Modeling a falling slinky

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Research Bite
7 June 2012

The fall of a plastic rainbow-coloured slinky.
Overview

**Background**

*Slinky physics and the fall*

*Waves on slinkies*

**Modeling the fall**

*Improving an existing model*

*Comparison with real slinkies*

**Conclusions**
Background: Slinky physics

- Slinkies are useful physics demonstration devices
  - hanging configuration (e.g. Mak 1993)
  - vertical modes of oscillation when suspended (e.g. Young 1993)
  - wave propagation/dispersion (e.g. Crawford 1987; Vandegrift et al. 1989)
  - peculiar dynamics when falling (e.g. Calkin 1993; Aguirregabiria et al. 2007)

- Slinkies are tension springs\(^1\)
  - under tension subject to Hooke’s law but not compression
  - they collapse to a state with turns in contact
  - tension is required to separate collapsed turns

- A slinky suspended from its top and dropped ...
  - collapses from the top down
  - the bottom remains hanging (≈ 0.3 s) during the collapse!

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\(^1\) Compression springs (the other type of spring) may be under compression or tension according to \(F = -kx\). They have separated turns in a relaxed state, with zero tension.
For movies, see: www.physics.usyd.edu.au/~wheat/slinky/.
Background: Waves on slinkies

- Collapse of tension occurs from the top down
  - a (tension) wave propagates down the slinky
    - the tension remains ahead of the wave front
    - turns collapse behind the wave front
- Uncollapsed slinky turns are described by a wave equation:

\[
m \frac{\partial^2 x}{\partial t^2} = k \frac{\partial^2 x}{\partial \xi^2} + mg
\] (1)

- \( m \) is slinky mass, \( k \) is spring constant
- \( x = x(\xi, t) \) is the vertical location of a point on slinky
- coordinate \( \xi \) defines mass fraction: \( dm = m d\xi \) and \( 0 \leq \xi \leq 1 \)
  - so \( \xi_i = i/N \) is the end of turn \( i \) (for an \( N \)-turn slinky)
- Eq. (1): waves in turn spacing propagate
  - characteristic propagation time along slinky:

\[
t_p = \sqrt{m/k}
\] (2)

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\(^2\)For typical slinkies \( m \approx 0.2 \text{ kg} \) and \( k \approx 0.8 \text{ N/m} \) giving a characteristic time \( t_p \approx 0.5 \text{ s.} \)
Modeling the fall: Improving an existing model

- Solving the equation of motion directly is tricky
  - Eq. (1) applies until turns collide
  - the tension-spring behaviour complicates the description
- An earlier model used a semi-analytic approach: (Calkin 1993)
  - wave front assumed to be at \( \xi_c = \xi_c(t) \) at time \( t \)
  - behind the front: the turns are collapsed
  - ahead of the front: the hanging configuration (right)
  - calculate the total momentum implied by this
  - set equal to the impulse \( mgt \) at time \( t \) and solve for \( \xi_c \)
  - total collapse time (bottom starts to fall):\(^a\)

\[
t_c = \sqrt{\frac{m}{3k} \xi_1^3} = \sqrt{\frac{1}{3} \xi_1^3} t_p
\]

- \( 1 - \xi_1 \) is fraction of hanging slinky collapsed at bottom

\(^a\)For typical slinky parameters, with \( \xi_1 = 0.9 \), the total collapse time is \( t_c \approx 0.24 \text{ s} \).
Problem: turns collapse instantly at the front in the model
   for real slinkies: a finite time for turns to come together

An improved model: (Cross & Wheatland 2012)
   same semi-analytic approach but ...
   ... including a finite time for collapse behind the front
   tension relaxes linearly over a fixed number of turns
   the total collapse time is unchanged

Movies of the slinky fall and the model:

Modeling the fall: Comparison with real slinkies

- Rod dropped slinkies and filmed them at 300 frames/s
  - position of turns with time extracted from frames
  - position of top fitted to model to determine parameters:
    - spring constant, collapse parameter, time of release
- Two slinkies with different properties considered
  - slinky A is a metal slinky
  - slinky B is the plastic rainbow-coloured slinky

<table>
<thead>
<tr>
<th></th>
<th>Slinky A</th>
<th>Slinky B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
<td>215.5</td>
<td>48.7</td>
</tr>
<tr>
<td>Collapsed length (mm)</td>
<td>58</td>
<td>66</td>
</tr>
<tr>
<td>Stretched length (m)</td>
<td>1.26</td>
<td>1.14</td>
</tr>
<tr>
<td>Number of turns</td>
<td>86</td>
<td>39</td>
</tr>
</tbody>
</table>
An independent test of the model parameters:

- the spring constant $k$ is a fitted parameter
- fundamental period $T_0$ depends on this: \( (\text{Young 1993}) \)

\[
T_0 = 4\sqrt{\frac{m}{k}} = 4t_p
\]

Rod measured $T_0$ for each slinky ...

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<tr>
<td>Model $T_0$ (s)</td>
<td>2.23</td>
<td>1.88</td>
</tr>
<tr>
<td>Observed $T_0$ (s)</td>
<td>2.18</td>
<td>1.77</td>
</tr>
</tbody>
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Consistent after taking uncertainties into account
Conclusions

► Slinkies are useful physics demonstration devices
► **Falling slinkies exhibit peculiar physics**
  ► tension collapses from the top down
  ► the bottom remains suspended until the top hits it
  ► a wave must propagate downwards before the bottom falls
► An improved model of the fall is developed *(Cross & Wheatland 2012)*
  ► based on an existing semi-analytic model ... *(Calkin 1993)*
  ► ... modified so the collapse of slinky turns takes a finite time
► The new model is fitted to data from high-speed movies
  ► good qualitative fit to data achieved
  ► values of spring constant checked independently