9 June 2015
TIME ALLOWED: 3 hours

THE UNIVERSITY OF SYDNEY

FACULTY OF SCIENCE

PHYSICS 4
and GRADUATE DIPLOMA IN SCIENCE
PAPER 1

ADVANCED ELECTROMAGNETIC THEORY

ANSWER ALL THE QUESTIONS.
ALL QUESTIONS HAVE THE VALUE SHOWN.
Q 1.
Consider a conducting sphere of radius \( a \) with its center at the origin. The hemispheres are held at constant potentials \( -V \) for \( z > 0 \) and \( +V \) for \( z < 0 \). An electric dipole of magnitude \( p \) is placed at the origin, pointing in the \( z \)-direction.

a) Calculate the electric potential inside the sphere. Comment on your results; what are the nature of the first two non-zero multipoles (note, \((-1)!! = 1\)).

b) Calculate the electric field along \( z \)-axis and argue that it is in the \( z \)-direction. Find the potential energy of the dipole, identifying the self energy term, and the force acting on the dipole. Show that the dipole is in a stable equilibrium along the \( z \)-direction, and find the effective spring constant assuming \( p \ll a^2V \). Comment on the ability of the first two non-zero multipoles to trap molecules which are neutral but have an electric dipole moment.

c) How does the electric field acting on the dipole change when it is rotated around the \( x \)-axis by an angle \( \theta \)? Find the torque acting on the dipole in this position. How much work needs to be done to rotate the dipole by \( 90^\circ \) and \( 180^\circ \)? How would your answers to torque and work done change if \( V = 0 \)?

(20 marks)

Q 2.
An iron sphere of radius \( a \) has a charge \( q \) and a uniform magnetization \( M = M \hat{z} \). The sphere is at rest initially.

a) Find the electric and magnetic fields everywhere (remember that iron is a conductor).

b) Calculate the Poyting vector, the angular momentum density and the total angular momentum stored in the electromagnetic field.

c) The sphere is slowly heated so that it loses its magnetization, that is, \( M \rightarrow 0 \) in a time period \([0, T]\). Find the electric field induced during this process in terms of the rate of change in \( M \).

d) Calculate the torque imparted on the sphere during the heating process, and the angular momentum of the sphere at the end of heating. Show that the total angular momentum (fields plus mechanical) is conserved.

(20 marks)
Q 3.
Consider a particle with mass $m$ and charge $q$ in free space. The particle is subjected to a linearly polarized electromagnetic wave propagating in the $z$-direction with fields, $\mathbf{E}_0 = E_0 e^{i(kz-\omega t)} \hat{x}$ and $\mathbf{B}_0 = (E_0/c)e^{i(kz-\omega t)} \hat{y}$.

a) Write down the equation of motion for the particle using a damping force due to radiation given by, $-\gamma mv$. Describe the trajectory of the particle on the $x$-$z$ plane schematically during a steady-state cycle. Assuming the particle moves non-relativistically ($v/c \ll 1$), solve the equation of motion using the ansatz, $x(t) = x_0 e^{-i(\omega t+\delta)}$, and determine $x_0$ and $\delta$.

b) Find the radiation fields due to the motion of the particle, the angular distribution of the power radiated, and the total power radiated. Describe the effect of the particle on the original electromagnetic wave. Could this effect be used to detect charged particles?

c) Calculate the average power absorbed by the particle in one cycle of the driving force (assume $\gamma \ll \omega$). Using energy conservation, that is, the energy absorbed goes into the energy radiated in a steady-state cycle, derive an expression for the damping constant $\gamma$. Justify the charge, mass and frequency dependence in this expression. Show that the approximation, $\gamma \ll \omega$, is justified for electrons even for very high frequencies of $\omega \sim 10^{18}$ s$^{-1}$. ($c = 3 \times 10^8$ m/s, $\epsilon_0 = 8.9 \times 10^{-12}$, mass and charge of electrons: $m = 0.9 \times 10^{-30}$ kg, $e = -1.6 \times 10^{-19}$ C.)

(20 marks)