L5 More on Polarizability and Dielectrics

Lecture outline:
- Relationship of Polarizability to the Dielectric Constant
- Effect of temperature.
- Effect of frequency.
- Piezoelectricity.
- Ferroelectrics.

L5.1 : Polarizability and Dielectric Constant

Relation of Polarizability to the Dielectric Constant:

We have
\[ p = \alpha E, \quad P = n p \]

Dipole moment  
Polarization  
Number/volume

So it looks like
\[ P = n \alpha E = (K - 1) \varepsilon_0 E \]

and so
\[ \alpha = \frac{\varepsilon_0}{n} (K - 1) \]

This appears to relate the microscopic quantity polarizability ($\alpha$) to the macroscopic quantity dielectric constant ($K$).

L5.2 : Polarizability and Dielectric Constant

In fact it only applies in the case of a very dilute gas.

For denser media the local electric field experienced by an atom or molecule, which determines its dipole moment, differs from the total field in the material which determines the macroscopic polarization and dielectric constant. This is because the local field is modified by the fields of the surrounding dipoles.

More careful analysis leads to the “Clausius-Mossotti” equation:
\[ \alpha = \frac{3 \varepsilon_0 (K - 1)}{n (K + 2)} \]

L5.3 : Polarizability and Dielectric Constant

Note that if $K = 1$
\[ \alpha = \frac{3 \varepsilon_0 (K - 1)}{n (K + 2)} \]

reduces to the dilute medium form
\[ \alpha = \frac{\varepsilon_0}{n} (K - 1) \]

Examples: 1. Nitrogen gas, $n=2.52 \times 10^{25} \text{ m}^{-3}$, $K=1.00058$.

Then $\alpha/4 \pi \varepsilon_0 = 1.83 \times 10^{-30} \text{ m}^3$ using both formulas (since dilute). Experimentally, $\alpha/4 \pi \varepsilon_0 = 1.74 \times 10^{-30} \text{ m}^3$.

L5.4 : Polarizability and Dielectric Constant

2. Diamond.

If $P = 8 \times 10^{-6} \text{ Cm}^{-2}$, how far is centre of cloud of electrons from the nucleus? Atomic wt=12, atomic number = 6

density = $3.51 \times 10^3 \text{ kg m}^{-3}$

Now $n=3.51 \times 10^{3}/(12 \times 1.66 \times 10^{-2}) = 1.76 \times 10^{29} \text{ m}^{-3}$ & $P=np$, so $p=8 \times 10^{-6}/(1.76 \times 10^{29}) = 4.55 \times 10^{-36} \text{ Cm} \equiv p_d$

where $q=6 \times 1.6 \times 10^{-19} \text{ C}$,

so $d=4.55 \times 10^{-15}/(6 \times 1.6 \times 10^{-19}) = 4.7 \times 10^{-17} \text{ m}$.

If $K = 5.5$,

\[ \alpha/4 \pi \varepsilon_0 = 0.81 \times 10^{-30} \text{ m}^3 \text{ (Clausius-Mossotti)} \]

\[ \alpha/4 \pi \varepsilon_0 = 2.03 \times 10^{-30} \text{ m}^3 \text{ (dilute formula) – different} \]

L5.5 : More on Dielectrics

Some molecules have an intrinsic dipole moment – called polar molecules – an example is the water molecule, as we have seen.

Effect of temperature: the orientations of the molecules may be random due to their thermal motion, so there is no net polarization. An applied electric field will tend to line up the dipoles.
L5.6: More on Dielectrics

The resulting polarization is given by the Langevin equation:
\[ P = np \left( \coth \frac{pE}{kT} - 1 \right) \frac{pE}{kT} \]

(k = Boltzmann’s constant)

For big T, \( P \approx np \frac{pE}{3kT} \), small alignment, so small P.

For small T, \( P \approx np \), strong alignment, so large P.

L5.7: More on Dielectrics

Polarization depends on the frequency of E.

High frequency means only electrons respond.

Example of water: for DC, \( K = 80 \) but for light wavelength (500nm, 600THz), \( K = 1.77 \).

E oscillates too fast for molecules to rotate, so small polarization.

Refractive index \( n: K = n^2 \), so for water at visible wavelength, \( n = 1.33 \).

L5.8: More on Dielectrics

Polarization depends on the frequency of E.

High frequency means only electrons respond.

Example of water: for DC, \( K = 80 \) but for light wavelength (500nm, 600THz), \( K = 1.77 \).

E oscillates too fast for molecules to rotate, so small polarization.

Refractive index \( n: K = n^2 \), so for water at visible wavelength, \( n = 1.33 \).

L5.9: More on Dielectrics

Applications:

- Gas lighter
- Quartz crystal (watch, computers)
- Mechanical oscillations (definite frequency) give voltage in circuit.
- Feedback and a battery keeps it ringing.
- Electromechanical transducers: microphone, speakers, record player pickup, scanning tunnelling microscope.

L5.10: More on Dielectrics

Piezoelectricity:

Mechanical stress \( \rightarrow \) polarization \( \rightarrow \) electric field

Under stress, for suitable crystal structures (e.g. quartz, PZT), charges in crystal are asymmetrical so get dipole moment:

\[ D = K \varepsilon_0 E + eS \]

\[ T = cS - eE \]

where \( e \) = piezoelectric constant, \( S \) = strain, \( T \) = stress, \( c \) = elastic constant (Young’s modulus).

L5.11: More on Dielectrics

STM & nanotechnology

Working principle of the scanning tunnelling microscope: When the tip approaches the surface of a specimen up to a distance of few atomic widths and a voltage is applied, a so-called tunnelling current flows between the tip and the specimen. While the surface is being scanned (x direction), the distance of the tip via a feedback loop by keeping the tunnelling current (I) constant. The movement of the tip produces the elevation profile of atoms lined up in series. By shifting the tip sideways (y direction) a field of scanned lines is produced. Computer processing is used to turn this into a three-dimensional image of the surface.
L5.12 : More on Dielectrics
STM & nanotechnology

Xenon on Nickel, spelling out IBM

L5.13 : More on Dielectrics
STM & nanotechnology

Title : Carbon Monoxide Man
Media : Carbon Monoxide on Platinum

L5.14 : More on Dielectrics
STM & nanotechnology

Two point defects adorning the copper (111) surface. The point defects (possibly impurity atoms) scatter the surface state electrons resulting in circular standing wave patterns.

L5.15 : More on Dielectrics
STM & nanotechnology

"Quantum Corral" being constructed from Fe atoms.
Note the ripple-like quantum wavefunction of an electron trapped inside the corral

L5.16 : More on Dielectrics
STM & nanotechnology

2 Gold atoms on NaCl surface
Atom on the left was deliberately charged with a single electron using STM tip, hence appears lower.

Single electron memories?