L9 Magnetic forces

Lecture outline:
• Force on a current.
• Torques and forces on magnetic dipoles.
• Potential energy of magnetic dipoles.
• Applications of magnetic forces.
• Atomic magnetic dipole moments.

L9.1 Magnetic forces

The magnetic force on a current: \( d\mathbf{F}_B = i d\mathbf{L} \times \mathbf{B} \)

Consider a wire bent into a loop:

The field looks like that from a magnet, with north and south poles – a dipole field.

So the loop is called a magnetic dipole.
L9.2 Magnetic forces

Forces and Torques on magnetic dipoles.
Any loop can be considered to be made up of a number of rectangular loops.

Consider the torque on a rectangular current loop

\[ \tau_B = s_1 F \sin \theta \hat{i}, \text{ and } F = I s_2 B. \]

(If B is different on each side, there is a net force.)

L9.3 Magnetic forces

The torque \( \tau_B = s_1 F \sin \theta \hat{i}, \) and \( F = I s_2 B. \)

(for N turns, I \( \rightarrow \) NI)

Then \( \tau_B = (I s_1 s_2) B \sin \theta \hat{i} = \mu B \sin \theta \hat{i} \)

i.e., \( \tau_B = \mu \times B \)

where the direction of \( \mu \) is perpendicular to the loop, and determined by a right hand rule with respect to the direction of the current.

The magnitude of \( \mu \) is \( \mu = I s_1 s_2 = I \text{ A Amp m}^2, \)

where \( A = s_1 s_2 \) is the area of the loop. \( \mu \) is called the magnetic dipole moment of the loop.
L9.4 Magnetic forces

The torque acts to line the dipole up with the magnetic field. The dipole therefore has potential energy $U$ in the magnetic field: as for the electric dipole,

$$U = -\int_{90^\circ}^{0^\circ} \tau_B d\theta = \int_{90^\circ}^{0^\circ} \mu B \sin \theta d\theta = -\mu B \cos \theta$$

ie $U = -\mu \cdot B$

The energy is least when $\mu$ is parallel to $B$

The energy is most when $\mu$ is anti-parallel to $B$

L9.5 Magnetic forces

Examples:

The DC motor: current produces mechanical rotational motion.

Loudspeaker – example of transducer.
L9.6 Magnetic forces

Atomic magnetic dipole moments.

Consider an electron revolving around a nucleus:

The current is \( I = \frac{e}{T} \) where \( T \) is the period, so \( \mu = \frac{e}{T} \pi r^2 \)

The orbital angular momentum is \( L = mvr \), and \( T = \frac{2\pi r}{v} \) so \( \mu = \frac{e}{(2\pi r/v)} \pi r^2 = \frac{e vr}{2} = \frac{e}{2m} L \)

The electron also spins with angular momentum \( S \), so it has a spin magnetic moment \( \mu_s = 2 \frac{e}{2m} S \). The factor 2 comes from quantum physics.