Experiment 22. Gamma Radiation

1 Safety First

During this experiment we are using radioactive sources. All sources are well shielded or are of low intensity, nevertheless you should avoid any unnecessary exposure by keeping away from the sources ($r^{-2}$ dependence) and limit the time of being in proximity to them. If not in use, radioactive sources should stay in a lead container provided.

Another safety concern is the high voltage power supply used to power on detectors. Under any circumstances do not switch on the high voltage power supply if high voltage cables are not connected to the detector. Do not disconnect high voltage cables if the high voltage power supply is active.

If you notice any exposed or damaged electrical wires, please notify laboratory staff immediately.
2 Objectives

It is the purpose of this experiment to examine gamma decay using a scintillation counter. A source of $^{137}\text{Cs}$ is examined first; its spectrum shows, amongst other things, evidence of the Compton effect. The latter is further examined with the use of another scintillation counter in coincidence. The experiment ends with an examination of the gamma rays from $^{22}\text{Na}$ and from the naturally occurring radionuclides in the environment.

3 Equipment

- A 75 mm × 75 mm cylindrical sodium iodide crystal activated with thallium NaI(Tl) detects the gamma rays by their secondary reactions: atomic photoelectric effect, Compton effect and (if the photon’s quantum energy exceeds twice the rest mass energy of an electron), pair production. All these reactions produce high energy electrons which create ionisation tracks in the crystal. The latter responds by emitting optical photons; the number of these, several thousand for a typical secondary electron, is proportional to the secondary electron’s energy.

A photomultiplier tube is optically coupled to the end of the scintillator cylinder with special transparent grease. A good proportion of the optical photons produced in the crystal find their way to the photocathode of the photomultiplier tube where they eject photoelectrons. The probability that a photon will emit a photoelectron, the quantum efficiency, is about 15% so that some hundreds of photoelectrons leave the photocathode for a typical event.

The action of the dynode chain in the photomultiplier tube is as follows: at each secondary emitting dynode, for each incident electron, about four secondary electrons are emitted. (This factor of about four is called the secondary emission ratio). As a result of many amplifications by this factor, a large cloud of electrons reaches the anode of the photomultiplier tube producing a pulse lasting several nanoseconds. This is amplified in the preamplifier and passed to the multichannel pulse analyser. Apart from fluctuations, the height of this pulse is proportional to the energy of the secondary electron released in the scintillator by the gamma ray.

The pulse height also depends on the high voltage on the photomultiplier tube; increased high voltage means increased kinetic energy for the electrons approaching each dynode. The effect of the increased secondary emission ratio from every stage causes a large increase in the output pulse.

Fluctuations occur in the number of optical photons in the crystal, in the number of photoelectrons leaving the photomultiplier’s photocathode and in the number of secondary electrons leaving dynodes in the earlier stages of the photomultiplier. The central limit theorem of statistics tells us that the superposition of all these fluctuations will turn the pulse height spectrum coming from a source of photoelectrically absorbed gamma rays, from a sharp ‘spike’ into a normal or Gaussian distribution.

The sodium iodide crystal is efficient in stopping low energy gamma rays but the mean free path for the production of secondary electrons increases as the gamma energy increases. At the end of the experiment you will record quanta of 2.61 MeV. Most of these photons will pass through the scintillator and not be recorded.\footnote{More information about radiation detectors can be found in [1].}
• A 50 mm × 25 mm cylindrical plastic scintillator acts as a scattering medium for the gamma rays in the Compton scattering section. Because the scintillator is organic - abounding in light nuclei there is a high probability that the Compton effect will occur and a high probability for the scattered gamma rays to escape from this scintillator. However the chance of any scattered electron escaping is fairly small and hence this system can be used to trigger a gate to the analyser thus increasing the signal to noise ratio for the recording process. The plastic scintillator’s output goes through an amplifier, then a discriminator and monostable multivibrator arrangement thence to the gate input of the analyser. This is to make a proper logic signal of the correct width for the analyser.

• The UCS30 multichannel analyser (MCA) sorts signals according to pulse height (and hence energy deposited in the crystal) and puts them in bins (also called channels) corresponding to given energies (apart from the fluctuations described above). That is, the x-axis represents energy, and the y-axis represent gamma counts. The MCA is connected to the computer by means of an USB port. This allows us to get data from the MCA and also to set up its parameters (using the USX application). The spectrum can later be displayed on the computer for plotting or further analysis.

4 Initial calibration

In order to use the MCA we have to calibrate it first, i.e. find the energy corresponding to each channel. To do this we will use well known source of the gamma radiation, $^{137}$Cs.

![137Cs spectrum for calibration](image)
1. Switch on all equipment used in this experiment.
2. Run the “USX” application.
3. Set Mode → PHA(AmpIn).
5. Use Settings → Amp/HV/ADC to access the MCA settings. Check that the settings are as listed below:
   - Amp In Polarity: Negative,
   - Coarse Gain: 8,
   - Fine Gain: 1,
   - Conversion Gain: 1024,
   - LLD: 18,
   - ULD: 1024,
   - Peak Time: $2 \mu$s.
6. Make sure that the GATE IN cable is not connected to the MCA. Place the small $^{137}$Cs source in front of the detector and start data acquisition. In the spectrum you should notice well formed peak around channel 240. It is total absorption peak corresponding to gamma radiation of energy 661.659 keV. The decay chain giving rise to the 661.659 keV photons is as follows:
   \[ ^{137}\text{Cs} \rightarrow ^{137}\text{Ba}^m + e^- + \bar{\nu}_e \quad (t_{1/2} = 30.05 \text{ a}) \]
   \[ ^{137}\text{Ba}^m \rightarrow ^{137}\text{Ba} + \gamma \quad (E_\gamma = 661.659 \text{ keV}, t_{1/2} = 2.552 \text{ min}) \]
7. Collect the spectrum to get a smooth peak around channel 240 (corresponding to 662 keV) so that all expected peaks during the whole experiment will be in MCA range 1024 channels (up to 2.6 MeV peak). If the position of the 662 keV is very different from channel 240, check again settings of the MCA. If it does not solve the problem, ask lab technician for assistance.
8. Use Settings → ROI’s → Set ROI to set regions of interest around 662 keV peak. When you place a cursor inside the ROI then information about mean position of the peak and its width are displayed in the information bar (Centroid: and FWHM:). You can also use View → Peak Finder to mark position of the peaks automatically, or just look for position of the maximum number of counts in the peak.
9. Use Settings → Energy Calibrate → 2Point to calibrate the spectrum in terms of energy rather than channel number. As a second point use channel 0 corresponding to 0 keV.
10. Erase the current spectrum and replace the $^{137}$Cs source with the $^{60}$Co source and acquire the spectrum.
11. Determine the energies of the two gamma ray peaks from $^{60}$Co and compare it with accepted values: 1173.2 keV and 1332.5 keV.

5 Examination of the $^{137}$Cs spectrum

Apart from the total absorption peak, there are other features of note in this spectrum:

- Compton electron edge.

If a photon of incident energy $E_\gamma$ makes a Compton collision with an electron in the scintillator and the scattered photon emerges at an angle $\theta$ with energy $E_{\gamma'}$, conservation
of momentum and energy give

\[
\frac{1}{E_{\gamma'}} - \frac{1}{E_\gamma} = \frac{1}{E_e}(1 - \cos \theta)
\]

(1)

where \(E_e = 511\ \text{keV}\) is the rest mass energy of the electron. See the appendix [A] for the derivation of the equation (1). The recoil electron has an energy \(E_\gamma - E_{\gamma'}\) which is dissipated in the crystal by ionising collisions. The absolute maximum recoil energy given to the electron occurs when \(\theta = 180^\circ\). At this value of \(\theta\), \(E_{\gamma'} = 184\ \text{keV}\) and \(E_\gamma - E_{\gamma'} = 478\ \text{keV}\). This condition shows up as an edge on the spectrum. No single process (apart from total absorption) can dump greater energy in the crystal.

- Compton backscattered peak.

Gamma rays can be scattered into the crystal from the surrounding material. The energy of the scattered ray \(E_{\gamma'}\) as calculated above is 184 keV at \(\theta = 180^\circ\). In fact \(E_{\gamma'}\) varies slowly at large \(\theta\) and for these cases has a value near above 184 keV. A peak will thus develop in the spectrum at above 184 keV.

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\[\begin{array}{ll}
\text{7/2}^+ & 0\ \text{keV} \\
{^{137}\text{Cs}}_55 & 30.05\ \text{a} \\
\text{94.36\%} & 661.659\ \text{keV} \\
\text{11/2}^- & 283.5\ \text{keV} \\
0.00061\% & 0\ \text{keV} \\
5.64\% & \text{137 Ba}^m_{56} \\
\text{137 Ba}^m_{81} & 2.552\ \text{min} \\
\text{5+} & 0\ \text{keV} \\
{^{60}\text{Co}}_{33} & 5.2710\ \text{a} \\
\beta^{-} & 100\% \\
99.88\% & 2505.748\ \text{keV} \\
0.002\% & 0.30\ \text{ps} \\
0.12\% & 2158.61\ \text{keV} \\
0.0075 & 0.59\ \text{ps} \\
0.002 & 1332.508\ \text{keV} \\
0.000002 & 0.713\ \text{ps} \\
0.0076 & 0.0012 \\
99.98 & 26 \\
\end{array}\]

\[\begin{array}{ll}
\text{0+} & 0\ \text{keV} \\
{^{60}\text{Ni}}_{28} & 0.713\ \text{ps} \\
\text{0.713 ps} & 0.0012 \\
\text{0.59 ps} & 0.002 \\
\text{0.30 ps} & 0.0075 \\
\text{0.000002} & 0.0076 \\
\text{99.9826} & 0.0012 \\
\end{array}\]

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**Fig. 22-2** \(^{137}\text{Cs}\) and \(^{60}\text{Co}\) decay scheme.
**Question 1:** Why does the total energy absorption peak and the Compton edge, as shown on your plot, exhibit a rounded profile instead of being completely sharp?

**Question 2:** Compare the energies observed for Compton electron edge and backscatter peaks with the predicted values.

### 6 Compton scattering

The aim of this section is to verify the dependence of $E_{\gamma'}$ on $\theta$ as in equation (1).

1. Use the strong Cs source and the MCA in the coincidence (gated) mode (connect the cable going from the output of the amplifier/monostable circuit to the GATE IN input on the MCA).

2. Set $\theta$ to 60° and start accumulating a spectrum of the pulses from the NaI scintillator. You should notice that the counting rate decreases a lot compared with the gated mode counting. The reason is that the pulse from NaI scintillator is counted only if at the same time (more precisely up to 1 $\mu$s after), the pulse from organic scintillator occurs. The pulse in the organic scintillator is produced by the electron that gains energy from the deflected gamma ray. Thus each of the two detectors registers a pulse for the same primary gamma ray. The Compton scattered gamma ray has energy lower than 662 keV and you should notice a peak around 402 keV to be formed. Measure the energy of this peak the same way as you did during calibration procedure.

3. Repeat your measurements for $\theta$ changing from 0° to 110° in 10° steps. The optimum counting time varies with $\theta$; wait until the Compton scattered peak appears reasonably free of the statistical fluctuations (in practice $\approx$ 4 minutes). After each measurement take short spectrum of $^{137}$Cs without gating (using weak $^{137}$Cs source) to check if your calibration is correct. Write down position of 662 keV peak to use it later for correction of your data if needed.

4. Compare your results with equation (1) by plotting $1/E_{\gamma'}$ against $(1 - \cos \theta)$. Estimate $E_e$ from the slope and $E_{\gamma'}$ from the intercept. Use the spread in the data points to estimate errors for both quantities (“QtiPlot” calculates it automatically during the Fit Linear... procedure, in “LibreOffice Calc” or “MS Excel” use the LINEST function).

**Question 3:** When the scattering angle of the gamma photon $\theta = 0^\circ$, the electron does not gain any energy and therefore it does not produce any voltage pulse on the organic scintillation detector’s output. Why we observe that MCA still works in the coincidence mode at $\theta = 0^\circ$?

### 7 Examination of the spectrum from $^{22}$Na

The decay scheme for $^{22}$Na is as follows:

$^{22}$Na $\rightarrow ^{22+}$Ne + $e^+$ + $\nu_e$ (E$_{\text{max}}$ = 1821.02 keV, t$_{1/2}$ = 2.6029 a)

$^{22+}$Ne $\rightarrow ^{22}$Ne + $\gamma$ (E$_{\gamma}$ = 1274.577 keV, t$_{1/2}$ = 5.24 ps)
The positron emitted in the source soon slows down and annihilates via the reaction:

\[ e^+ + e^- \rightarrow 2\gamma \] (each of 511 keV)

These photons are emitted in opposite directions so that only one of each pair will be detected in the detector. However, in this experiment we can also use organic scintillator detector to observe very strict (time and direction) correlation between those two 511 keV gammas.

1. Place the $^{22}$Na source in front of the detector and run the MCA in the non-gated mode, until a smooth spectrum is obtained.
2. Identify and estimate the energy of the annihilation quanta (511 keV) and the high energy gamma ray from neon de-excitation (1275 keV).
3. Also note the Compton edges for each absorption peak and the backscatter peak.
4. Place the $^{22}$Na source in the middle in between the NaI detector and the organic scintillator detector. Run the MCA in the gated (coincidence) mode, until a smooth spectrum is obtained.
5. Compare this spectrum with the previous one and explain the differences.

**Fig. 22-3** $^{22}$Na decay scheme.

**Question 4:** Why in $^{22}$Na spectrum is only one backscatter peak evident when we have two of different energy (511 keV and 1275 keV) gamma rays in the spectrum?

**Question 5:** What are practical uses of electron-positron annihilation?

### 8 Natural radioactivity, the background

There is a surprisingly large concentration of naturally occurring radionuclides in the environment. The nuclides concerned fall into two main classes.

The first class comprises the nuclei with atomic number $Z > 82$ (lead) which are all radioactive with various half lives, emitting alpha particles, beta particles and gamma rays. Alpha decay
causes $Z$ to drop by 2 and the mass number $A$ to drop by 4. Beta decay changes $Z$ by one unit (up for $\beta^-$ and down for $\beta^+$); there is no change in $A$ (see Fig. 22-4). Gamma emission leaves both $Z$ and $A$ unchanged. Thus there are four separate decay chains of nuclei with $Z > 82$ with $A$ given by

$$A = 4n, \quad A = 4n + 1, \quad A = 4n + 2 \quad \text{and} \quad A = 4n + 3$$

where $n$ is a natural number.

For a chain to occur naturally, it is necessary that the nuclide at the head of the chain have a half life that is not too short compared with the age of the earth. It is currently believed that the heavy elements in the solar system were formed in a supernova explosion which is thought to have triggered the condensation of the sun and its system of planets. This event would have preceded the earth’s formation, making the criterion on minimum half lives of nuclei at the head of the chains even more stringent.

The result of the minimum half-life condition is that there are two major chains: $A = 4n + 1$ is headed by the nuclide $^{233}\text{U}$ (0.16 million years) whose concentration is 1/20000 times smaller than $^{238}\text{U}$ and thus can be neglected as a naturally occurring chain as far as this experiment is concerned; $A = 4n + 3$ is headed by the nuclide $^{235}\text{U}$ (704 million years) whose concentration is 140 times smaller than $^{238}\text{U}$ and for the present purposes this chain can also be neglected. We are left with $A = 4n + 2$ headed by $^{238}\text{U}$, half life 4468 million years and $A = 4n$, headed by $^{232}\text{Th}$, half life 14100 million years.

In the present experiment we will examine the $A = 4n$ chain. The longest half life in the chain

![Fig. 22-4 $^{232}\text{Th}$ decay chain.](image-url)
(apart from the chain’s head) is the 5.75 year $^{228}\text{Ra}$. Consider a source consisting exclusively of $^{232}\text{Th}$. As time progresses some $^{228}\text{Ra}$ is formed, then $^{228}\text{Ac}$ and so on, until all the daughter nuclides are present right down to $^{208}\text{Pb}$.

If the gaseous nuclide $^{220}\text{Rn}$ is kept in the source until it decays and if the source is kept for a time long compared with the life of $^{228}\text{Ra}$ (i.e. $\gg 6$ years) then we have secular equilibrium where the rate of production of each member of the chain is equal to the rate of its decay (excluding $^{232}\text{Th}$ and $^{208}\text{Pb}$ of course). This situation holds for the thorium chain in the floor and ceiling of the laboratory and for the thorium powder calibration source to be used later.

The second class of naturally occurring radionuclides in the environment have $Z < 83$. They also were presumably formed in the supernova explosion. By far the most important member of this class is $^{40}\text{K}$ with a half life of 1300 million years. It produced the large peak in the background spectrum at or near 1.46 MeV.

![Fig. 22-5](image-url) Simplified $^{208}\text{Tl}$ decay (only prominent energy levels and $\gamma$ transitions are shown).

![Fig. 22-6](image-url) $^{40}\text{K}$ decay scheme.

1. Record the spectrum from the thorium source. A single 400 s run (in the non-gated mode) will probably be enough.
2. Analyse the spectrum. The table [22-2] in the appendix shows the most prominent gammas from the thorium chain. The peak in or near 2.6 MeV is the gamma ray coming from the very last stage of the decay chain where $^{208}\text{TI}$ decays by beta emission, not to the ground state of $^{208}\text{Pb}$ but to its excited states. Try to assign isotopes from the $^{232}\text{Th}$ decay chain responsible for observed peaks, and comment on result.

3. Remove all radioactive sources from surroundings of NaI detector or place them in the lead shield.

4. Acquire the background spectrum during at least one hour. This spectrum also shows the 2.61 MeV gamma ray, which comes from nuclides of the $^{232}\text{Th}$ decay chain in the cement, sand and aggregate of the concrete of the laboratory floor and ceiling. A small proportion of the gammas come from other materials nearby, including those of people in the laboratory!

5. Analyse the peak produced by $^{40}\text{K}$ and compare its energy with the accepted value 1460.851 keV.

9 Calculation of dose equivalent from body Potassium (Optional)

$^{40}\text{K}$ undergoes beta type decay; one branch involves an electron and goes to $^{40}\text{Ca}$ (89% of decays), the other is by orbital electron capture and goes to $^{40}\text{Ar}$ (11% of decays). $^{40}\text{Ar}$ is nearly always produced in its 1.46 MeV first excited state and the gamma ray follows 1.6 ps later. An insignificant number of decays of the $^{40}\text{Ar}$ branch go to the ground state, some with positron emission.

It has been estimated that 2/3 of the heat flow out of the earth’s hot interior is due to the decay of the uranium chain, the thorium chain, and $^{40}\text{K}$.

The body contains about 0.2% of potassium of which 0.0117% is $^{40}\text{K}$. When decaying to $^{40}\text{Ca}$ the nuclide emits an electron and neutrino. The latter goes straight out of the body and, on the average, about half the disintegration energy (i.e. 1.31/2 MeV) goes into the electron. As the latter’s range is short, we can assume that all this energy is dumped into body tissue.

The other branch of the decay deposits energy into the body via positron emission if it occurs but, as mentioned above, this is insignificant.

Electron capture deposits almost no energy in the body. However, we might expect some energy dumped from the 1.46 MeV gamma ray. We could, for a rough calculation, assume that the gamma ray puts all its energy in the body. (In fact a large proportion of these gammas will exit from the body).

In order to compare radiation doses from different particles it is necessary to take into account the density of the ionisation produced by the particles and their secondaries. The so called

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2 More detailed information on radioactive nuclides and their radiation can be found in [2] and [3].

3 The large separation between the ground and first excited states 2614.552 keV is connected with the fact that $^{208}\text{Pb}$ has 82 protons and 126 neutrons. Both 82 and 126 are “magic numbers” for the nucleons in a nucleus so $^{208}\text{Pb}$ is “doubly” stable. Recall that in atomic physics one also has magic numbers; atoms with 2, 10, 18, 36, 54 and 86 electrons, namely the noble gas elements, have stable atoms, resulting in chemical inertness.
quality factor \( (Q) \) allows for this. The dose equivalent is a unit called the sievert (abbreviation Sv) which is the absorbed dose multiplied by Q. For alpha particles Q is 20 but for gamma rays and electrons it is unity.

<table>
<thead>
<tr>
<th>Source</th>
<th>Dose/(mSv/yr)</th>
<th>Percent of total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radon</td>
<td>2.0</td>
<td>55</td>
</tr>
<tr>
<td>Cosmic</td>
<td>0.27</td>
<td>8</td>
</tr>
<tr>
<td>Terrestrial</td>
<td>0.28</td>
<td>8</td>
</tr>
<tr>
<td>Internal</td>
<td>0.39</td>
<td>11</td>
</tr>
<tr>
<td>Total natural</td>
<td>3</td>
<td>82</td>
</tr>
<tr>
<td>Artificial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medical x-ray</td>
<td>0.39</td>
<td>11</td>
</tr>
<tr>
<td>Nuclear medicine</td>
<td>0.14</td>
<td>4</td>
</tr>
<tr>
<td>Consumer products</td>
<td>0.1</td>
<td>3</td>
</tr>
<tr>
<td>Occupational</td>
<td>&lt; 0.01</td>
<td>&lt; 0.3</td>
</tr>
<tr>
<td>Nuclear fuel cycle</td>
<td>&lt; 0.01</td>
<td>&lt; 0.03</td>
</tr>
<tr>
<td>Fallout</td>
<td>&lt; 0.01</td>
<td>&lt; 0.03</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>&lt; 0.01</td>
<td>&lt; 0.03</td>
</tr>
<tr>
<td>Total artificial</td>
<td>0.63</td>
<td>18</td>
</tr>
<tr>
<td>Total all</td>
<td>3.6</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 22-1 Annual effective dose equivalent

1. Using these assumptions, calculate the energy dumped per kilogram of body tissue per year from the potassium radioactivity in the body. This is called the absorbed dose and is usually given in the unit called the gray (abbreviation Gy). 1 Gy = 1 J/kg. Consider just one kilogram of normal body material and take one year as your unit of time (to facilitate the calculations).

2. Compare the dose equivalent from your body potassium with the contribution from other sources (see Table 22-1).

**Question 6:** Why does electron capture deposit almost no energy in the body?
References


Appendix

A Further notes on Compton scattering

The Compton effect is the dominant process of gamma ray absorption for gammas of intermediate energy (around 1 MeV quantum energy). The photoelectric effect is dominant at lower energies and pair production at higher energies (energies several times the threshold for this: 1022 keV).

The incident photon in a Compton event, say of energy \( E_\gamma \), interacts with a quasi free electron on an atom in an absorber and an electron of energy \( E_e \) emerges: the Compton electron. This comes out of the interaction region with the photon which has now energy \( E_\gamma' \): the Compton photon.

Because only two particles come out we can say that the Compton electron and the Compton photon lie in the same plane and that plane has the incident photon in it too. In the analysis below we’ll use \( \theta \) for the angle in the plane, between the incoming and outgoing photon’s momentum.

A full investigation of the Compton effect would involve tracking all three particles and measuring their energies, the total cross-section for the reaction and the differential cross-section (Cross-section dependence on the angle \( \theta \)). However we will focus here on the formula for the dependence of \( E_\gamma' \) on the angle \( \theta \).

A set of two simultaneous equations (equation (3) is in vector form and it contains momentum conservation of both: longitudinal and transversal components) are used and the end result (equation (6)) is the same equation (1) that you investigated (or you will investigate) experimentally in Section 6.

\[
\begin{align*}
E_\gamma + E_e &= E_\gamma' + E_e' \\
\vec{p}_\gamma = \vec{p}_\gamma' + \vec{p}_e' \quad \text{(3)}
\end{align*}
\]

![Fig. 22-7 Compton scattering: (a) before and (b) after \( \gamma \) and \( e^- \) collision.](image)

From energy conservation:

\[
E_\gamma + E_e = E_\gamma' + E_e' \quad \text{(2)}
\]

From momentum conservation:

\[
\vec{p}_\gamma = \vec{p}_\gamma' + \vec{p}_e' \quad \text{(3)}
\]
Substituting $E_{e'} = \sqrt{(cp_{e'})^2 + E_e^2}$ in (2) and solving for $(cp_{e'})^2$ we have:

$$(cp_{e'})^2 = E_e^2 + E_{e'}^2 + 2E_e E_{e'} - 2E_e E_{e'} - 2E_e E_{e'}$$

(4)

From (3) we have $\vec{p}_{e'} = \vec{p}_e - \vec{p}_{e'}$. Now we square it:

$$p_{e'}^2 = p_e^2 + p_{e'}^2 - 2p_e p_{e'} \cos \theta$$

Let us multiply both sides by $c^2$ and for photons substitute $cp_e$ and $cp_{e'}$ by their energies $E_e$ and $E_{e'}$ respectively:

$$(cp_{e'})^2 = E_e^2 + E_{e'}^2 - 2E_e E_{e'} \cos \theta$$

(5)

The right sides of equations (4) and (5) must be equal. A few terms cancel from both sides and after simplifying we have

$$E_e E_{e'} - E_e E_{e'} = E_e E_{e'} - E_e E_{e'} \cos \theta$$

Now divide both sides of the above equation by $E_e E_{e'}$ and the result is:

$$\frac{1}{E_{e'}} - \frac{1}{E_e} = \frac{1}{E_e} (1 - \cos \theta)$$

(6)
\textbf{B} \textsuperscript{232}Th chain gammas

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Isotope & Energy/keV & Intensity/\% \\
\hline
\textsuperscript{228}Ac & 99.509 & 1.26 \\
& 129.065 & 2.42 \\
& 209.253 & 3.89 \\
& 270.245 & 3.46 \\
& 328.000 & 2.95 \\
& 338.320 & 11.27 \\
& 409.462 & 1.92 \\
& 463.004 & 4.40 \\
& 755.315 & 1.00 \\
& 772.291 & 1.49 \\
& 794.947 & 4.25 \\
& 835.710 & 1.61 \\
& 911.204 & 25.80 \\
& 968.971 & 15.80 \\
& 1588.190 & 3.22 \\
& 1630.627 & 1.51 \\
\hline
\textsuperscript{228}Th & 84.373 & 1.22 \\
\hline
\textsuperscript{224}Ra & 240.986 & 4.10 \\
\hline
\textsuperscript{212}Pb & 238.632 & 43.30 \\
& 300.087 & 3.28 \\
\hline
\textsuperscript{212}Bi & 39.858 & 1.09 \\
& 727.330 & 6.58 \\
& 785.370 & 1.10 \\
& 1620.500 & 1.49 \\
\hline
\textsuperscript{212}Po & 583.191 & 2.00 \\
& 2614.533 & 2.60 \\
\hline
\textsuperscript{208}Tl & 277.37 & 6.6 \\
& 510.770 & 22.60 \\
& 583.191 & 84.50 \\
& 763.130 & 1.81 \\
& 860.564 & 12.42 \\
& 2614.533 & 99.00 \\
\hline
\end{tabular}
\caption{Gammas from \textsuperscript{232}Th chain which possibly can be observed in this experiment.}
\end{table}