L10 Magnetic fields due to currents

Lecture outline:
- The magnetic field of a current element.
- Ampere’s law and the Biot-Savart law.
- The field of a current loop.
- The field of the earth.
- The field of a magnetic dipole.

L10.1 Magnetic fields due to currents

Magnetic fields are produced by currents.

\[ dB = \frac{\mu_0}{4\pi} \frac{i ds \times r}{r^3} \]

Ampere’s law

\[ \oint B \cdot ds = \mu_0 i \]

Example:

\[ \oint B \cdot ds = B 2\pi r = \mu_0 i \]

so

\[ B = \frac{\mu_0 i}{2\pi r} \]
L10.2 Magnetic fields due to currents

A solenoid:

\[ \oint B \cdot ds = BL = \mu_0 inL \quad \text{(n is number of turns/length)} \]

Therefore \( B = \mu_0 ni \quad \text{(inside)} \)

L10.3 Magnetic fields due to currents

Use the Biot-Savart law to derive the magnetic field on the axis of a current loop:

\[ dB = \frac{\mu_0 i ds}{4\pi r^2} \quad dB_z = \frac{\mu_0 i ds}{4\pi r^2} \sin \phi \]

\[ B_z = \frac{\mu_0 i 2\pi a}{4\pi r^2} \sin \phi \]

\( r^2 = a^2 + z^2 \), and \( \sin \phi = a/r \)

Therefore \( B_z = \frac{\mu_0 i 2\pi a^2}{4\pi r^3} = \frac{\mu_0 ia^2}{2(z^2 + a^2)^{3/2}} \)
L10.4 Magnetic fields due to currents

Magnetic field of the Earth

L10.5 Magnetic fields due to currents

The magnetic field of a magnetic dipole:

This magnetic field has the same shape as the electric field of an electric dipole: do the exercise in the Exercise Set.