L18 Electromagnetic Waves

Lecture outline:
- Electromagnetic waves.
- Energy and momentum transfer in EM waves.
- Antennas.
- Possible hazards of EM radiation.

L18.1 : Electromagnetic Waves
Consider fields in a vacuum, with no free charges or currents. Maxwell’s equations are:
\[ \oint E \cdot dA = 0 \quad \oint B \cdot dA = 0 \quad \oint E \cdot ds = -\frac{d\Phi_B}{dt} \]
\[ \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]
Consider a disturbance in \( \mathbf{E} \) and \( \mathbf{B} \) moving in the x-direction:

L18.2 : Electromagnetic Waves
From Faraday’s law:
\[ \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta B_x}{\Delta t} \alpha a \Delta x \]
\[ \therefore \frac{\Delta \Phi_B}{\Delta t} = \frac{\Delta B_x}{\Delta t} \alpha a \Delta x \]
and \( \oint E \cdot ds = (E_y + \Delta E_y)a - E_ya = \Delta E_ya \)
\[ \therefore \Delta E_ya = -\frac{\Delta B_x}{\Delta t} a \Delta x \]
\[ \therefore \frac{\Delta E_y}{\Delta x} = -\frac{\Delta B_x}{\Delta t} \]
or \[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_x}{\partial t} \]

L18.3 : Electromagnetic Waves
From Ampere’s law:
\[ \frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta E_y}{\Delta t} a \Delta x \]
\[ \therefore \frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta E_y}{\Delta t} a \Delta x \]
\[ \therefore \Delta E_ya = -\frac{\Delta B_z}{\Delta t} a \Delta x \]
\[ \therefore \frac{\Delta B_z}{\Delta x} = -\frac{\mu_0 \varepsilon_0}{\Delta t} \frac{\Delta E_y}{\Delta t} \]
or \[ \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]

L18.4 : Electromagnetic Waves
Combining the equations, we get the electromagnetic wave equation in vacuum:
\[ \frac{\partial^2 E_y}{\partial x^2} - \frac{\mu_0 \varepsilon_0}{\partial t^2} \frac{\partial^2 E_y}{\partial t^2} = 0 \]
This has the solution
\[ E_y = E_0 \cos(\omega t - kx) \]
which corresponds to a wave with phase velocity
\[ v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c \quad \text{and } c \text{ is the speed of light in a vacuum} \]
The magnetic field is
\[ B_z = \frac{E_0}{c} \cos(\omega t - kx) = B_0 \cos(\omega t - kx) \]
i.e. in magnitude
\[ E = cB \]

L18.5 : Electromagnetic Waves
The electric and magnetic fields are in phase, and linearly polarized. We can also have circularly polarized fields:
\[ \mathbf{E} = E_0(\cos(\omega t - kx)\hat{\mathbf{y}} + \sin(\omega t - kx)\hat{\mathbf{z}}) \]
L18.6 : Electromagnetic Waves

Energy transfer by electromagnetic waves:

The Poynting vector:

\[ \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} = \mathbf{E} \times \mathbf{H} \]

The amplitude gives the power crossing unit area. The direction gives the direction of the wave.

\[ |\mathbf{S}| = \text{energy /A per unit time} \]

Example: wave in x-direction with \( E_y \) and \( B_x \)

\[ \mathbf{E} \times \mathbf{B} = (0, E_y, 0) \times (0, 0, B_x) = (E_y B_x, 0, 0) = E_y B_x \hat{x} \]

\[ |\mathbf{S}| = \frac{1}{\mu_0} E_y B_x = \frac{1}{\mu_0} E_y \frac{c}{c} = \frac{1}{\mu_0} E_y \]

L18.7 : Electromagnetic Waves

Energy density in a cylinder length dx along the x-direction is:

\[ \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \frac{dU}{Adx} \]

\[ = \frac{1}{2c^2 \mu_0} E^2 + \frac{1}{c^2 \mu_0} \frac{E^2}{c^2} = \frac{1}{c^2 \mu_0} E^2 \]

So

\[ S = \frac{dU}{Adt} = \frac{dU}{Adx} \cdot \frac{dx}{dt} = \frac{1}{c^2 \mu_0} \frac{E^2}{c} = \frac{1}{c \mu_0} E^2 \]

L18.8 : Electromagnetic Waves

Now \( S = \frac{1}{c \mu_0} (E_0 \cos(\omega t - kx))^2 \) is the instantaneous energy flow rate.

More useful is the time average over a period \( T \).

Use

\[ \frac{1}{T} \int_0^T \cos^2(2\pi t/T) dt = \frac{1}{2} \]

Then

\[ S_{\text{average}} = \frac{1}{\mu_0 c} \frac{E_0^2}{2} \text{W m}^{-2} = I \quad \text{Intensity} \]

L18.9 : Electromagnetic Waves

Momentum transfer by EM waves:

There are 2 situations:

\[ \begin{align*}
\text{(absorbed)} & \quad \text{Let } \Delta U \text{ be the energy transferred in time } \Delta t. \text{ The theory of relativity says that for light } U = pc, \text{ where } p \text{ is the momentum. The momentum transferred is} \\
\text{(reflection)} & \quad \Delta p = \frac{\Delta U}{c} (\text{absorption}) \text{ or } = \frac{2\Delta U}{c} (\text{reflection})
\end{align*} \]

L18.10 : Electromagnetic Waves

The force exerted is

\[ F = \frac{\Delta p}{\Delta t} = \frac{1}{c} \frac{\Delta U}{\Delta t} \quad (\text{absorbed}) \]

Now \( \Delta U = IA \Delta t \) so

\[ F = \frac{IA}{c}, \quad \left( \frac{2IA}{c} \right) \]

The force per unit area, or radiation pressure is then

\[ P_{\text{rad}} = \frac{I}{c}, \quad \left( \frac{2I}{c} \right) \]

The radiation pressure in powerful lasers can lift small objects. The radiation pressure from the Sun causes dust tails in comets.

L18.11 : Electromagnetic Waves

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L18.12 : Electromagnetic Waves

If electromagnetic waves propagate in non-conducting matter rather than vacuum: \( E = vB \)

where \( v = \frac{1}{\sqrt{\mu \epsilon}} \)

The energy density becomes \( \frac{1}{2} \epsilon E^2 + \frac{1}{2\mu} B^2 = \frac{1}{v^2\mu} E^2 \)

The Poynting vector is \( S = \frac{1}{\mu} E \times B \)

and the intensity \( I = \frac{1}{\mu v} \frac{E^2 g_2}{2} W m^{-2} \)

L18.13 : Electromagnetic Waves

To produce EM radiation, we need accelerating charges, such as in oscillating currents:

Eg, a dipole antenna:

L18.14 : Electromagnetic waves

EM fields and health.

There may be a greater incidence of cancers near power lines (50Hz), but difficult statistics. The photons are of too low energy to ionize organic matter. Heating is negligible compared with heat generated by body. Perhaps cell membranes are disturbed, nerves stimulated etc. The typical magnetic field encountered from power lines or VDUs is \( \sim 10 \mu T \) (1/4 of Earth’s field). To minimise field from power lines, use 3-phase power transmission.

Fields partially cancel; total \( \sim 1/d^2 \)

If you swap A’ & C’, total \( \sim 1/d^3 \)

L18.15 : Electromagnetic waves

Mobile Phones: Frequency is 900 – 1900MHz. Energy in this range is non-ionising. The most apparent effect of RF energy at these frequencies is heating of tissue – but this is not a problem with low-powered phones – 125 mW (digital) to 600 mW (analogue). Normal heating of the body is in this range. The US standard limits peak exposure to 1.6mW/g – can be approached by phones. The evidence and research into effects of RF from phones is controversial.

L18.16 : Electromagnetic waves

A lawsuit in Florida in 1993 alleged brain cancer arising from mobile phone use, but was dismissed in 1995 due to lack of evidence. Epidemiology studies (large population statistics) have been mostly negative. Some animal studies have shown effects (breaking of DNA), but not repeatable.

Research at Sydney: Maybe the pulse nature of the signals damages cells – transient power is much bigger than averaged power. Hands-free phone – the speaker wire acts as an antenna.