L18 Electromagnetic waves

Lecture outline:
- Electromagnetic waves.
- Energy and momentum transfer in EM waves.
- Antennas.
- Possible hazards of EM radiation.

L18.1 Electromagnetic waves

Consider fields in a vacuum, with no free charges or currents. Maxwell’s equations are:

\[ \oint E \cdot dA = 0 \quad \oint B \cdot dA = 0 \quad \oint E \cdot ds = -\frac{d\Phi_B}{dt} \]

\[ \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

Consider a disturbance in \( E \) and \( B \) moving in the \( z \)-direction:

Rectangles fixed in space
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From Faraday's law: \( \Delta \Phi_B = \Delta B_y d \Delta z \)

\[ \therefore \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta B_y}{\Delta t} d \Delta z \]

and \( \oint \mathbf{E} \cdot d\mathbf{s} = (E_x + \Delta E_x) d - E_x d = \Delta E_x d \)

\[ \therefore \Delta E_x d = -\frac{\Delta B_y}{\Delta t} d \Delta z \]

\[ \therefore \frac{\Delta E_x}{\Delta z} = -\frac{\Delta B_y}{\Delta t} \]

or

\[ \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \]

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From Ampere's law: \( \Delta \Phi_E = \Delta E_x d \Delta z \)

so

\[ \frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta E_x}{\Delta t} d \Delta z \]

\( \oint \mathbf{B} \cdot d\mathbf{s} = -(B_y + \Delta B_y) d + B_y d = -\Delta B_y d \)

\[ \therefore -\Delta B_y d = \mu_0 \varepsilon_0 \frac{\Delta E_x}{\Delta t} d \Delta z \]

\[ \therefore \frac{\Delta B_y}{\Delta z} = -\mu_0 \varepsilon_0 \frac{\Delta E_x}{\Delta t} \]

so

\[ \frac{\partial B_y}{\partial z} = -\mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t} \]
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Combining the equations, we get

\[ \frac{\partial^2 E_x}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0 \]

This has the solution \( E_x = E_{x0} \cos(\omega t - k z) \)

which corresponds to a wave with phase velocity

\[ v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \]

and \( c \) is the speed of light in a vacuum

The magnetic field is

\[ B_y = \frac{E_{x0}}{c} \cos(\omega t - k z) = B_{y0} \cos(\omega t - k z) \]

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The electric and magnetic fields are in phase, and linearly polarized. We can also have circularly polarized fields:

\[ E = E_0 (\cos(\omega t - k z)\hat{x} + \sin(\omega t - k z)\hat{y}) \]
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Energy transfer by electromagnetic waves:

The Poynting vector: \( \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \mathbf{E} \times \mathbf{H} \)

The amplitude gives the power crossing unit area.
The direction gives the direction of the wave.

\[ |\mathbf{S}| = \frac{\text{energy}}{\text{A per unit time}} \]

Example: wave in z-direction with \( E_x \) and \( B_y \):

\[ \mathbf{E} \times \mathbf{B} = (E_x, 0, 0) \times (0, B_y, 0) = (0, 0, E_x B_y) = E_x B_y \hat{z} \]

\[ |\mathbf{S}| = \frac{1}{\mu_0} E_x B_y = \frac{1}{\mu_0} E_x \frac{E_x}{c} = \frac{1}{c \mu_0} E^2 \]

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Energy density in a cylinder length \( dz \) along the z-direction is:

\[
\frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \frac{dU}{Adz} = \text{energy}
\]

\[
= \frac{1}{2c^2 \mu_0} E^2 + \frac{1}{2\mu_0} \frac{E^2}{c^2} = \frac{1}{c^2 \mu_0} E^2
\]

so

\[
S = \frac{dU}{Adt} = \frac{dU}{Adz} \cdot \frac{dz}{dt} = \frac{1}{c^2 \mu_0} E^2 \cdot c = \frac{1}{c \mu_0} E^2
\]
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Now \( S = \frac{1}{c\mu_0} (E_0 \cos(\omega t - kz))^2 \) is the instantaneous energy flow rate.

More useful is the time average over a period \( T \).

Use \( \frac{1}{T} \int_0^T \cos^2(2\pi t/T)dt = \frac{1}{2} \)

Then \( S_{\text{average}} = \frac{1}{\mu_0 c} \frac{E_0^2}{2} Wm^{-2} = I \quad \text{Intensity} \)

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Momentum transfer by EM waves:

There are 2 situations:

- \( \sim \) Absorbed
- \( \sim \) Reflected

Let \( \Delta U \) be the energy transferred in time \( \Delta t \). The theory of relativity says that for light \( U = pc \), where \( p \) is the momentum.

The momentum transferred is

- \( \Delta p = \frac{\Delta U}{c} \) (absorption), or
- \( \Delta p = \frac{2\Delta U}{c} \) (reflection)
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The force exerted is

$$F = \frac{\Delta p}{\Delta t} = \frac{1}{c} \frac{\Delta U}{\Delta t}$$

(absorbed)

Now \( \Delta U = IA \Delta t \) so

$$F = \frac{IA}{c}, \quad \left( \frac{2IA}{c} \right)$$

The force per unit area, or radiation pressure is then

$$P_{rad} = \frac{I}{c}, \quad \left( \frac{2I}{c} \right)$$

The radiation pressure in powerful lasers can lift small objects. The radiation pressure from the Sun causes dust tails in comets.

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L18.12 Electromagnetic waves

To produce EM radiation, we need accelerating charges, such as in oscillating currents:

Eg, a dipole antenna:

\[ E(t) = E_0 \cos \omega t \]

\[ B(t) = -\frac{E(t)}{c} \]

L18.13 Electromagnetic waves

EM fields and health.

There may be a greater incidence of cancers near power lines (50Hz), but difficult statistics. The photons are of too low energy to ionize organic matter. Heating is negligible compared with heat generated by body. Perhaps cell membranes are disturbed, nerves stimulated etc. The typical magnetic field encountered from power lines or VDUs is \(\sim 10 \mu T\) (1/4 of Earth’s field). To minimise field from power lines, use 3-phase power transmission.

Fields partially cancel, total \(\sim 1/d^2\)
L18.14 Electromagnetic waves

Mobile Phones: Frequency is 900 – 1900MHz. Energy in this range is non-ionising. The most apparent effect of RF energy at these frequencies is heating of tissue but this is not a problem with low-powered phones 125 mW (digital) to 600 mW (analogue). Normal heating of the body is in this range. The US standard limits peak exposure to 1.6mW/g can be approached by phones. The evidence and research into effects of RF from phones is controversial.

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A lawsuit in Florida in 1993 alleged brain cancer arising from mobile phone use, but was dismissed in 1995 due to lack of evidence. Epidemiology studies (large population statistics) have been mostly negative. Some animal studies have shown effects (breaking of DNA), but not repeatable. Research at Sydney: Maybe the pulse nature of the signals damages cells transient power is much bigger than averaged power. Hands-free phone the speaker wire acts as an antenna.