Lecture 6

Interference and Superposition

Pre-reading: §15.6

*Please take a clicker*

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Wave Function and Wave Equation

- Wave function gives displacement as function of space and time
- 1-D periodic wave: \( y(x,t) = A \cos(\omega t \pm kx) \)
- Wave equation relates changes in wave shape to its speed
- Wave equation is true statement for all waves

\[
\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2}
\]

§15.3
**Speed of Mechanical Waves**

- To find $v$, consider the forces, use Newton’s 2nd law, calculate derivatives (complicated!)
- From wave eqn:
  \[ v \approx \sqrt{\text{Acceleration} / \text{Curvature}} \]
- Another way:
  \[ v \approx \sqrt{\text{Restoring force} / \text{Inertia}} \]
- 1-D transverse wave on string: $v = \sqrt{F/\mu}$
- Longitudinal wave in fluid: $v = \sqrt{B/\rho}$
- Sound wave in a gas: $v = \sqrt{\gamma RT/M}$

§15.4, 16.2

**Example 15.3**

- A box with mass 20.0 kg hangs at end of 80.0 m rope. Rope has mass 2.00 kg. Person at bottom sends a transverse wave to top.
  - What is speed of wave?
  - If wave is periodic with frequency 2.00 Hz, how many cycles are there in the rope length?
Reflections

- Waves reflect at a boundary
- Hard boundary: wave is inverted
- Soft boundary: wave is NOT inverted

<table>
<thead>
<tr>
<th>None</th>
<th>Hard</th>
<th>Soft</th>
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Superposition

- Waves functions add together linearly, e.g.
  \[ y(x,t) = y_1(x,t) + y_2(x,t) \]
- Waves can go in same or opposite directions
- Interference: Two or more waves pass through same region at same time
Standing waves

• What happens when two identical waves travelling in opposite directions are superimposed?

Standing Waves

• Formed through reflection + superposition of waves moving in opposite directions

• Contains ‘nodes’ (no displacement) and ‘anti-nodes’ (maximum displacement)
Normal modes

- When the string is fixed at both ends, only certain standing waves are allowed: **normal modes**
- A pattern with particular $\lambda$ (or $f$) is a ‘mode’
- Mode with lowest frequency is ‘fundamental’; higher frequency modes are ‘harmonics’ or ‘overtones’

Stretched string

$$f_n = n\nu/(2L), \ n = 1, 2, 3, \ldots$$
Chladni plates

- Two dimensional surfaces can vibrate in normal modes

Next lecture

Standing waves
and
Normal modes

Read §15.7–15.8, 16.1