EVOLUTION OF QUANTUM THEORY
Chapter 28
Hecht: Calculus 2nd ed.

More hydrogen lines

• Other series of hydrogen lines were discovered

\[ \frac{1}{\lambda} = R \left( \frac{1}{n^2} - \frac{1}{n^3} \right) \]

Paschen series (in the IR) for \( n \geq 4 \)

\[ \frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) \]

Balmer series (starts in Vis) for \( n \geq 3 \)

\[ \frac{1}{\lambda} = R \left( \frac{1}{1^2} - \frac{1}{n^2} \right) \]

Lyman series (in the UV) for \( n \geq 2 \)

All approach a "series limit" as \( n \to \infty \)

• However, nobody understood the significance of these simple relationships.

Ho hum, some definitions...

• Total Absorptivity \( \alpha \) (or "Absorptance")

fraction (or %) of total incident EM radiation absorbed by a surface averaged over a large range of \( \lambda \)
e.g. "Total solar absorptance" is the fraction of total solar radiation absorbed

• Absorption Coefficient \( \alpha_\lambda \):

fraction (or %) of incident EM radiation absorbed at a particular \( \lambda \)

If I leave out the \( \lambda \) bit, it means "total absorptivity"

…and so what is Black?

• An object looks black (e.g. soot) because visible reflectance \( R \approx 0 \) (i.e. visible absorptance \( \alpha \approx 1 \) or 100%)

Black road gets hotter in sun than white footpath

• Perfectly black \( \Rightarrow \alpha_\lambda = 1 \) for all wavelengths \( \lambda \), hard to do for all \( \lambda \) with pigments!

(Note: Perfectly white \( \Rightarrow \alpha_\lambda = 0 \) & \( R_\lambda = 1 \) or 100%)

• Cavity with a small aperture is a virtually perfectly black object ("black body") even if not black inside

The aperture behaves exactly like a highly absorbing surface.

Red-hot Physics

• You can feel "radiant heat" (IR) from hot objects

Human skin highly absorbing (\( \alpha_{IR} \approx 1 \)) in the IR

• As T\( ^\uparrow \) greater power emitted

• As T\( ^\uparrow \) above ~400°C objects emit noticeable intensity of visible light:

red hot \( \rightarrow \) orange hot \( \rightarrow \) yellow hot \( \rightarrow \) white hot

• What does this spectrum look like?

And why?
Stefan-Boltzmann Law

- Total power (Watts) emitted by a black body is:
  \[ P = \varepsilon \alpha AT^4 \] (S-B law)
  \[ A(m^2), \quad \alpha = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4) \text{ (S-B constant)} \]
  For non-black objects power is smaller; \( P = \varepsilon \alpha AT^4 \)
  \( \varepsilon \) = "total emissivity" or "emittance" so units (\( \varepsilon = 1 \) for blackbody)

- Kirchoff's radiation law
  "@ equilibrium, power absorbed = power emitted" implies that \( \varepsilon = \alpha \), where \( \alpha \) is absorptance averaged over the spectrum emitted by a black body @ temperature \( T \)

\[ \varepsilon \leq 1 \text{ always!} \]

Text Example 28.1

- 10 cm cube \( T = 400^\circ \text{C} \), total emissivity = 0.97
  Find rate at which it radiates energy from each face

- Solution:
  \[ P = \varepsilon \alpha AT^4 \]
  Area per face \( A = (0.1 \text{ m})^2 = 0.01 \text{ m}^2 \)
  \[ T = 400^\circ \text{C} + 273 = 673 \text{ K}, \quad \varepsilon = 0.97 \]
  \[ P = 0.97 \times 5.67 \times 10^{-8} \times 0.01 \times 673^4 \]
  \[ = 112.8 \approx 110 \text{ W} \]

But what COLOUR is BB radiation?

BB spectrum is continuous with broad peak @ \( \lambda_p \)

- Wien's displacement law; \( \lambda_p = \frac{b}{T} \) (empirical result)
  where \( b = 2.898 \times 10^{-8} \)

Rayleigh & Jeans failed to explain BB spectrum using "classical" theory. Their equations predicted that intensity would blow up to \( \infty \) as \( \lambda \) decreased

ultraviolet catastrophe

Text Example 28.2

- Assume human skin is BB in IR & \( T_{\text{skin}} = 33^\circ \text{C} \).
  Find wavelength at which humans radiate their maximum intensity

- Solution:
  \[ \lambda_p = \frac{b}{T} \]
  \[ b = 2.898 \times 10^{-8}, \quad T = 33^\circ \text{C} + 273 = 306 \text{ K} \]
  \[ \lambda_p = 2.898 \times 10^{-8}/306 = 9.5 \times 10^{-8} \text{ m} = 9.5\mu m \]

A Hint of Revolution

- Planck also tried to explain BB spectrum applying classical theory to cavity radiator.
- Eventually (1900) he obtained correctly-shaped spectrum by assuming that walls of cavity absorbed and emitted EM radiation only in packets or "quanta" of energy;
  \[ E = hf \]
  where Planck's constant \( h = 6.626 \times 10^{-34} \text{ Js} \)
- Huh?! Nothing in classical physics predicted this
Environmental: Radiation Module 17
Einstein & the Photoelectric effect

• Planck derived a formula for the shape of the BB spectrum which fitted all experimental data...

\[ I = \frac{2\pi hc}{\lambda^5} \left[ \frac{1}{e^{hc/\lambda kT} - 1} \right] \]

...but he regarded \( E = hf \) as a mathematical convenience and not a deep truth about light

Note: Sometimes we use a different version of Planck's constant:

\[ \hbar = \frac{h}{2\pi} = 1.0546 \times 10^{-34} \text{Js} \]

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So what's the final answer?

• but he regarded \( E = hf \) as a mathematical convenience... of the BB spectrum which fitted all experimental data...

Note: Sometimes we use a different version of Planck's constant:

\[ h = \hbar = 1.0546 \times 10^{-34} \text{Js} \]

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Photoelectric effect; What the..?!

• Reversing the voltage ("stopping potential") proved the \( e^- \) have a maximum energy \( (KE_{max} = V_i e^-) \)

\[ f_{max} \text{ depends on which metal is used} \]

\[ KE_{max} \text{ increases linearly with frequency} \]

• NO photocurrent for light frequency < \( f_{min} \) no matter how intense

• \( KE_{max} \) increases linearly with frequency.

• Huh! How very unclassical

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Einstein & the Photoelectric effect

• \( e^- \) need a minimum energy ("work function" + \( \phi \)) to leave metal. Each metal has different \( \phi \)

If \( h f < \phi \) then NO \( e^- \) ejected (i.e. \( h f_{min} = \phi \)).

If \( h f > \phi \) then some energy leftover from \( \phi \rightarrow KE \)

\[ eV_i = KE_{max} = hf - \phi \]

(For \( e^- \); \( E = eV \))

Plot \( KE_{max} \) vs \( f \); then slope = \( h \)

If \( h f > \phi \) then intensity means \( \uparrow \) \( e^- \) photons \( \uparrow \) current.

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Another crisis! Photoelectric effect

Hertz; X-rays, UV or Visible light causes \( e^- \) to be ejected from surface of metals (1887)

• Photocurrent proportional to light intensity (seems to make sense; more intensity, more energy, more \( e^- \) get enough energy to be ejected)

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That young upstart Einstein

Einstein went wild with Planck's formula. He supposed that light is actually composed of a stream of particles (now called photons) each with energy

\[ E = hf \]

• \( \uparrow \) light intensity means \( \uparrow \) \( e^- \) photons per second

• \( \uparrow \) light frequency means \( \uparrow \) energy per photon

Like atoms in the walls of Planck's BB cavity, \( e^- \) in the metal could only absorb one single photon at a time.

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Example: See problem 27 (Ch28)

• Find threshold frequency \( (f_{min}) \) if 220nm photons incident on a metal liberate electrons with a stopping voltage of 3.81V. (Remember; \( c = f \lambda \))

Solution:

\[ KE_{max} = eV_{stop} = hf - \phi = hf - h f_{min} \]

\[ KE_{max} = 1.60 \times 10^{-19} \cdot 3.81V = 6.10 \times 10^{-19}J \]

\[ h f_{min} = hf - KE_{max} \Rightarrow f_{min} = f - (KE_{max})/h \]

\[ f_{min} = 3.00 \times 10^{9}/220 \times 10^{-9} - 6.10 \times 10^{-19}/6.63 \times 10^{-34} \]

\[ f_{min} = 4.43 \times 10^{14} \text{Hz} \]
Left with embarrassing problem

- *But I thought light was a wave*

- Einstein;
  *Sometimes light behaves as particles (when being absorbed or emitted) & sometimes as waves (when travelling through open space)*

- "Wave-particle duality"

Yet another definition

- Energy of particles is often measured electrically using $E = qV$ so it is often convenient to express energy in electrical units
- 1 electron volt (or eV) is energy gained by an electron (or any singly charged particle) when accelerated though potential difference of 1 volt
- To convert from energy in Joules to eV;

  $$E(eV) = E(Joules)/e$$

  where $e = 1.602 \times 10^{-19}$ coulombs

Remember this

You should remember the following conversions;

- Wavelength of visible spectrum ranges from

  780nm (red end) to 390nm (violet end)

- In eV, this is approximately;

  1.6eV (red end) to 3.2eV (violet end)