Problem

A meteor crater in Arizona is thought to have been formed by the impact of a meteor with the earth some 20,000 years ago. The mass of the meteor is estimated at $5 \times 10^{10}$ kg and its speed 7200 m s$^{-1}$. What speed would such a meteor impart to the earth in a head on collision? Assume that the pieces of the shattered meteor stayed with the earth as it moved. Mass of earth = $5.98 \times 10^{24}$ kg.

Solution

We use the fact that the total momentum of the Earth+Meteor system is conserved i.e. the same before and after the impact. This is an inelastic collision.

Before:

Take the positive direction to be to the right in the figure. Magnitude of the momentum of the meteor, $p_M$

$$p_M = m_M v_M$$

Magnitude of the momentum of the Earth, $p_E$

$$p_E = m_E v_E = 0$$

since Earth is stationary (at least for our purposes here!).

Hence

$$p_{Total(before)} = p_M + p_E = m_M v_M + 0 = m_M v_M$$
After:

The Earth and Meteor are now “stuck together” and move as a single object.

\[ p_{\text{Tot}(\text{after})} = p_{M+E} = m_{M+E} v_{M+E} = (m_M + m_E) v_{M+E} \]

But by conservation of momentum

\[ p_{\text{Tot}(\text{after})} = p_{\text{Tot}(\text{before})} \]

i.e.

\[ (m_M + m_E) v_{M+E} = m_M v_M \]

Thus, solving for \( v_{M+E} \),

\[ v_{M+E} = \frac{m_M v_M}{(m_M + m_E)} = \frac{5 \times 10^{10} \text{ kg} \times 7200 \text{ m s}^{-1}}{5.98 \times 10^{24} \text{ kg} + 5 \times 10^{10} \text{ kg}} = \frac{5 \times 10^{10} \text{ kg} \times 7.2 \times 10^3 \text{ m s}^{-1}}{5.98 \times 10^{24} \text{ kg}} = 36 \times 10^{13} \text{ kg m s}^{-1} \]

If this is converted to more familiar everyday units, you will find that this speed amounts to about 2 mm/year.

Note that in this example, the mass of the meteor was insignificant when added to the mass of the Earth. This would not be the case in many collision problems.

Note also that we quote the final answer to only one significant figure. This is because the mass of the meteor (5 \times 10^{10} \text{ kg} ) which is used in the denominator is only given to us to one significant figure.