In the last episode of the show...

- Photons (massless) are both particles & waves;
- Particles of matter are both particles & waves;

\[ E = hf \quad \text{&} \quad p = \frac{h}{\lambda} \]

& for particles (de Broglie):

\[ p = mv = \sqrt{2mK} = \frac{h}{\lambda} \]

Text Example 29.1

- Calculate de Broglie \( \lambda \) for \( \text{e}^- \) accelerated through 110V. Compare with typical X-ray wavelength.

Assume non-relativistic speed i.e. \( K = \frac{1}{2}mv^2 \)

- Solution: 1st find electron kinetic energy:

\[ K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 1.60 \times 10^{-19} \cdot 1.76 \times 10^{-17} = 1.76 \times 10^{-34} \text{J} \]

\[ v = \sqrt{2K/m} = \sqrt{2 \cdot 1.76 \times 10^{-34}/9.11 \times 10^{-31}} = 6.22 \times 10^6 \text{ms}^{-1} \]

\[ p = mv = 6.22 \times 10^6 \cdot 9.11 \times 10^{-11} = 5.66 \times 10^{-34} \text{kg ms}^{-1} \]

\[ \lambda = \frac{h}{p} = 6.63 \times 10^{-34}/5.66 \times 10^{-34} = 1.17 \times 10^{-10} \text{m} \]

- same wavelength range as X-rays \( \therefore \) can do "wave-like"\( \text{e}^- \) diffraction experiments with a crystal

Follow-up Example

- Calculate de Broglie \( \lambda \) for a tennis ball (57 g) with speed 50.0ms\(^{-1}\).

- Solution:

\[ p = mv = 5.7 \times 10^{-2} \text{kg} \cdot 50 \text{ms}^{-1} = 28.5 \text{ kg ms}^{-1} \]

\[ \lambda = \frac{h}{p} = 6.63 \times 10^{-34}/28.5 = 2.33 \times 10^{-34} \text{ m} \]

which is so small (atom spacing ~10\(^{-10}\)m) that one could never measure its wave properties. Definitely "particle-like"

Complementarity Principle (Bohr)

- Particle-like properties and wave-like properties are complementary, 2 sides of a coin i.e. not apparent at the same time. 🧵

- Mathematical example:

Remember; \( p = \frac{h}{\lambda} \) & \( E = hf = h/T \) (\( T \) = wave period)

\[ E \& p \text{ are particles-like properties. } T \& \lambda \text{ are wave-like properties. If } E \& p \text{ are large then } T \& \lambda \text{ are too small to detect and vice-versa} \]

Schrödinger's Wave Equation

Schrödinger developed a more general equation than de Broglie's equations; \( E = hf \) & \( p = \frac{h}{\lambda} \)

The function which describes a de Broglie wave is called "wavefunction" \( \psi(x) \) (or \( \psi \) for simplicity)

Start with classical mechanics equations;

- \( K = \frac{1}{2}mv^2 = p^2/2m \)
- \( E = E_{\text{kin}} + E_{\text{pot}} = p^2/2m + U \)

But \( p^2 = h^2/\lambda^2 \) so \( E_{\text{kin}} = \frac{h^2}{2m\lambda^2} \)

- To find \( E_{\text{kin}} \) we need a method to extract \( \lambda^2 \) from \( \psi(x) \)

Schrödinger's Wave Equation 2

- Use simplest example of a wavefunction \( \psi \):

\[ \text{e.g. } \psi = \sin(\frac{2\pi x}{\lambda}) \]

- Let's try taking "double derivative":

\[ \frac{d^2\psi}{dx^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \sin \left(\frac{2\pi x}{\lambda}\right) = -\frac{4\pi^2}{\lambda^2} \psi \quad \text{but} \quad E_{\text{kin}} = \frac{h^2}{2m\lambda^2} \]

\( \times \) both sides of equation by \( \frac{\hbar^2}{2m} \):

\[ \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E_{\text{kin}} \psi \]

\( K \equiv -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \) ("kinetic energy operator") & so...
Schrödinger's Wave Equation!
\[ \frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi = E\psi \]
- We used a simple sine wave to show this, but it's true for all quantum wavefunctions.
- To find a wavefunction \( \psi(x) \), substitute in a function to represent potential energy \( U \) and solve. "differential equation"

**DON'T PANIC...**

P.S. This treatment is only meant to be qualitative. You don't need to derive or solve this equation in the exam.

Wavefunction & probability
- Consider a simple wavefunction \( \psi \) for a single particle
- Square it to find probability density \( \psi^2 \)
- Black arrows \( \downarrow \) mark positions where the particle is most likely to be found
- Red arrows \( \uparrow \) mark positions where the particle is least likely to be found
- "Found" means, absorbed, detected, scattered etc. in other words, any experiment which reveals the particle-like property "position".

What is a wavefunction?
- A "ripple" is a wave on the surface of water
- Sound is a pressure wave in air
- Light is wave in the electromagnetic field

\( \psi \) So what kind of wave is a quantum wavefunction?
\( \psi \) Max Born 1926; \( \psi(x) \) is a wave of probability. The probability of finding the particle at a particular position is proportional to the square of the wavefunction \( \psi^2(x) \) at that position.
- \( \psi^2 \) is called "probability density"

In 2 places at once..
- Young's double slit:
  - Reduce intensity until only one photon (particle) at a time passes through slits then appears on the screen. (Permanently record photon positions).
  - At first pattern of photons appears random, but after a long time, all the individual photons build up to reveal an interference pattern (wave property).
  - Which slit did each photon go through? BOTH at the same time! Can only get interference if wave passes through both slits. HUH?!!
- Also works with beams of \( \text{e}^- \), individual atoms and (theoretically?) human beings

Please stop freaking me out
- Exactly when \& where photon (or \( \text{e}^- \) etc.) appears is random, but its probability is proportional to \( \psi^2 \).
  - On small scale a particle "trajectory" has little meaning.
  - @ the moment photon is detected (converted to a flash) its wave-like, spread-out wavefunction "collapses" to be replaced by a concentrated particle-like wavefunction.
  - "Collapse of the wavefunction" is not yet understood but is believed to be instantaneous.
- The act of measuring particle position causes the wavefunction to collapse into a definite position.
- Is the moon really there if no one's there to see it?