"Quantum Many-body Information" Workshop, Jan. 17 (2011)

Quantum computational capability of a two-dimensional valence bond solid phase

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Aims of the talk

1. quantum computation, harnessing many-body correlation of a 2D condensed matter system.

   measurement-based quantum computation (MQC): entanglement is consumed to build up complexity via measurements

2. ubiquitous usefulness as a computational resource in an entire phase (valence bond solid phase).

   - renormalization of many-body correlations
   - quantum computational matter

Aim

Perspective to intrinsic complexity of 2D quantum systems

$$\left|\Psi\right\rangle = \sum_{\alpha} \mathrm{tr}\left[\prod_{k}^{\mathrm{vertex}} A[\alpha_k]\left|\alpha_k\right\rangle\right]$$

$$\frac{\mathrm{tr}\, O\, e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}} \qquad \frac{\left\langle\Psi\right|O\left|\Psi\right\rangle}{\left\langle\Psi\middle|\Psi\right\rangle} \qquad \left\langle 0^z \ldots 0^z\right| U^{\dagger} S_1 U \left|0^z \ldots 0^z\right\rangle$$

classical 2D statistical (vertex) model

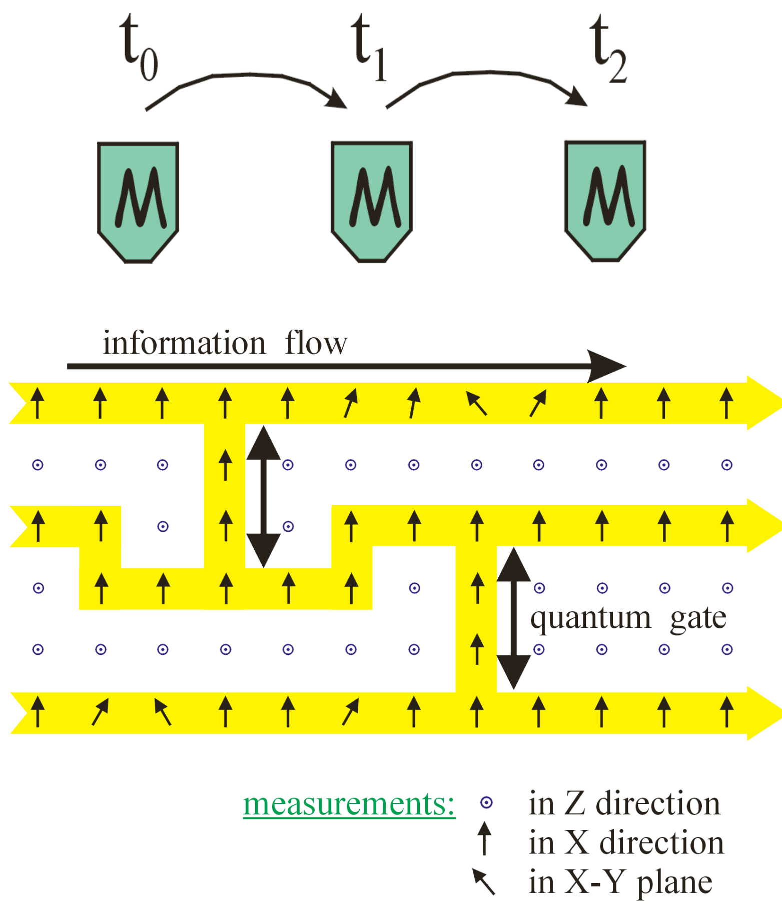
classical variational algorithm for 2D quantum system (cf. DMRG in 1D, …)

quantum transition magnitude of arbitrary *U* (quantum computer)

Measurement-based quantum computation 2

universal QC model by

- many-particle entanglement
- single-particle measurements
- communication of outcomes

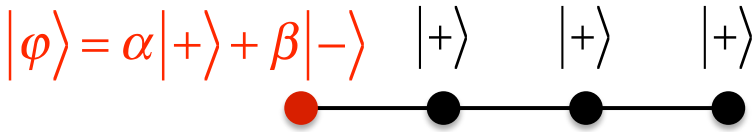


significant questions

- characterization of entanglement which enables one to simulate universal quantum computation (BQP)?
- practical implementation? (large-scale entanglement)

[MQC on a 2D cluster state: Raussendorf & Briegel, PRL '01; Raussendorf, Browne, Briegel, PRA '03]

measured correlation may give a unitary map



cluster state

[Briegel, Raussendorf, PRL'01]

$$CZ|\varphi\rangle_1|+\rangle_2 = \left(\alpha|+\rangle_1 + \beta|-\rangle_1\right)|0\rangle_2 + \left(\alpha|-\rangle_1 + \beta|+\rangle_1\right)|1\rangle_2$$

$$= |+\rangle_1\left(H|\varphi\rangle\right) + |-\rangle_1\left(XH|\varphi\rangle\right)$$

$$= |+,\eta\rangle_1\left(H\underbrace{\begin{bmatrix} 1 & \\ & e^{-i\eta}\end{bmatrix}}_{R_z(\eta)}|\varphi\rangle\right)_2 + |-,\eta\rangle_1\left(XH\begin{bmatrix} 1 & \\ & e^{-i\eta}\end{bmatrix}|\varphi\rangle\right)_2$$

$R_z(\eta)$ rotation around z axis

$|\pm,\eta\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\eta}|1\rangle)$

$\sigma(\eta) \equiv \cos\eta\,\sigma_x + \sin\eta\,\sigma_y$

$$|Cl\rangle = \sum_{\alpha_1,\ldots,\alpha_N=\pm} \mathrm{tr}\left[M[\alpha_N]\cdots M[\alpha_1]\right]|\alpha_1\ldots\alpha_N\rangle$$

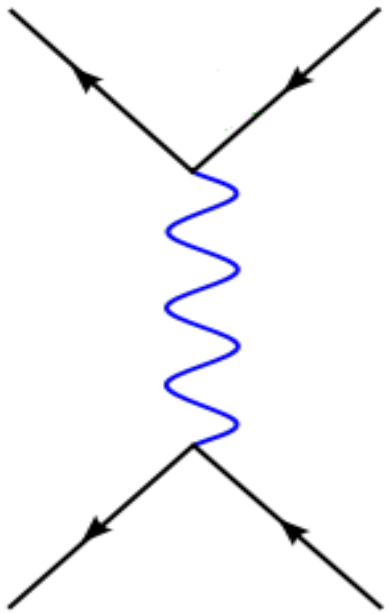
$$M[+] = H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1\end{pmatrix},\quad M[-] = XH$$

$$\cdots P^{vertex}_{\pm,\eta}|Cl\rangle \approx \cdots HR_z(\eta)|\varphi\rangle$$

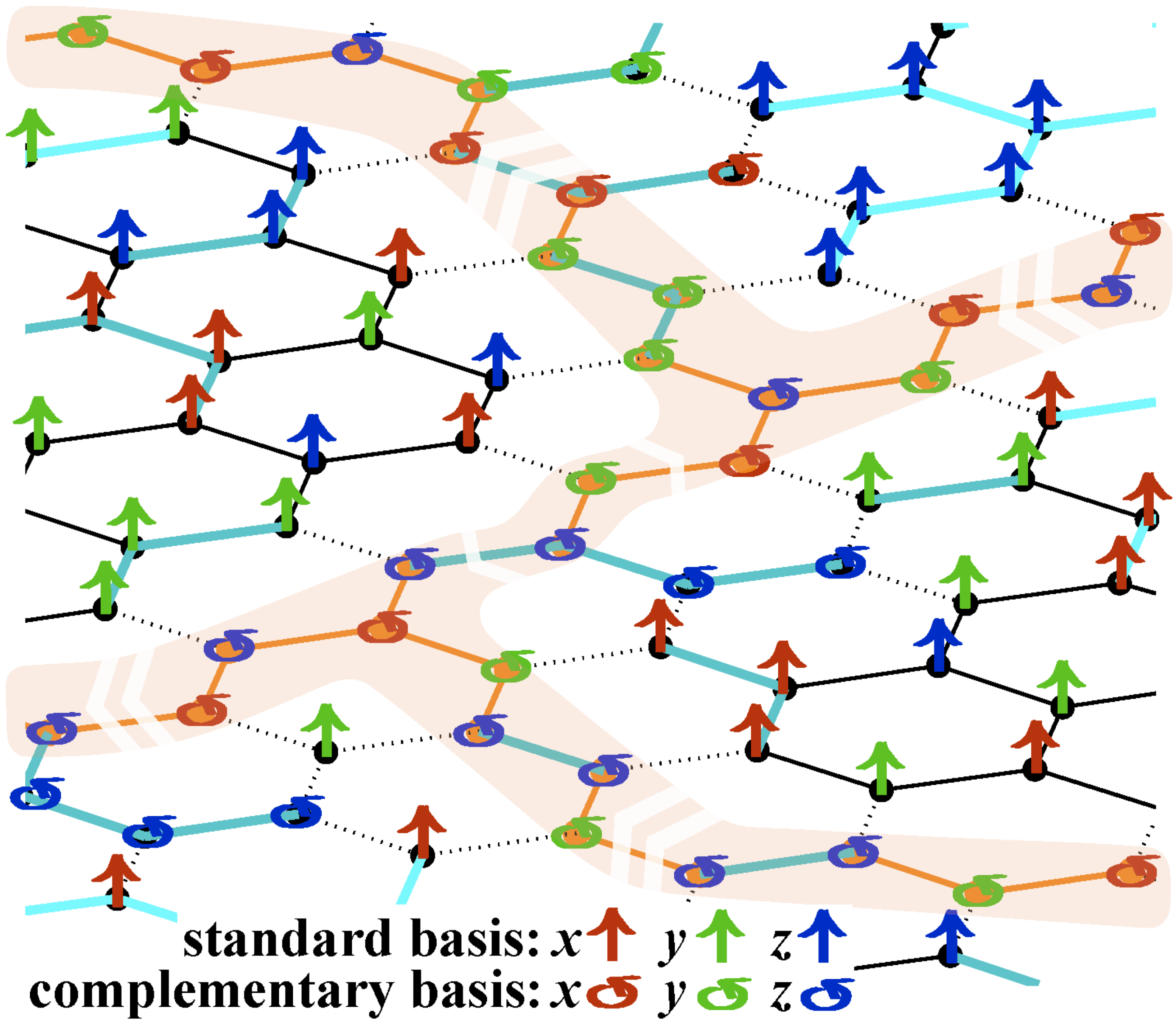
single-site quantum measurement allows tensors to tilt!

How does MQC look like?

microscopic information processing machine for our real world



scattering matrix



2D valence bond solid (VBS) phase

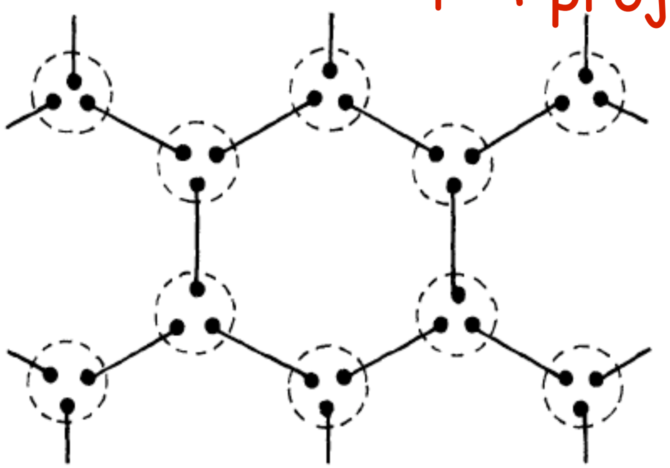
natural resource(?): a preparation by cooling
stability of a gapped ground state

quantum antiferromagnet of spin 3/2's on 2D hexagonal lattice

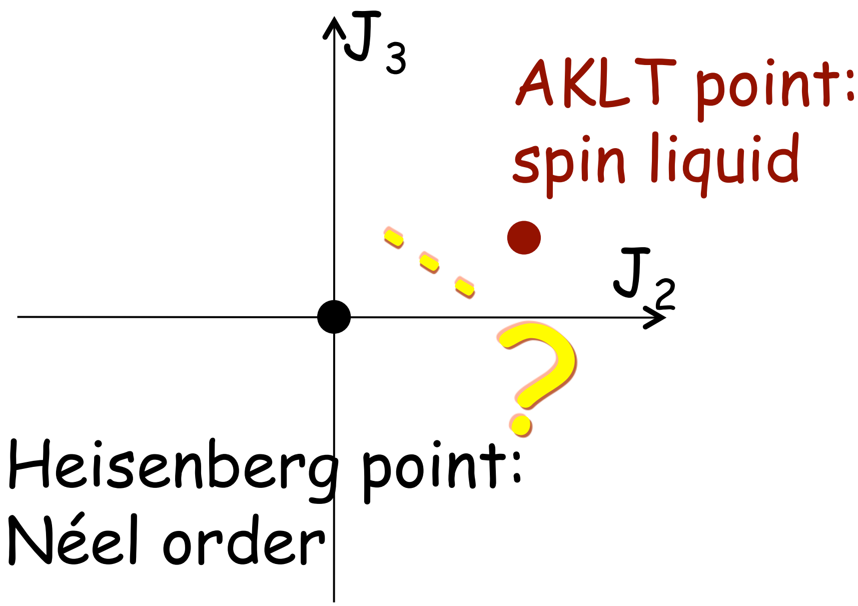
SU(2) invariant, J > 0

$$H = J \sum_{(k,k')}^{\text{n.n.}} \left[\mathbf{S}_k \cdot \mathbf{S}_{k'} + \underbrace{\frac{116}{243}}_{J_2} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^2 + \underbrace{\frac{16}{243}}_{J_3} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^3 \right],$$

P^3 : projector to total spin 3 for every pair



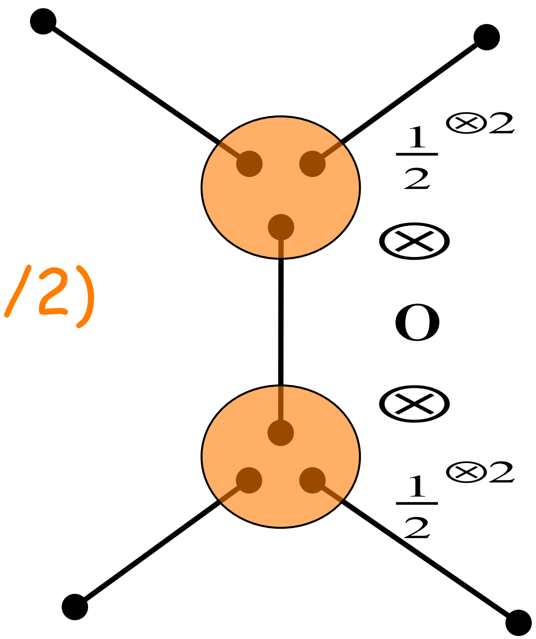
[Affleck, Kennedy, Lieb, Tasaki, PRL '87; CMP '88]
[Kirillov, Korepin,'89]



2D AKLT ground state: VBS construction


singlet of <u>virtual</u> two spin $\frac{1}{2}$'s

$$\frac{1}{\sqrt{2}}(|1^z\rangle \otimes |0^z\rangle - |0^z\rangle \otimes |1^z\rangle)$$



site: mapping to su(2) irrep (spin 3/2)

antisymmetric tensor per bond,
followed by symmetrization per site

$$P^3_{i,i+1}|g\rangle = 0$$: unique g.s.

[cf. optimal g.s. approach by Klümper et al.]

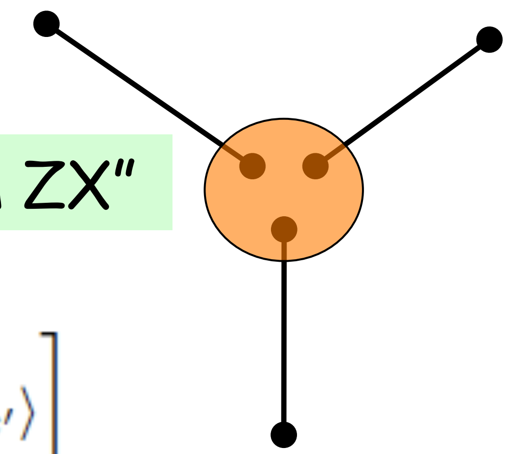
Schwinger boson method (total # bosons per site is 3)

[Arovas, Auerbach, Haldane'88; AKLT'88; Kirillov, Korepin,'89]

$$|\mathcal{G}\rangle = \sum_{\alpha_k,\alpha_{k'}} \mathrm{tr}\left[B \prod_{k\in\top} A_\top[\alpha_k]\,|\alpha_k\rangle \prod_{k'\in\perp} A_\perp[\alpha_{k'}]\,|\alpha_{k'}\rangle\right]$$

tensors

$$\frac{1}{\sqrt{2}}(|1^z\rangle \otimes |0^z\rangle - |0^z\rangle \otimes |1^z\rangle)$$



"symmetrization followed by left-side action ZX"

$$|\mathcal{G}\rangle = \sum_{\alpha_k,\alpha_{k'}} \mathrm{tr}\left[B \prod_{k\in\top} A_\top[\alpha_k]\,|\alpha_k\rangle \prod_{k'\in\perp} A_\perp[\alpha_{k'}]\,|\alpha_{k'}\rangle\right]$$

$$A[\pm\tfrac{3}{2}^{\mu}] \sim 000^{\mu}$$

$$A[\pm\tfrac{1}{2}^{\mu}] \sim \tfrac{1}{\sqrt{3}}\left(001+010+100^{\mu}\right)$$

$$A_\top[\alpha^x] = A_\top[\alpha^z]|_{z\mapsto x}, \quad A_\perp[\alpha^x] = -A_\perp[\alpha^z]|_{z\mapsto x},$$

$$A_\top[\alpha^y] = -iA_\top[\alpha^z]|_{z\mapsto y}, \quad A_\perp[\alpha^y] = A_\perp[\alpha^z]|_{z\mapsto y},$$

$A_\top[\tfrac{3}{2}$ … $| \otimes |0^z\rangle,$

$$A_\top[\tfrac{1}{2}^z] = \frac{1}{\sqrt{3}}(|0^z\rangle\langle 1^z| \otimes \langle 0^z| + Z \otimes \langle 1^z|), \quad A_\perp[\tfrac{1}{2}^z] = \frac{1}{\sqrt{3}}(-|0^z\rangle\langle 1^z| \otimes |1^z\rangle + Z \otimes |0^z\rangle),$$

$$B = \prod_\ell \Upsilon_\ell$$

$A_\top[-\tfrac{1}{2}^z] = \frac{1}{\sqrt{3}}$ … $\rangle\langle 0^z| \otimes \langle 1^z| + Z \otimes \langle 0^z|)$, $A_\perp[-\tfrac{1}{2}^z] = \frac{1}{\sqrt{3}}(|1^z\rangle\langle 0^z| \otimes |0^z\rangle + Z \otimes |1^z\rangle),$

$A_\top[-\tfrac{3}{2}^z] = |1^z\rangle\langle 0^z| \otimes \langle 0^z|$ … $A_\perp[-\tfrac{3}{2}^z] = |1^z\rangle\langle 0^z| \otimes |1^z\rangle,$

boundary tensor associated with a pair of edge states that are contracted in the case of a periodic condition

edge states

what is a physical entity of 0^z and 1^z ?
localized collective mode at boundary

$$\xi = 1/\ln(3/2) \approx 2.47$$

- area law of entanglement
  [Katsura et al. 2010]

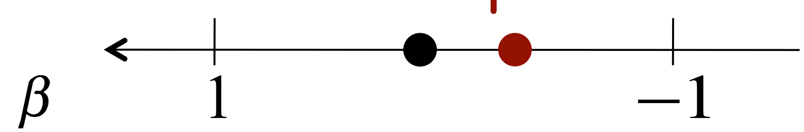
- degeneracy in gapped ground states
  (cf. topological feature)

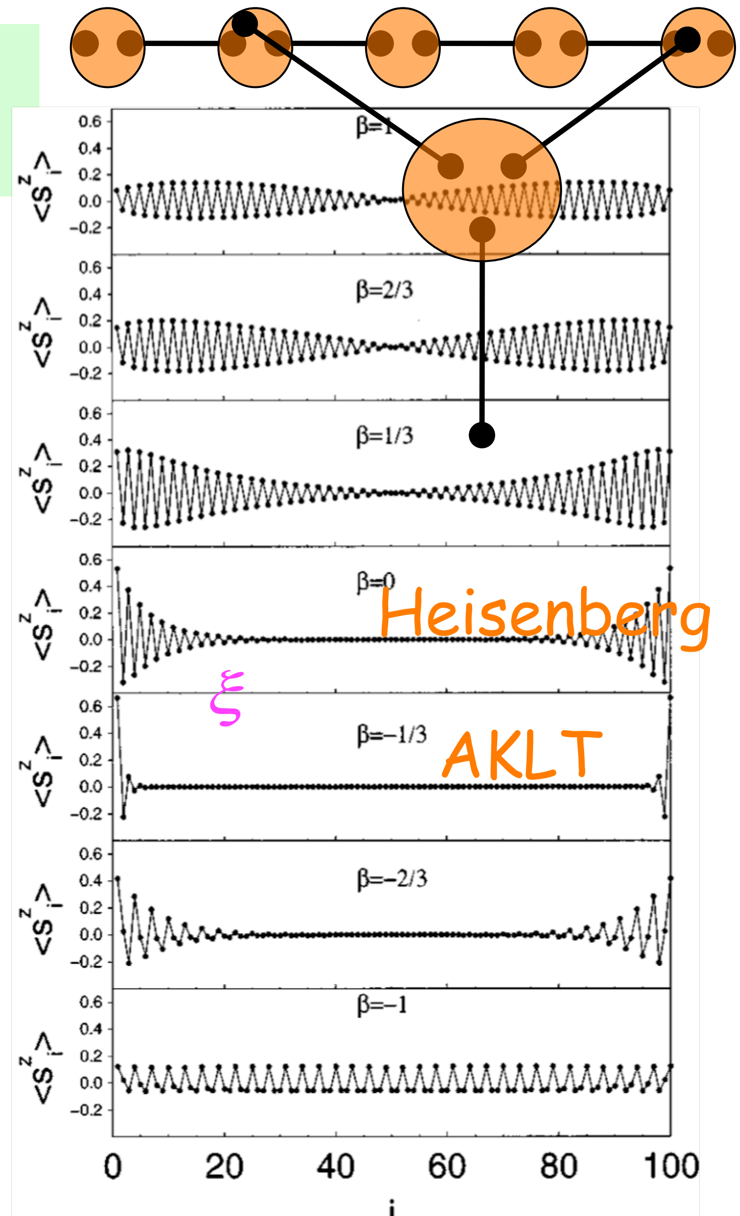
- ubiquitous in the VBS phase

1D SU(2)-invariant spin-1 chain

$$H = J \sum_{k=1}^{N-1} [\mathbf{S}_k \cdot \mathbf{S}_{k+1} - \beta(\mathbf{S}_k \cdot \mathbf{S}_{k+1})^2]$$

1D VBS phase





[Polizzi, Mila, Sorensen, PRB1998]

Challenge to construct MQC Protocol

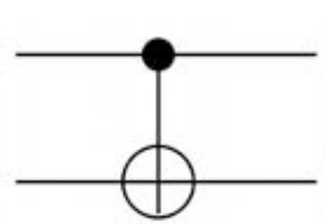
- entanglement network to gate-teleport quantum information

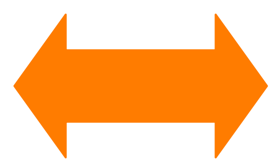
cluster-state has a VBS-like entanglement structure (PEPS)

[Gottesman, Chuang, Nature'99]
[Raussendorf, Briegel, PRL'01]
[Verstraete, Cirac, PRA'04]
[Childs, Leung, Nielsen PRA'05]
[Gross, Eisert, PRL'07;
Gross, Eisert, Schuch, Perez-Garcia, PRA'07] ....

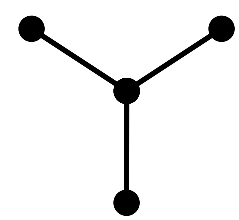
- steering quantum information in a controllable (quantum-circuit) manner

1+1D quantum circuit
= backbone





3-way symmetric !



How to get unitary maps? How to distinguish space and time?

Outline of MQC Protocol

How to get unitary maps and composed them?

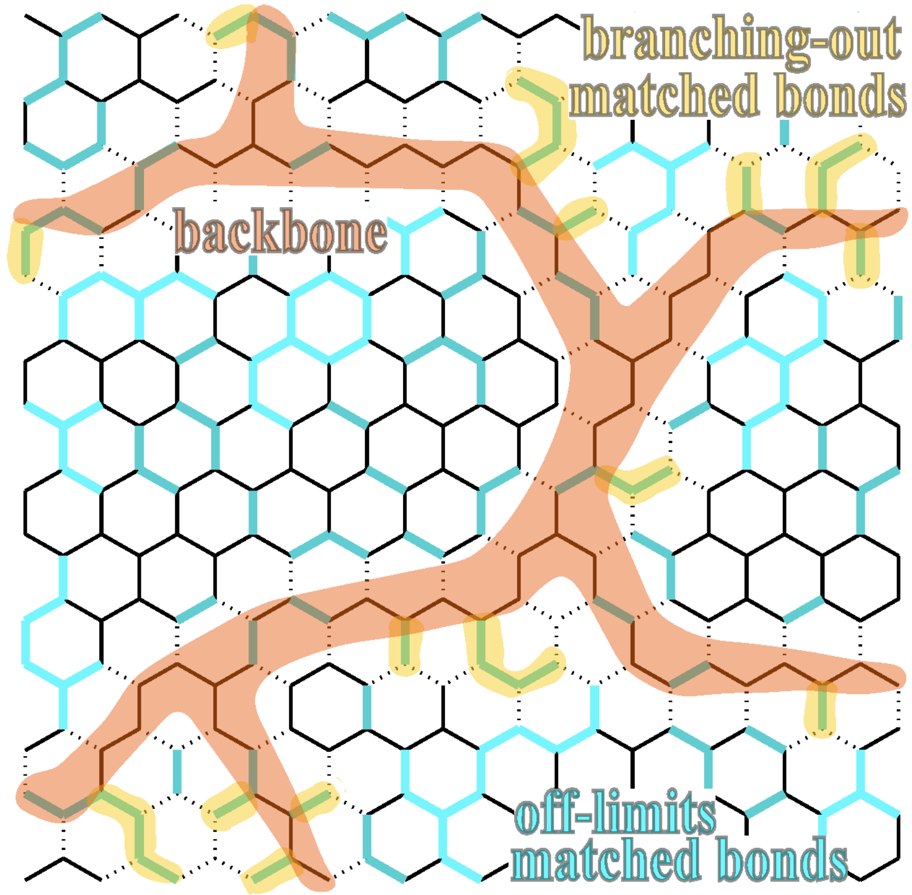
1. measurement at every site, depolarizing randomly into one of the three axes

$$\{M^x, M^y, M^z\}$$

$$M^\mu = \sqrt{\tfrac{2}{3}}(|\tfrac{3}{2}^\mu\rangle\langle\tfrac{3}{2}^\mu| + |-\tfrac{3}{2}^\mu\rangle\langle-\tfrac{3}{2}^\mu|)$$

$$\sum_{\mu=x,y,z} M^{\mu\dagger} M^\mu = 1$$

matched bond: $\mu_k = \mu_{k'}$

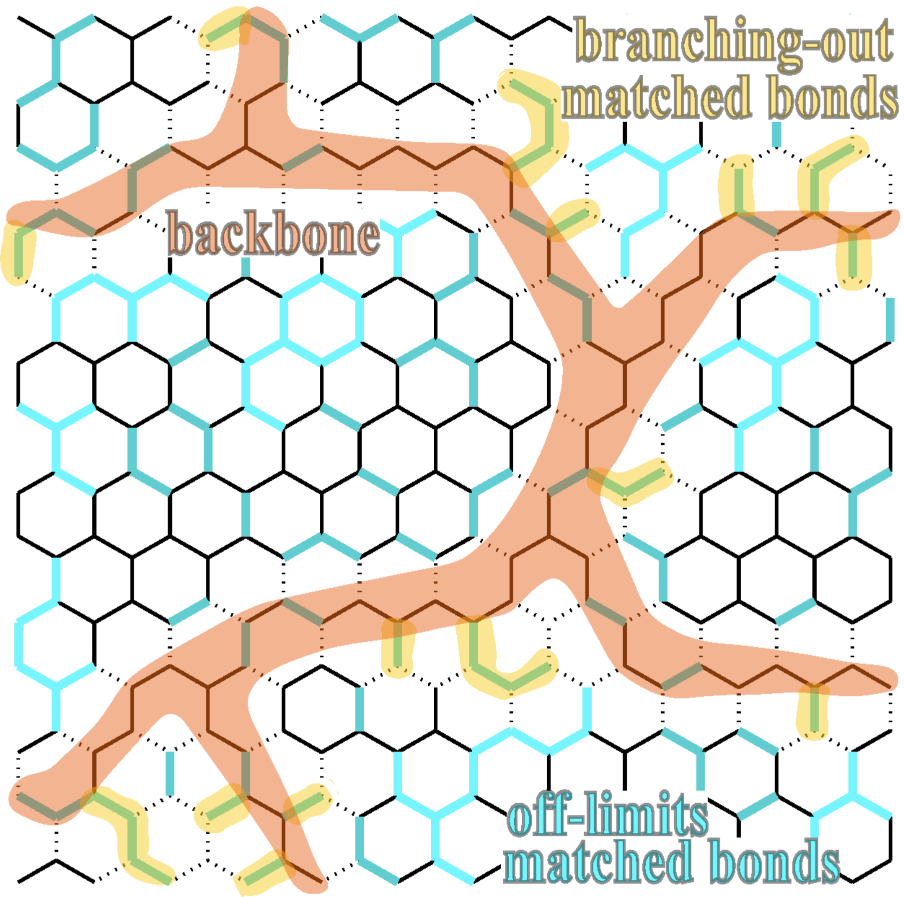


1′. classical side-computation:
in a typical configuration of matched bonds, identifying a backbone (which excludes all sites with triple matched bonds)

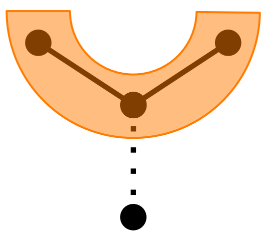
2. deterministic quantum computation

Ideas behind MQC Protocol

How to get unitary maps and composed them?



- a (mutually-unbiased) pair of standard and complementary measurements
  <= non-matched bond

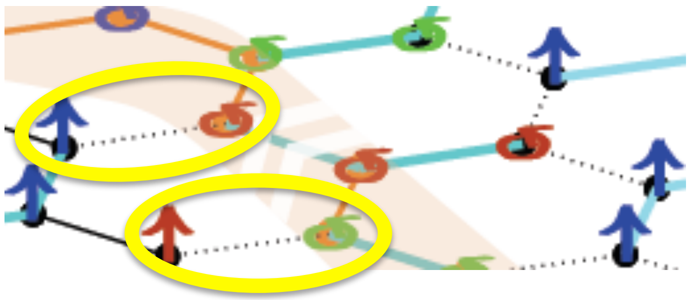


unitary!

- 2 bits of information per site

- "concentration" from 2D (3-way symmetric) correlation
  = classical statistical correlation (via random sampling)
  + "more rigid" quantum correlation

Unitary logical gates



if a bond is not matched, one site of the pair can be used in the backbone

backbone site: $\mu = z$ non-backbone site: $\nu = x, \pm\frac{3}{2}^x$

standard basis: x

$$\tilde{A}_{\top}[\tfrac{3}{2}^z] = |0^z\rangle\langle 1^z| \otimes \langle 1^z|0^x\rangle = \tfrac{1}{\sqrt{2}}|0^z\rangle\langle 1^z|,$$
$$\tilde{A}_{\top}[-\tfrac{3}{2}^z] = |1^z\rangle\langle 0^z| \otimes \langle 0^z|0^x\rangle = \tfrac{1}{\sqrt{2}}|1^z\rangle\langle 0^z|.$$

$\{\frac{1}{\sqrt{2}}(\langle\frac{3}{2}^z| + \langle -\frac{3}{2}^z|), \frac{1}{\sqrt{2}}(-\langle\frac{3}{2}^z| + \langle -\frac{3}{2}^z|)\}$ **complementary basis: z**

$$X (= \tilde{A}_{\top}[\tfrac{3}{2}^z] + \tilde{A}_{\top}[-\tfrac{3}{2}^z])$$

$$XZ (= -\tilde{A}_{\top}[\tfrac{3}{2}^z] + \tilde{A}_{\top}[-\tfrac{3}{2}^z])$$

1-qubit gates

Euler angles: $SU(2) = R^z(\theta_3)R^x(\theta_2)R^z(\theta_1)$

$$R^\mu(\theta) \; = \; |0^\mu\rangle\langle 0^\mu| + e^{i\theta}|1^\mu\rangle\langle 1^\mu|$$

complementary basis: z

one-parameter freedom: θ

$$\langle\gamma^{z|x}(\theta)| = \frac{1}{2\sqrt{2}}\left[\left(1+(-1)^b e^{i\theta}\right)\left(\langle \tfrac{3}{2}^z| + \langle -\tfrac{3}{2}^z|\right) + \left(1-(-1)^b e^{i\theta}\right)\left(-\langle \tfrac{3}{2}^z| + \langle -\tfrac{3}{2}^z|\right)\right]$$

$$\langle\gamma^{z|y}(\theta)| = \frac{1}{2\sqrt{2}}\left[\left(1+(-1)^b e^{i\theta}\right)\left(-i\langle \tfrac{3}{2}^z| + \langle -\tfrac{3}{2}^z|\right) + \left(1-(-1)^b e^{i\theta}\right)\left(i\langle \tfrac{3}{2}^z| + \langle -\tfrac{3}{2}^z|\right)\right]$$

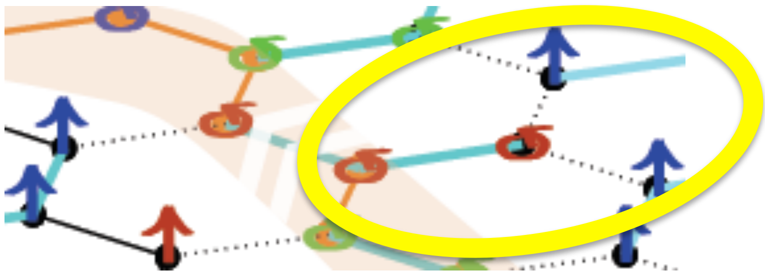
$$\sum_\alpha \tilde{A}[\alpha]\langle\gamma^{z|\nu}(\theta)|M^z|\alpha\rangle = XZ^{b\oplus c}R^z(\theta)$$

non-backbone outcome: $c \in \{0,1\}$

backbone outcome: $b \in \{0,1\}$

Pauli byproduct: $\Upsilon = XZ^{b\oplus c}$

branching-out matched bonds



if a backbone site is matched to its immediate non-backbone site

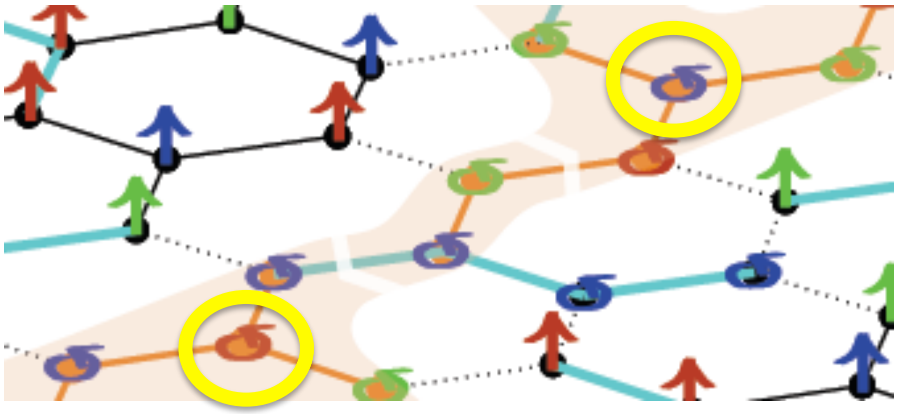
Measure all sites connected by matched bonds
in a complementary basis

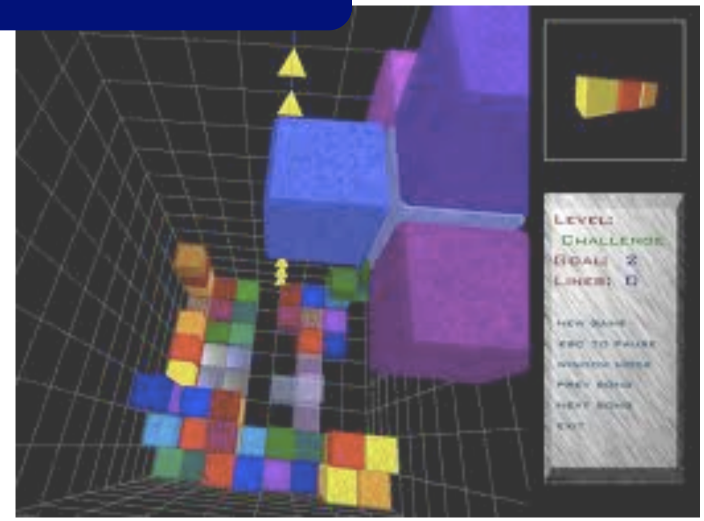
backbone site: $\mu = x$

branching-out part: $\Upsilon R^{x}(0)\left|0/1^{z}\right\rangle$

They can be a standard (z)-complementary (x) pair, as far as no site with triple matched bonds is attached to the backbone!

2-qubit gate: Controlled NOT





a pair of sites that share no matched bond

$$A_{\top}[\tfrac{3}{2}^{z}] = |0^z\rangle\langle 1^z| \otimes \langle 1^z|$$
$$A_{\top}[-\tfrac{3}{2}^{z}] = |1^z\rangle\langle 0^z| \otimes \langle 0^z|$$

$$A_{\top}[\gamma^{z|x}(0)] = \frac{1}{\sqrt{2}} \Upsilon_{\top} \left(1 \otimes \langle 0^x| + Z \otimes \langle 1^x|\right)$$
$$A_{\perp}[\gamma^{x|z}(0)] = \frac{1}{\sqrt{2}} \Upsilon_{\perp} \left(1 \otimes |0^z\rangle + X \otimes |1^z\rangle\right)$$

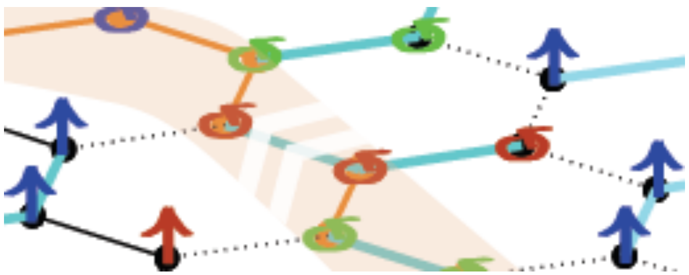
$$\text{CNOT } \left(= \tfrac{1}{2}\left(1 \otimes 1 + 1 \otimes X + Z \otimes 1 - Z \otimes X\right)\right)$$

Emergence of space and time

2-bit of classical information per backbone site: $\Upsilon = X^{a^x} Z^{a^z}$

μ	a^x	a^z
$\gamma^{x\|\nu}$	$b \oplus c$	1
$\gamma^{y\|\nu}$	$b \oplus c$	$b \oplus c \oplus 1$
$\gamma^{z\|\nu}$	1	$b \oplus c$

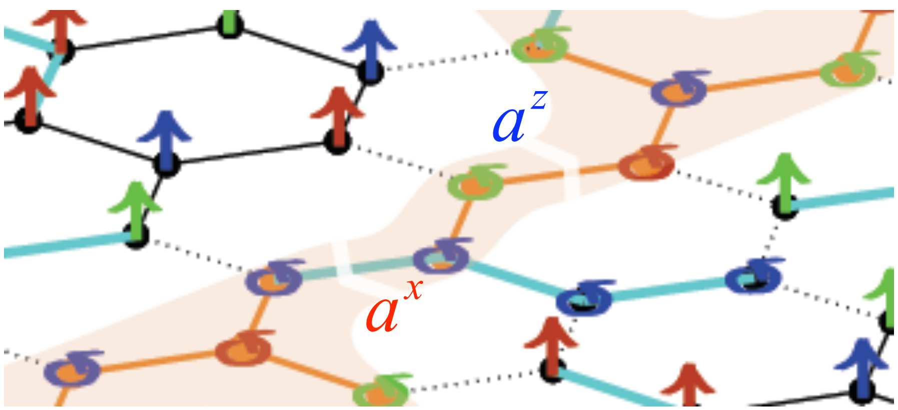
time-like:



adaptation for determinism

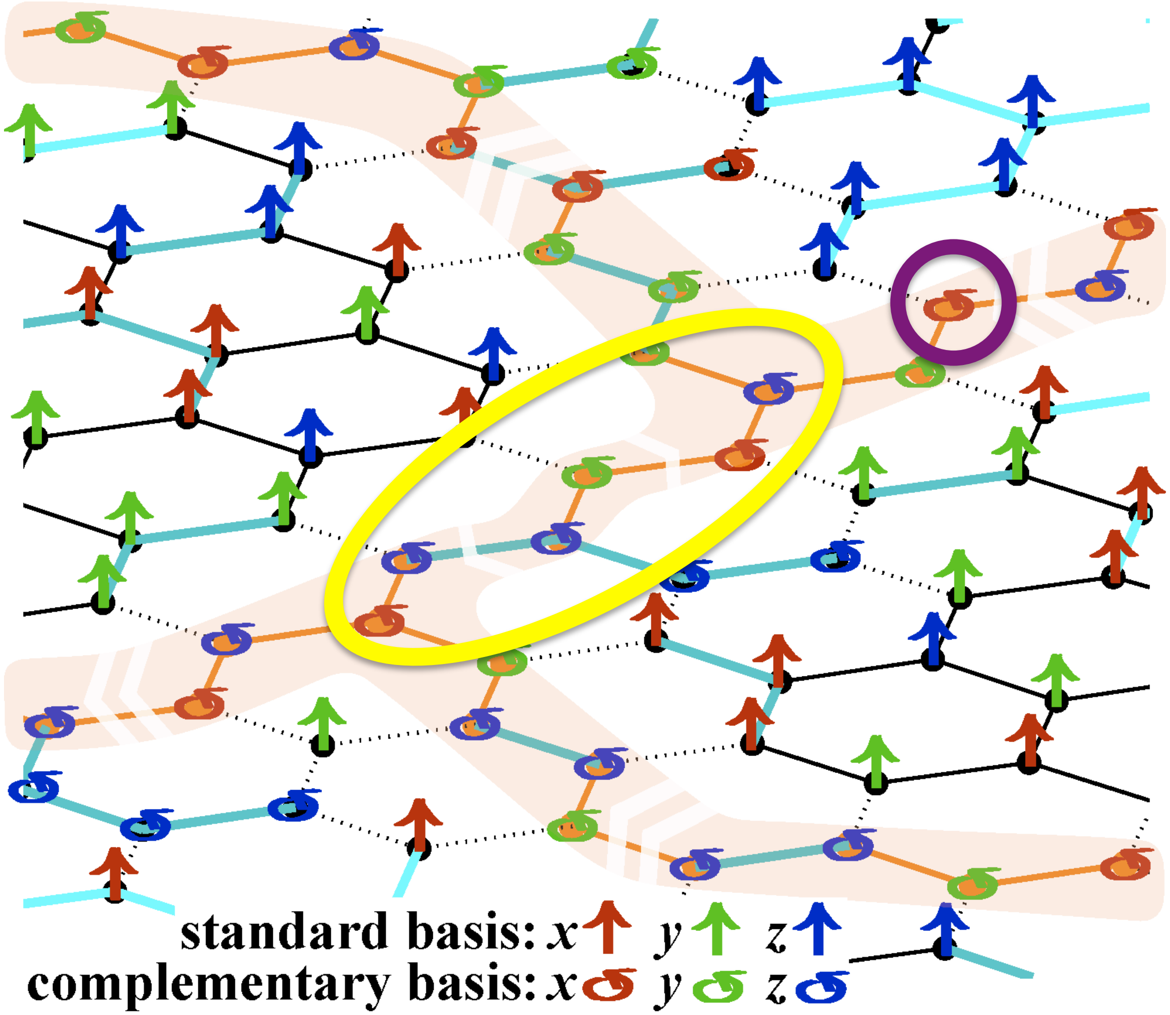
$$(X^{b'\oplus c'} Z R^x(\theta^x))(X Z^{b\oplus c} R^z(\theta^z)) = X^{b'\oplus c' \oplus 1} Z^{b\oplus c\oplus 1} R^x((-1)^{b\oplus c}\theta^x) R^z(\theta^z),$$

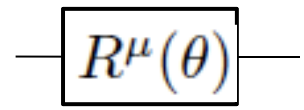
space-like:

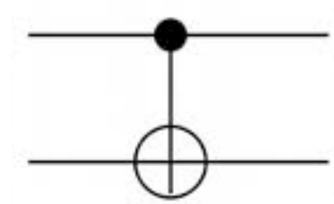


no adaptation because of "identity only" in between

Emergence of space and time







classical information at backbone site:

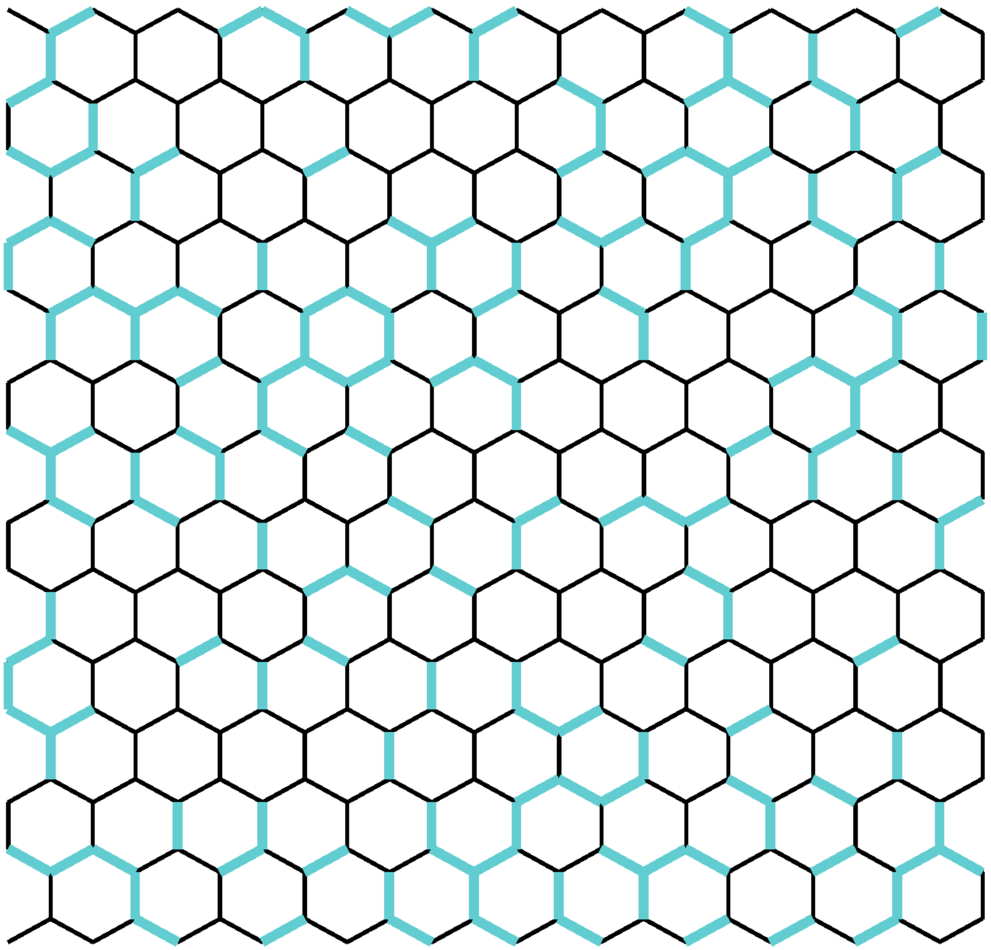
$$\Upsilon = X^{a^x} Z^{a^z}$$

time: two bits sent in the <u>same</u> direction

space: two bits sent in <u>opposite</u> directions (no net asymmetry in directions)

Identification of the backbone

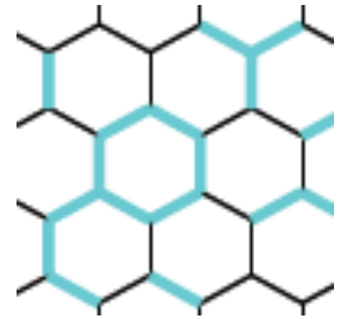
a classical problem: can we circumvent sites with triple matched bonds ?



intuition: it should be easier than avoiding all matched bonds.

approximation by bond percolation model

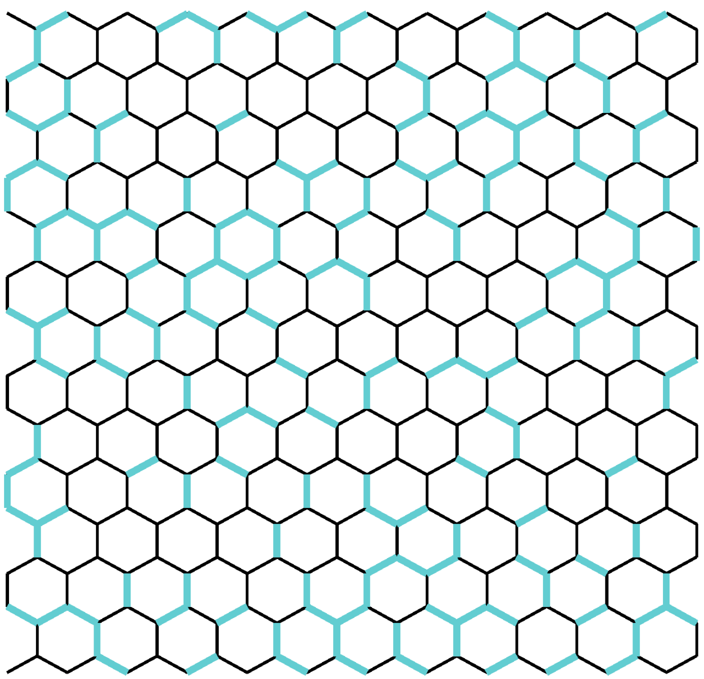
$\{M^x, M^y, M^z\}$ each probability 1/3 per site

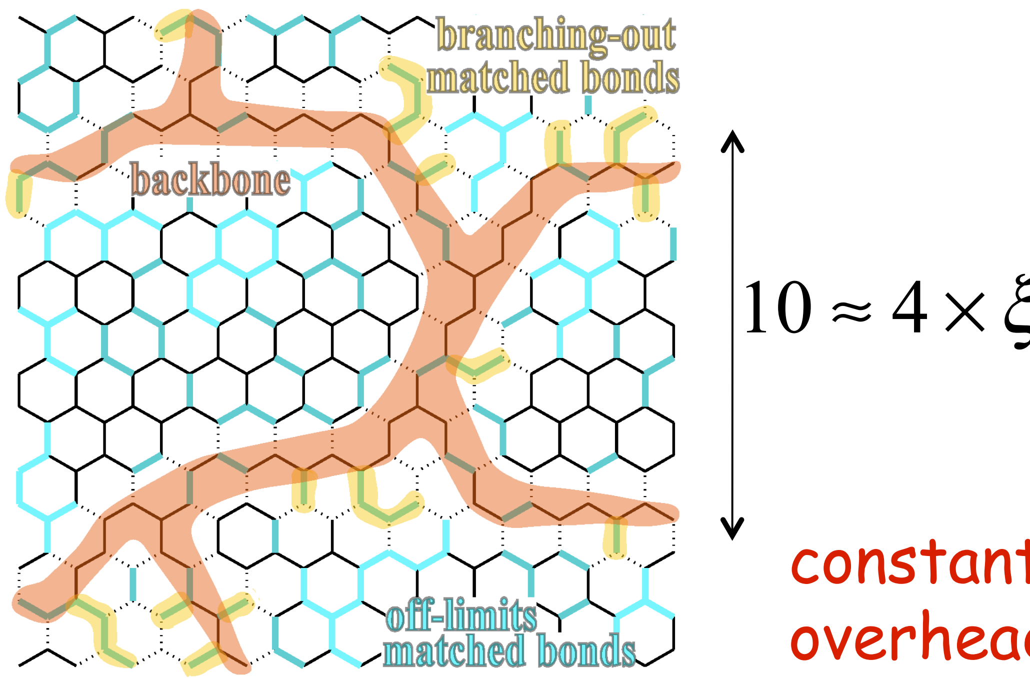


matched bond: $\mu_k = \mu_{k'}$

macroscopically analogous to bond percolation with p = 2/3

[cf. 2D hexagonal threshold: $p_c = 1 - 2\sin(\frac{\pi}{18}) \approx 0.652\ldots$]





constant overhead

Aim +

Does quantum computational capability (observed in the AKLT state) persist in an entire valence bond solid phase?

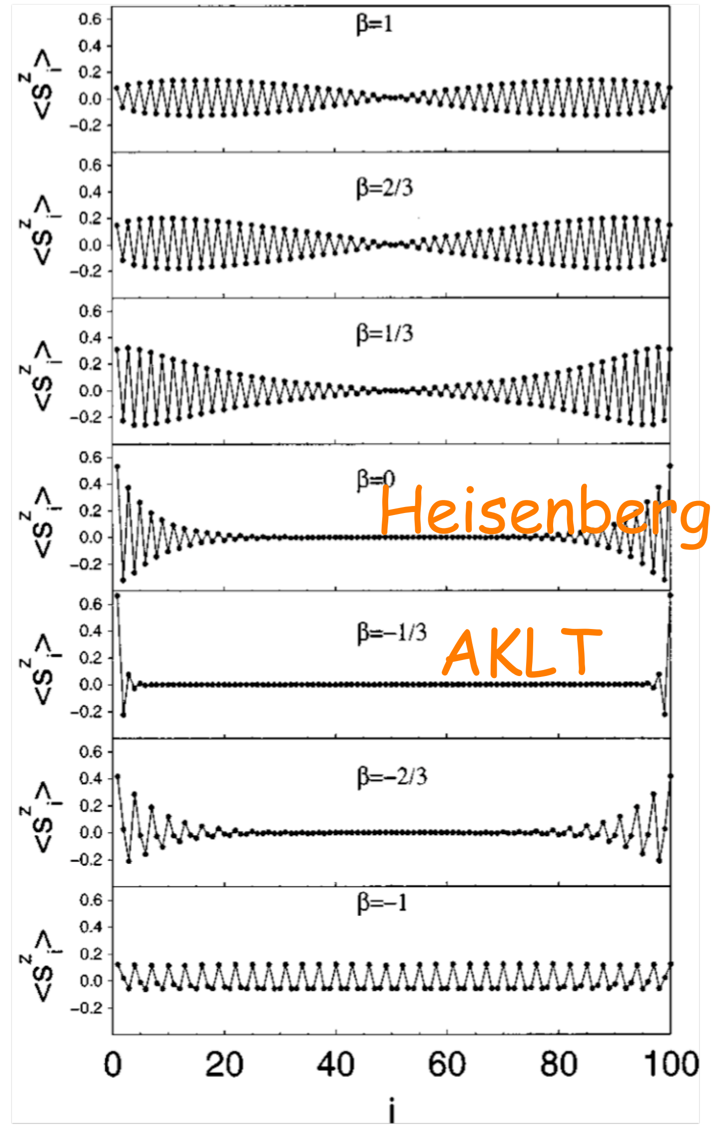
cf: cluster state is singular?
its epsilon neighborhood with epsilon ~ 0.01 is only available by fault-tolerance application

Persistence of computational capability

Analysis in 1D VBS phase

$$|G(\beta)\rangle, -1 < \beta < 1$$

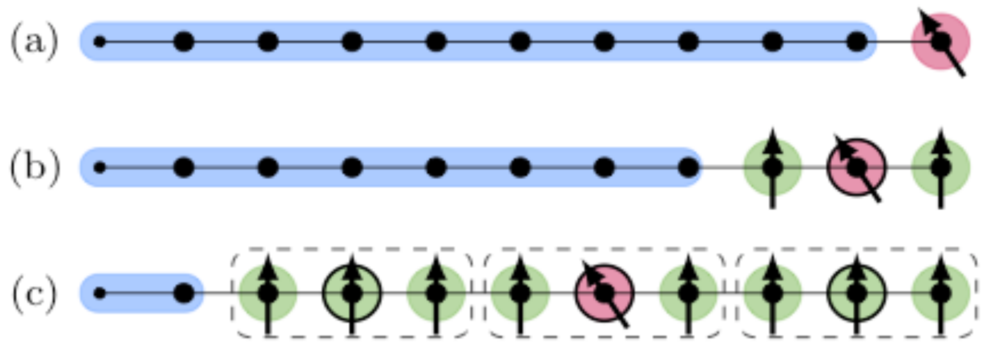
no known exact description except
beta = -1/3
(though the numerical approximation by MPS is possible)



Two possible solutions 1

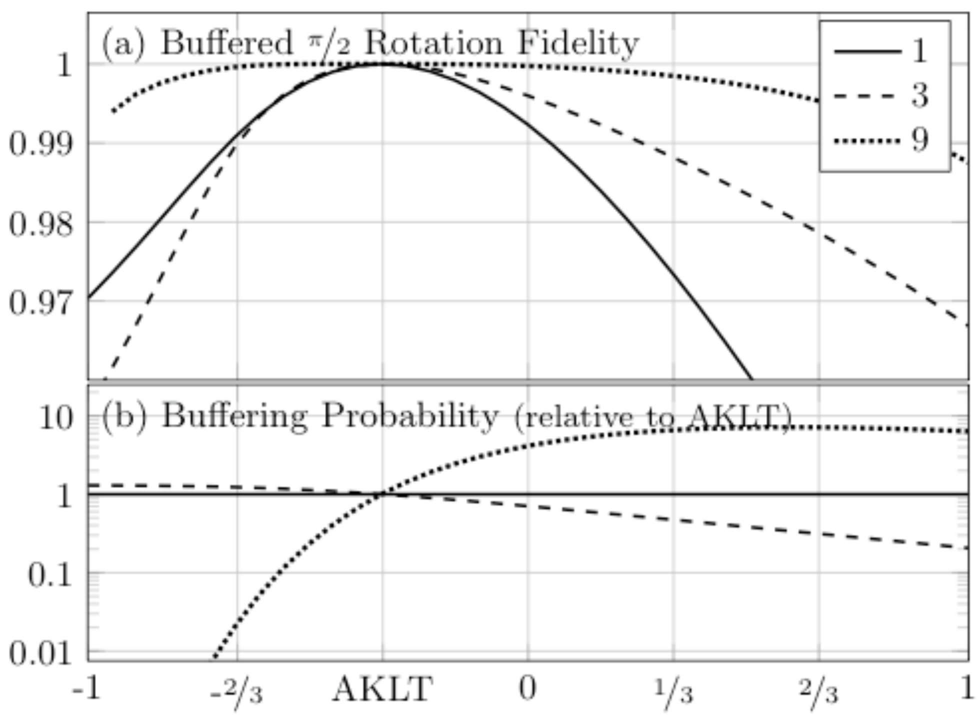
quantum computational renormalization

[Bartlett, Brennen, AM, Renes, PRL **105**, 110502 (2010)]



single-spin
measurements only

[compare CORE-DMRG]



probabilistic nature of
buffering is compensated
by a physical overhead
in length

Two possible solutions 2

adiabatic evolution by control of boundary Hamiltonian

[AM, PRL **105**, 040501 (2010)]

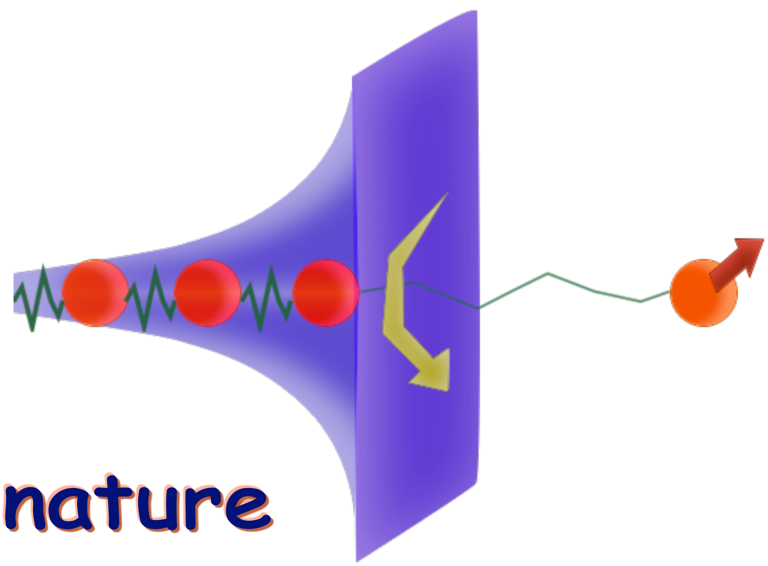
$$H(j) = J\sum_{k=j}^{N-1}[\mathbf{S}_k \bullet \mathbf{S}_{k+1} - \beta(\mathbf{S}_k \bullet \mathbf{S}_{k+1})^2]$$

$$H(j;t) = \left(1 - c(t)\right)h_{j,j+1} + H(j+1)$$

c(t) is monotonically increasing during a constant period *T*

entanglement persists by a property of (symmetry-protected) topological order

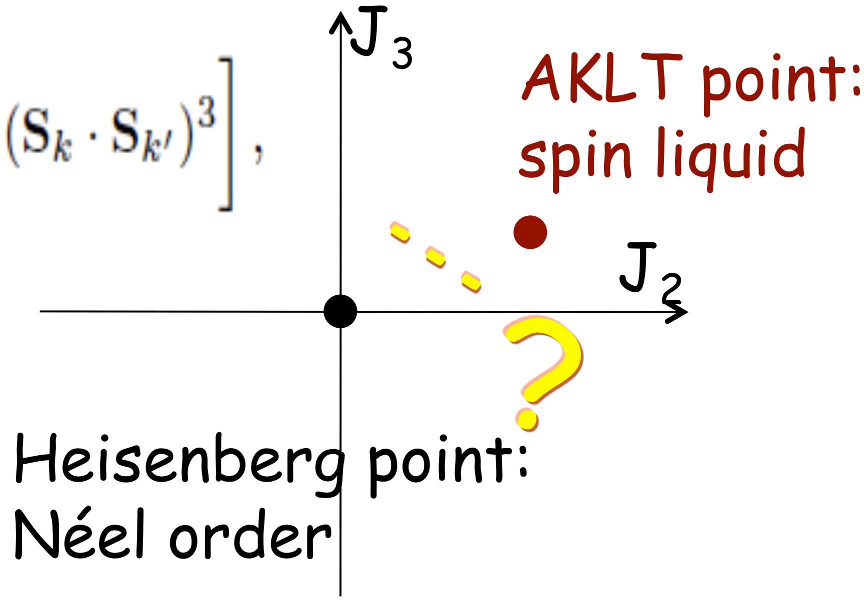
boundary correlation of 1D VBS phase is "renormalized" to that of the AKLT (frustration-free) point



holographic nature

2D counterpart?

$$H = J \sum_{(k,k')}^{\text{n.n.}} \left[\mathbf{S}_k \cdot \mathbf{S}_{k'} + \overset{J_2}{\frac{116}{243}} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^2 + \overset{J_3}{\frac{16}{243}} (\mathbf{S}_k \cdot \mathbf{S}_{k'})^3 \right],$$



2. such a computational capability may persist in an entire phase (valence bond solid phase).

renormalization of many-body correlation (to encode logical information in common low-energy/macroscopic physics)

computational usefulness as the characteristic of a certain phase ("quantum computational phase")

Summary and outlook

1. quantum computational capability is available in a 2D condensed matter system.

   new perspective to an intrinsic complexity of 2D systems

2. such a computational capability may persist in an entire phase.

   possible realization of a quantum computer without much fine engineering of microscopic parameters.

- A. Miyake, quantum computational capability of a two-dimensional valence bond solid phase, arXiv:1009.3491

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