Exercises for Chapter 9

Fusion reactions

1. Suppose the distance between the centres of the two nuclei must be within 3 fm for the nuclear force to be effective. Use this to estimate the ignition temperature. Comment. The temperature you obtain will be too high because: the nuclei have a finite size, quantum mechanical tunnelling through the Coulomb barrier can occur, the fast nuclei at the tail of the Maxwellian distribution are most important.

Magnetic confinement fusion

2. Ohmic heating

(i) Find an expression for the $p_{\text{ohmic}}$, the power per unit volume for ohmic heating of a toroidal plasma, in terms of resistivity, plasma current and plasma dimensions.

(ii) Write down the expression for $p_{\text{lost}}$, the power lost per unit volume by the escaping hot D and T ions, in terms of temperature and the energy confinement time.

(iii) The maximum temperature that can be reached by ohmic heating is set by $p_{\text{ohmic}} > p_{\text{lost}}$. Use the parameters for JET, $I = 7 \times 10^6$ A, $a = 1$ m and the empirical value for the containment time $\tau_E \approx \frac{n}{2 \times 10^{20}} a^2$ to show that the maximum temperature is far below that for fusion.

Inertial confinement fusion

3. The Lawson criterion $n \tau_E > 10^{20}$ m$^{-3}$ s and a $T$ of 10 keV must be satisfied.

(i) Estimate $n$ for solid hydrogen (Take $\rho = 200$ kg m$^{-3}$). Use this definition of the energy containment time $\tau_E$, the time for the plasma to expand freely, $\tau_E \approx \frac{1}{4} \frac{R}{U}$ where $R$ is the target radius and $U$ is the sound speed (about $10^6$ m s$^{-1}$ for $T = 10$ keV).

(ii) Estimate $R$.

(iii) Estimate the energy required if all the atoms in a sphere of radius $R$ were to have an energy of 10 keV. This is much more than can be provided by present-day lasers.

(iv) Suppose $\rho$ is 100$\times$ higher.