Lecture 3

Pendulums
and
Resonance

Pre-reading: §14.5–14.8

SHM: Pendulums

- Simple pendulum: point mass on a massless, unstretched string
- Exhibits SHM with
  \[ \omega = \sqrt{\frac{g}{L}} \]
  provided the amplitude (angle) is small (\(\theta \leq 15^\circ\))
- For real pendulum, need to know mass \((m)\), distance to center of mass \(d\), and moment of inertia \(I\) about rotation axis
  \[ \omega = \sqrt{\frac{mgd}{I}} \]
  provided the amplitude (angle) is small (\(\theta \leq 15^\circ\))

§14.5–14.6
Damped Oscillations

• In real world, friction causes oscillations to decrease in amplitude

• If friction force varies linearly with speed, we can solve for motion:

\[ x = Ae^{-(b^2/2m)t}\cos(\omega't + \phi) \]

\[ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \]

with $b$ describing the amount of damping

Effects of damping

• Amplitude decays exponentially.

• Energy decays exponentially.

• Angular frequency decreases.

• **Critical damping**
  
  if $b=2\sqrt{km} \Rightarrow \omega'=0$ returns to equilibrium without oscillating!

• **Underdamped**
  
  $b < 2\sqrt{km}$

• **Overdamped**
  
  $b > 2\sqrt{km}$
Forced Oscillations

• Now we add a **driving force** to a damped harmonic oscillator

• Suppose driving force is sinusoidal with driving frequency \( \omega_d \)

• Compare driving frequency with ‘natural frequency’

\[
\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}
\]

• If \( \omega_d = \omega' \) then system oscillates with **resonant** behaviour: amplitude gets very large

§14.8
Next lecture

Mechanical waves

Read §15.1–15.2