Pendulums and Resonance

Pre-reading: §14.5–14.8

SHM: Pendulums
- Simple pendulum: point mass on a massless, unstretched string
- Exhibits SHM with 
  \[ \omega = \sqrt{\frac{g}{L}} \]
  provided the amplitude (angle) is small (\(0 \leq 15^\circ\))
- For real pendulum, need to know mass \(m\), distance to center of mass \(d\), and moment of inertia \(I\) about rotation axis
  \[ \omega = \sqrt{\frac{mgd}{I}} \]
  provided the amplitude (angle) is small (\(0 \leq 15^\circ\))

§14.5–14.6

Damped Oscillations
- In real world, friction causes oscillations to decrease in amplitude
- If friction force varies linearly with speed, we can solve for motion:
  \[ x = Ae^{-\omega dt} \cos(\omega t + \phi) \]
  \[ \omega' = \sqrt{\frac{b}{m} - \frac{b^2}{4m^2}} \]
  with \(b\) describing the amount of damping

§14.7

Effects of damping
- Amplitude decays exponentially.
- Energy decays exponentially.
- Angular frequency decreases.
- Critical damping if \(b = 2\sqrt{km}\) \(\Rightarrow \omega' = 0\) returns to equilibrium without oscillating!
- Underdamped if \(b < 2\sqrt{km}\)
- Overdamped if \(b > 2\sqrt{km}\)

§14.8

Forced Oscillations
- Now we add a driving force to a damped harmonic oscillator
- Suppose driving force is sinusoidal with driving frequency \(\omega_d\)
- Compare driving frequency with ‘natural frequency’
  \[ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \]
- If \(\omega_d = \omega'\) then system oscillates with resonant behaviour; amplitude gets very large

§14.8
Next lecture

Mechanical waves

Read §15.1–15.2