Damped Oscillations: lecture 4

\[ F = ma = -kx - bv \]

\[ m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt} \]

\[ x = Ae^{-\left(\frac{b}{2m}\right)t} \cos(\omega t + \phi) \]

\[ \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \]

Critical Damping and Over Damping

\[ b = 2\sqrt{km} \quad (b) \]

Over Damping

\[ b > 2\sqrt{km} \quad (c) \]

\[ x = C_1 e^{-a_1 t} + C_2 e^{-a_2 t} \]
Examples

Forced Oscillations

Amplitude of a Forced Oscillator

- force is of form $F(t) = F_{\text{max}} \cos(\omega_d t)$
- driving frequency equals systems natural frequency when $\omega_d = \omega_o$ ⇒ amplitude peaks (RESONANCE)
- amplitude of oscillation varies as

$$A = \frac{F_{\text{max}}}{\sqrt{(k - m \omega_d^2)^2 + b^2 \omega_d^2}}$$