefly justify your answer.

(d) Using Clebsch-Gordan coefficients, predict the ratio of the probability of the \( \Delta^0 \) decaying to the \( p\pi^- \) final state as compared to the \( n\pi^0 \) final state. Describe your reasoning.

(e) Explain why the final decay is written in terms of a particle \( K_S \) rather than the produced particle \( K^0 \). Give another example of a way in which the kaon produced in the first scattering reaction could decay to a final state containing only hadrons.

The table of particle properties at the beginning of this section may be useful.

(15 marks)
10. **All students:**

When a tau lepton decays, it can produce either an electron, a muon, or a system of one or more charged mesons (represented by $h^-$)

\[
\begin{align*}
\tau^- & \rightarrow e^- + \overline{\nu}_e + \nu_{\tau} \\
\tau^- & \rightarrow \mu^- + \overline{\nu}_\mu + \nu_{\tau} \\
\tau^- & \rightarrow h^- + \nu_{\tau}
\end{align*}
\]

(a) Make a prediction for the relative probabilities for the above three final states to be produced, based purely on considerations of the two scenarios (a) quarks have no colour quantum number; (b) quarks are coloured. Briefly explain your reasoning.

(b) Consider the third type of decay, and consider the case where the decaying tau meson is “spin up” and a single meson is produced. What possible intrinsic spin quantum numbers $(S, S_z)$ could this meson possess, assuming that there is no orbital angular momentum between the meson and neutrino? Briefly justify your answer.

(c) Supposing that in fact the only mesons that could be produced in part (b) are the charged pion and charged kaon, draw a Feynman diagram for each case and make a prediction for the relative probabilities of these two being produced (you may ignore the fact that the kaon is heavier than the pion).

(d) If the tau lepton decays at rest to produce a charged kaon and neutrino, calculate the energy of the kaon and the energy of the neutrino, in GeV.

The table of particle properties at the beginning of this section may be useful.

*(15 marks)*
11. **Normal only:**

In the following, the term “neutrino” can refer either to a neutrino or an antineutrino.

(a) In no more than a few lines, explain the meaning of the phrase *parity conservation*.

(b) With the aid of a labeled diagram, in no more than a few lines describe how Wu’s experiment with radioactive cobalt revealed that the weak force violated parity conservation.

(c) Explain, with the aid of a labeled diagram, what neutrino properties are the source of this parity violation.

(d) Give an example of a reaction which could be used to detect the neutrino emitted in Wu’s experiment. Draw a Feynman diagram at the quark level for this reaction.

(e) In no more than a few lines, explain the meaning of the phrase *charge conjugation*.

(f) Write down the charge conjugate decay to the following

\[ K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \]

Briefly explain how the two decays may be used to communicate an absolute sense of particle versus antiparticle to a distant civilisation, using only neutral and unpolarised means of communication.

*(15 marks)*
12. **Advanced only:**

   (a) In no more than a few lines each, explain the meaning of the terms *parity conservation* and *charge conjugation symmetry*.

   (b) Draw a Feynman diagram which demonstrates how a neutral kaon can oscillate into its antiparticle state as it propagates through free space.

   (c) In no more than a few lines, describe the basic principles and results of the experiment of Christenson et al which demonstrated that $CP$ symmetry is violated by the weak force.

   (d) A violation of $CP$ symmetry is one of the necessary conditions for the appearance of a matter-antimatter asymmetry in the early Universe. State another of the necessary conditions, and briefly describe any experimental or observational consequences of this condition that you can think of.

   (e) Give one example of a limitation of the Standard Model of Particle Physics, and briefly describe the essential elements of a theoretical extension of the Standard Model which may allow this limitation to be overcome.

   *(15 marks)*
13. All students: Explain briefly (two or three sentences each) what is meant by each of the following.

(a) A microstate.
(b) The Gibbs free energy.
(c) Diffusive equilibrium.
(d) The quantum volume.
(e) The ultraviolet catastrophe.

(15 marks)

14. All students: Consider the reaction by which albite converts into jadeite and quartz:

\[
\text{NaAlSi}_3\text{O}_8 \leftrightarrow \text{NaAlSi}_2\text{O}_6 + \text{SiO}_2
\]

Thermodynamic properties of these crystals are given in the table below.

(a) In two or three sentences, with supporting equations as necessary, argue that you would expect to find albite, rather than jadeite and quartz, naturally occurring at standard temperature and pressure.

(b) Keeping the temperature constant, above what pressure would you expect to find jadeite and quartz as the stable form?

(c) Assuming that specific volume and entropy are constant, sketch the phase diagram of this system. Use \(T\) as the horizontal axis and \(P\) as the vertical axis. Label your diagram.

(d) In two or three sentences, comment on whether you would expect the above assumptions (that specific volume and entropy are constant) to be valid.

(15 marks)

<table>
<thead>
<tr>
<th>Crystal</th>
<th>(\Delta_f H) (kJ)</th>
<th>(\Delta_f G) (kJ)</th>
<th>(S) (J/K)</th>
<th>(V) (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaAlSi(_3)O(_8) (albite)</td>
<td>-3935.1</td>
<td>-3711.5</td>
<td>207.40</td>
<td>100.07</td>
</tr>
<tr>
<td>NaAlSi(_2)O(_6) (jadeite)</td>
<td>-3030.9</td>
<td>-2852.1</td>
<td>133.5</td>
<td>60.40</td>
</tr>
<tr>
<td>SiO(_2) (quartz)</td>
<td>-910.94</td>
<td>-856.64</td>
<td>41.84</td>
<td>22.69</td>
</tr>
</tbody>
</table>

Table 1: Data for albite, jadeite and quartz. All values are for one mole of material at room temperature and atmospheric pressure.
15. **Normal only:** A lithium nucleus has four independent spin orientations, conventionally labelled by the quantum number \( m = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2} \). In a magnetic field \( B \), the energies of these four states are \( E = -m \mu B \), where the constant \( \mu \) is \( 1.03 \times 10^{-7}\) eV/T.

(a) Write an expression for the partition function of this system. (You do not need to evaluate the sum.)

(b) If the magnetic field is \( B = 0.63 \) T and the temperature is \( T = 0 \), what is the mean energy? Explain your answer.

(c) If the magnetic field is \( B = 0.63 \) T and the temperature is 300 K, calculate the ratio of probabilities \( P(m = -\frac{3}{2})/P(m = \frac{3}{2}) \) of finding the system in the state \( m = -\frac{3}{2} \) compared with the state \( m = \frac{3}{2} \).

(d) If we quickly reverse the direction of the magnetic field, show that the probability distribution of the four states obeys the Boltzmann distribution for \( T = -300 \) K.

(15 marks)

16. **Advanced only:**

A collection of \( N \) non-interacting, spin-\( \frac{1}{2} \) fermions with mass \( m \) are confined to move in two dimensions. They are confined within a square area \( A = L^2 \), with \( L \) the length of each side of the square.

(a) Using box quantization in two dimensions, show that the energy of the mode labelled by \( \vec{n} = (n_x, n_y) \) is

\[
\varepsilon = \frac{\hbar^2}{8mL^2} n^2,
\]

where \( n^2 = (n_x^2 + n_y^2) \).

(b) The Fermi energy \( \varepsilon_F \) is the energy of the highest-energy occupied mode at \( T = 0 \). Given that there are \( N \) fermions in this area \( A \), show that the Fermi energy is given by

\[
\varepsilon_F = \frac{\hbar^2 }{4\pi m A} N.
\]

(c) Calculate the mean energy per fermion, expressed as a function of \( \varepsilon_F \).

(d) Although we are working in two dimensions, there is a sensible generalized definition of pressure for this system. Write down such a formal definition for pressure (as a partial derivative), and determine the degeneracy pressure of this system at \( T = 0 \) K.

(15 marks)

**THERE ARE NO MORE QUESTIONS.**