1 Magnetism

1. A blood flow meter is made to operate on the principle of the Hall effect. The blood flow through a magnetic field of 2 T generates a voltage of 600 $\mu$V across two electrodes 1 mm apart, placed across the stream and perpendicular to the field. Calculate the flow speed.

![Diagram of ion trajectories](image)

Figure 1: Ion trajectories for different isotopes in a mass spectrometer.

2. A mass spectrometer can be used to separate the isotopes of chlorine: $^{35}Cl$ and $^{37}Cl$, as in the figure. These ions have masses $54.85 \times 10^{-27}$ kg and $61.79 \times 10^{-27}$ kg respectively. The chlorine ions entering the slit have been accelerated through 10 kV. The magnetic field is 1.0 T. First, calculate the speed of the ions as they enter the slit from $eV = 1/2mv^2$, justifying this equation. Second, calculate the separation between the two beams at the detector.

3. Use the equation for the field of a long straight wire

\[
B(r) = \frac{\mu_0 i}{2\pi r}\]

(1)

to estimate the magnetic field due to mains wiring. Suppose there are two long
wires carrying a current of 10 A, and they are 5 mm apart. Estimate the field at
two points P and Q, 10 cm from the pair of wires, as shown.

\[ \text{10 A} \]

\[ \text{Q} \]

\[ \text{10 cm} \]

\[ \text{5 mm} \]

\[ \text{P} \]

Figure 2: Calculation of magnetic field due to two anti-parallel currents.

4. A superconducting magnet has 23500 turns, mean radius 100 mm. Take the current
as 90 A, and treat the coil as flat.

(a) Estimate the field 1 m and 2 m above the centre.

(b) Estimate the magnetic dipole moment.

5. Work through the derivation of the electric field of an electric dipole, and the mag-
netic field of a magnetic dipole.

(a) The Coulomb field at a point \( \mathbf{x} \) of a charge \( q \) can be written

\[ \mathbf{E} = \frac{q \mathbf{x}}{4\pi\varepsilon_0 r^3}, \]  \hspace{1cm} (2)
where \( r \) is the length of the position vector \( \mathbf{x} \). Apply this two charges \( \pm q \) at points \( \pm d/2 \mathbf{d} \). You should obtain:

\[
\mathbf{E} = \frac{q}{4\pi \varepsilon_0} \left[ \frac{(\mathbf{x} - d/2 \dot{\mathbf{d}})}{|\mathbf{x} - d/2 \mathbf{d}|^3} - \frac{(\mathbf{x} + d/2 \dot{\mathbf{d}})}{|\mathbf{x} + d/2 \mathbf{d}|^3} \right].
\]  

(3)

(b) Now let the two charges tend together (\( d \to 0 \), while preserving the product \( q \dot{d} \mathbf{d} \), the dipole moment \( \mathbf{d} \). Use the Binomial Theorem to justify the approximation

\[
\frac{q}{|\mathbf{x} - d/2 \mathbf{d}|^3} \approx \frac{1}{r^3} \left( q + \frac{3\mathbf{x} \cdot \mathbf{d}}{2r^2} \right).
\]  

(4)

(c) Hence obtain the field due to an electric dipole:

\[
\mathbf{E} = \frac{1}{4\pi \varepsilon_0 r^3} \left( \frac{3(\mathbf{x} \cdot \mathbf{d})\mathbf{x}}{r^2} - \mathbf{d} \right).
\]  

(5)

(d) Make the obvious changes to obtain the magnetic field of a magnetic dipole of moment \( \mu \):

\[
\mathbf{B} = \frac{\mu_0}{4\pi r^3} \left( \frac{(3\mathbf{x} \cdot \mu)\mathbf{x}}{r^2} - \mu \right).
\]  

(6)

6. Consider the Helmholtz coil shown in the figure: two circular coils each with \( N \) turns, radius \( a \) and current \( i \) in the same direction, with a separation equal to the radius. Evaluate the magnetic field on axis, as a function of \( z \), the coordinate along the axial line of the coils, with the origin of \( z \) measured at the middle of the line joining them. Show that

\[
B(z) = \frac{\mu_0 i \mathbf{l}}{2\pi} \left( \frac{1}{(z - a/2)^2 + a^2} + \frac{1}{(z + a/2)^2 + a^2} \right)^{3/2}.
\]  

(7)

Show also that

\[
\frac{B(z)}{B_{\text{max}}} = \frac{5^{3/2}}{2 \cdot 4^{3/2}} \left( \frac{1}{1 + (z/a - 1/2)^2} + \frac{1}{1 + (z/a + 1/2)^2} \right)^{3/2}.
\]  

(8)
Figure 3: Helmholtz coils.

Plot this function between $z = -a/2$ and $z = a/2$. 