

$$\pm \overline{z}_{\text{in}} \overline{z}_{\text{out}} | \alpha \rangle_{\mathcal{L}} = | \alpha \rangle_{\mathcal{L}} ,$$