Lecture 3: Simple pendulum

Consider a simple pendulum: a point mass suspended by a massless, unstretched string.

Take the length of the arc as our distance $x$, where $\theta = \frac{x}{L}$.

From the FBD, the restoring force is the component of the weight force perpendicular to the tension:

$$ F = -mg \sin \theta $$

where the sign is negative because it points opposite to $x$. This is not SHM, since $F \propto \sin x$ instead of $F \propto x$.

However, if $\theta$ is small, then $\sin \theta \sim \theta$ (in radians); so then

$$ F = -mg \theta = -mg \frac{x}{L} $$

so

$$ F = \frac{-mg}{L} x $$

i.e. SHM with $k = \frac{mg}{L}$. This will have oscillation frequency

$$ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{L}} \frac{1}{m} = \sqrt{\frac{g}{L}} $$

(simple pendulum, small amplitude). The frequency does not depend on $m$. 