

$$S=\langle P_1,P_2,\dots,P_{L^2-1},S_1,S_2,\dots,S_{L^2-1}\rangle$$