Exercises

Debye length and plasma frequency – characterising a plasma

1. For the radio-frequency discharge in the Senior Physics Lab, $T = 3$ eV, $n_e = 10^{17}$ m$^{-3}$ and diameter about 100 mm,
   (i) calculate $\lambda_D$,
   (ii) calculate $N_D$ the number of electrons in a Debye sphere.

2. Add some curves of constant Debye length and constant plasma frequency to the figure on p.2.

3. Suppose you take a conducting slab (fixed ion lattice with mobile electrons) of thickness $d$ and apply a potential of negative potential ($-V_0$) to one side while holding the other side at 0 Volts.
   a) What happens to the E field inside the conductor? In particular, how and how far from the edge is the electric field shielded out?
   b) Assume now that the electrons have some thermal energy (in one dimension $E_{\text{ave}} = \frac{1}{2} kT_e$). How far could an electron placed on the zero volt side move into an unshielded ion lattice before losing all it’s energy and being reflected back?

   $E=0$  
   $V=0$  

   $n_i=n_0$  
   $+$  
   $+$  
   $+$  
   $+$  
   $+$  

   $x=0$  
   $+$  
   $+$  
   $+$  
   $+$  
   $+$  
   $+$  

   $x$  

   *Hint*: Use Poisson’s equation to find the potential at distance $x$.

   c) Describe what this exercise tells you about the meaning of the Debye length.

4. The mean velocity of electrons is of order $v_{th} = \sqrt{\frac{kT_e}{m_e}}$. Show that $\lambda_D = \frac{v_{th}}{\omega_{pe}}$.

   Explain what this means in terms of the time scale over which shielding occurs in a plasma.

Boltzman Relation

5. Show that the Boltzman’s relation for electrons implies that there is balance between the electron pressure force and the electric force on the electron fluid everywhere in the plasma. *Hint*: write out the expressions for these two forces on a fluid element with electron density satisfying the Boltzman relation.

What have you implicitly assumed about the electron energy distribution?
Floating Potential

6. a) Explain how the floating potential arises and what physically determines its value? b) Why is it usually negative? c) How would you expect the floating potential of a planar electrode to change if a magnetic field is added parallel to the plane of the electrode surface?

High Voltage Sheath – Matrix and Child law sheaths

7. a) Explain how a matrix sheath transforms into a Child-law sheath b) What is the life-time of a matrix sheath?

8. The Child law sheath density, \( n = \frac{4 \epsilon_0 V_0}{9 e s^2} \left( \frac{x}{s} \right)^{-\frac{1}{2}} \), is singular at the sheath edge, \( x=0 \), while the potential, \( \phi = -V_0 \left( \frac{x}{s} \right)^{\frac{1}{2}} \), is not. Assuming that \( \phi = -V_0 \left( \frac{x}{s} \right)^{\frac{1}{2}} \) still holds and that all ions enter the sheath with the Bohm velocity, \( u_B \), find a non-singular expression for \( n(x) \) as a function of \( J_0, u_B, \phi(x), \) and other constants. Plot \( n/n_s \) versus \( x/s \) for \( eV_0/kT_e = 100 \). Plot \( n/n_s \) given by \( n = \frac{4 \epsilon_0 V_0}{9 e s^2} \left( \frac{x}{s} \right)^{-\frac{1}{2}} \) on the same graph to compare with your result.

9. Show that the time for an ion with zero initial energy to transit a collisionless Child law sheath with voltage \( V_0 \) and thickness \( s \) is \( t = 3s/v_0 \), where \( v_0 = \left( \frac{2eV_0}{m_i} \right)^{\frac{1}{2}} \).

10. A probe whose collecting surface is a square tantalum foil 2x2 mm in area immersed in a singly ionised Argon (atomic weight 40) plasma draws an ion saturation current of 100 \( \mu \)A. If \( kT_e = 2 \) eV, what is the approximate plasma density? (Hint: both sides of the foil collect ions).

11. A solar satellite consisting of 10 km\(^2\) of photovoltaic panels is placed in synchronous orbit around the earth. It is immersed in a 1-eV atomic hydrogen plasma with density \( 10^6 \) m\(^{-3}\). During solar storms the satellite is bombarded by energetic electrons, which charge it to a potential of \(-2\) kV. Calculate the flux of energetic ions bombarding each square metre of the panel.

12. An ion velocity analyser consists of a stainless steel cylinder 5 mm in diameter with one end covered with a fine tungsten mesh grid (grid 1). Behind this, inside the cylinder, are a series of insulated, parallel grids. Grid 1 is at “floating” potential – it is not electrically connected. Grid 2 is biased negative to repel all electrons coming through grid 1, but it transmits ions. Grid 3 is the analyser grid, biased so as to
decelerate ions accelerated by grid 2. Those ions able to pass through grid 3 are all collected by a collector plate. Grid 4 is a suppressor grid that turns back secondary electrons emitted by the collector. If the plasma density is too high, a space charge problem occurs near grid 3 because the ion density is so large that a potential hill forms in front of grid 3 and repels ions that would otherwise reach grid 3. Using the Child law, estimate the maximum meaningful He\(^+\) current that can be measured on a 4-mm-diameter collector if grids 2 and 3 are separated by 1 mm and 100 V.

**Diffusion and mobility**

13. A positive column of a glow discharge in helium at \( p = 1 \) torr, no magnetic field.

The helium and the helium ions are at room temperature, the electrons have an energy of 2 eV.

(i) Estimate \( n_n \).

(ii) Estimate \( v_{e\text{rms}}, v_{i\text{rms}} \).

The electron-neutral collision cross-section is about \( 5 \times 10^{-20} \) m\(^2\).

(iii) Estimate \( \lambda_m \).

(iv) Estimate \( v_{en} \).

(v) Hence estimate \( D_e \), and \( \mu_e \).

For the ions, \( D_i \) is around \( 0.02 \) m\(^2\) s\(^{-1}\) and \( \mu_i \) is around \( 1 \) m\(^2\) V\(^{-1}\) s\(^{-1}\).

(vi) Estimate \( D_{\alpha} \).

(vii) If the plasma density is \( 10^{16} \) m\(^{-3}\), the axial electric field is 10 kV m\(^{-1}\) and the column diameter is 1 cm, estimate the current.

14. Calculate the resistivity of a plasma. Take \( n = 10^{19} \) m\(^{-3}\) and \( T_e = 10^4, 10^5, 10^6, 10^7, 10^8, 10^9 \) K.

5. Consider a plasma of thermonuclear interest. \( n = 10^{19} \) m\(^{-3}\), \( T_e = 100 \) keV, \( B = 1 \) T.

(i) Calculate \( \eta \). High temperature plasmas are good conductors and ohmic \( (P = \dot{F}R) \) heating is no longer useful. Compare your value with the resistivity of stainless steel.

(ii) Compare the classical diffusion coefficient \( D_\perp \) and the Bohm diffusion coefficient for this plasma.