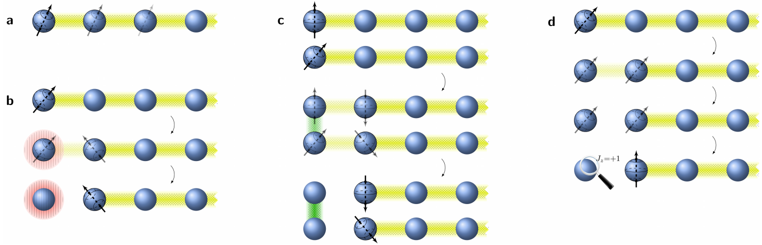
Quantum Phases for Quantum Computation

Renormalization & Symmetry-Protected Topological Order



Joseph M. Renes

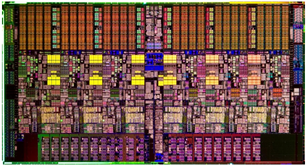
Stephen D. Bartlett, Gavin K. Brennen, and Akimasa Miyake

PRL 105 110502 (2010) & arXiv:11xx.xxxx?

Motivation

State of the Art Computers

Classical



Intel 6 core processor (Gulftown);
$\sim 10^9$ transistors

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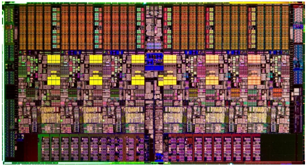

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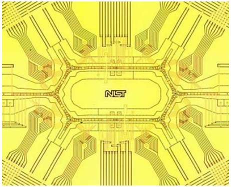
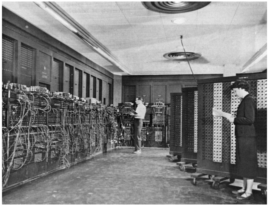


NIST Racetrack ion trap;
$\sim$ 100 ions (?)

Motivation

Trouble: noise and reliability



Motivation

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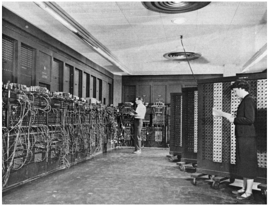
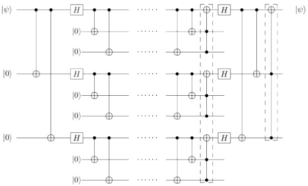
Trouble: noise and reliability



No problem: use error correction!



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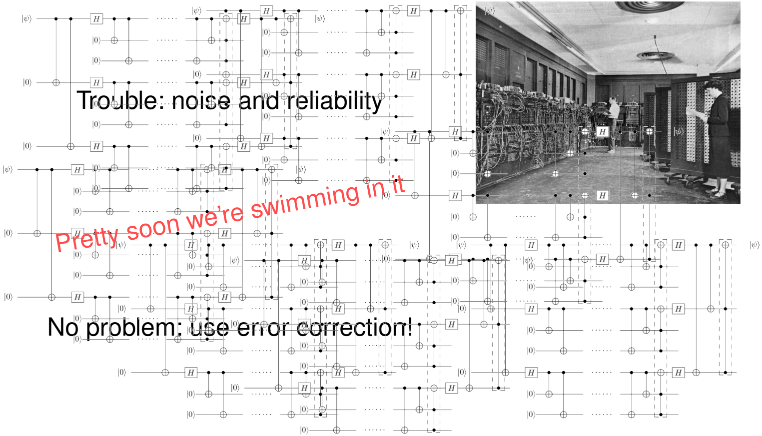

Trouble: noise and reliability

Pretty soon we're swimming in it

No problem: use error correction!

Motivation

Can we solve this problem in hardware?

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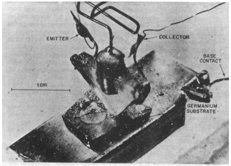

Fig. 1 The first transistor.[1]

Classical solution

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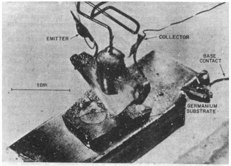

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Classical solution

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Quantum solution

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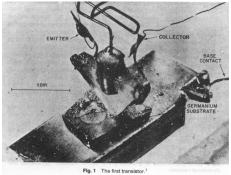
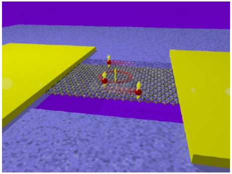


Classical solution



Quantum solution

Perhaps some exotic quantum phase of matter? Anyons?

(graphene FQHE, Andrei group Rutgers)

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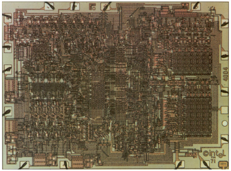
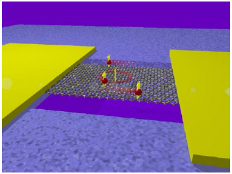


Ambitious

(Intel 4004, 1972)



Quantum solution

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Is there something a little easier to build?

Motivation

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Yes!

The Haldane phase of spin-1 chains offers several interesting ideas:

- MBQC renormalization
- Holonomic QC from symmetry-protected topological order

Quantum computational renormalization in the Haldane phase

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First, the short version

- Can define MBQC model at the AKLT point, in the Haldane phase
- Gate fidelities decay as we move away from AKLT
- But there's an RG flow towards AKLT, so just measure the block spins!

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- Gate fidelities decay as we move away from AKLT
- But there's an RG flow towards AKLT, so just measure the block spins!
- That would require multispin measurements, so you could do QC anyway
- Simulate block measurements with single-site measurement & postselection!
- QC ability is a property of the phase, in this sense

AKLT spin-chain

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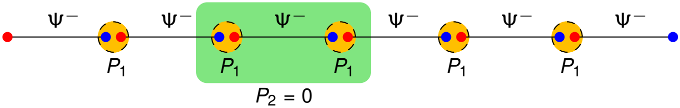
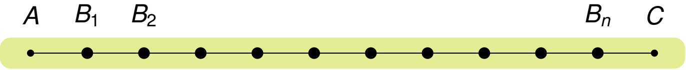
Affleck-Kennedy-Lieb-Tasaki nearest-neighbor Hamiltonian

$$H_{\mathrm{AKLT}} = \sum_{j=1}^{n} (\vec{S}_j \cdot \vec{S}_{j+1}) + \tfrac{1}{3}(\vec{S}_j \cdot \vec{S}_{j+1})^2 \simeq \sum_{j=1}^{n} (P_2)_{j,j+1}$$



- Ground state is unique periodic BCs or $n \to \infty$; 4fold degenerate open BCs and $n < \infty$
- Gap to first excited state (conjectured by Haldane, analytic example by AKLT)
- Ground state is a "valence bond solid" (VBS), frustration-free

MBQC with AKLT



$$|\mathcal{G}_0\rangle = \sum_{\{s_k\}} |s_1, s_2, \ldots, s_n\rangle_B \otimes \left(\sigma_{s_n}\sigma_{s_{n-1}} \cdots \sigma_{s_1}\right)_C |\Psi^-\rangle_{AC}$$

- Chain encodes one logical qubit (think of it at C); $|s\rangle \equiv |J_s = 0\rangle$ & $s = \hat{x}, \hat{y}, \hat{z}$.

MBQC with AKLT

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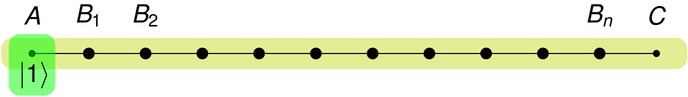

$$|\mathcal{G}_1\rangle = |1\rangle_A \otimes \sum_{\{s_k\}} |s_1, s_2, \dots, s_n\rangle_B \otimes \sigma_{s_n}\sigma_{s_{n-1}} \cdots \sigma_{s_1} |0\rangle_C$$

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- Initialize: Measure $|0\rangle$, $|1\rangle$ on end qubit A

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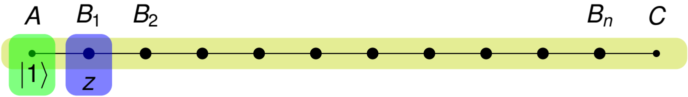

$$|\mathcal{G}_2\rangle = |1\rangle_A \otimes |\hat{z}\rangle_{B_1} \otimes \sum_{s_k} |s_2, \dots, s_n\rangle_B \otimes \sigma_{s_n}\sigma_{s_{n-1}} \cdots \sigma_{s_2}\sigma_z|0\rangle_C$$

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MBQC with AKLT

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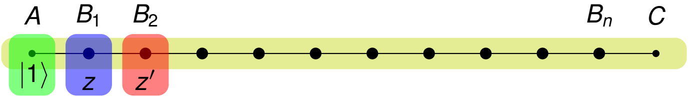

$$|\mathcal{G}_3\rangle = |1\rangle_A \otimes |\hat{z}\rangle_{B_1} |\hat{z}'\rangle_{B_2} \otimes \sum_{s_k} |s_3, \ldots, s_n\rangle_B \otimes \sigma_{s_n}\sigma_{s_{n-1}} \cdots \sigma_{s_3}\sigma_{\hat{z}'}\sigma_{\hat{z}}|0\rangle_C$$

- Chain encodes one logical qubit (think of it at C); $|s\rangle \equiv |J_s = 0\rangle$ & $s = \hat{x}, \hat{y}, \hat{z}$.
- Initialize: Measure $|0\rangle$, $|1\rangle$ on end qubit A
- Measuring in the $|s\rangle$ basis rotates the qubit by π around s
- Works for rotated basis $|s'\rangle$, too, by spherical symmetry
- Combine measurement in different bases to perform arbitrary rotations
- Compound rotations are probabilistic, but heralded

Haldane Phase Renormalization

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- Gate fidelity decreases when using non-AKLT ground states
- What to do? Renormalize!

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Renormalization recipe:

1. From three adjacent spins, extract the $J = 1$ components
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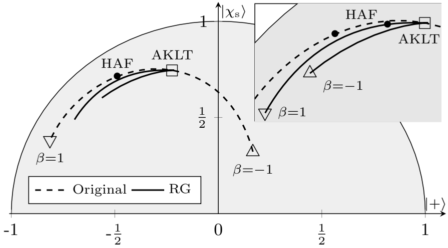

MBQC Renormalization

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Suppose we want to do a π rotation around $\hat{x}\cos\theta + \hat{y}\sin\theta$.

$$\left|z, \theta, z\right\rangle_{123} \propto \left|\theta\right\rangle_J \left|\chi_s\right\rangle_L + J \neq 1 \text{ component,}$$
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Buffered measurement effectively replicates the block spin measurement

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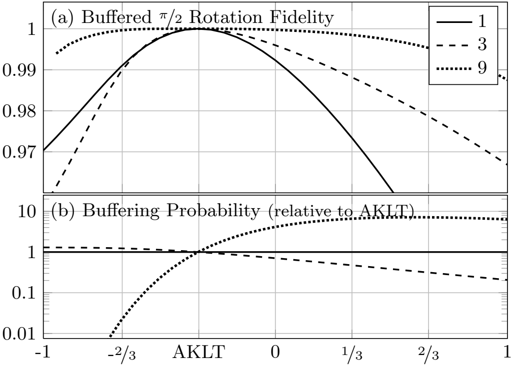
Buffering Works

But needs a lot of postselection



Holonomic quantum computation from symmetry-protected topological order

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First, the short version

- Haldane phase possesses SPTO
- Symmetries of SPTO also define qubit encoding, gates
- Architecture inherits some protection from SPTO

SPTO of 1D systems

- Topological order doesn't exist for 1D systems. All states are $\sim$ product states
- But in the presence of certain symmetries, distinct phases appear
- For spin-1 chains $\Rightarrow$ Haldane phase

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  - time-reversal
  - bond inversion

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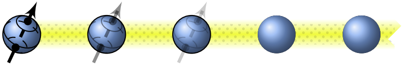
- For spin-1 chains $\Rightarrow$ Haldane phase
- What symmetries?
  - π rotations about orthogonal axes (D_2)
  - time-reversal
  - bond inversion
- What properties?
  - gapped ground state, fourfold degenerate
  - fractionalized spin-$\frac{1}{2}$ edge modes
  - nearest-neighbor, two-body couplings $H_0 = \sum h_{j,j+1}$



Holonomic Quantum Computing with SPTO

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D_2 symmetry doesn't just define the phase, also encoded qubits & gates

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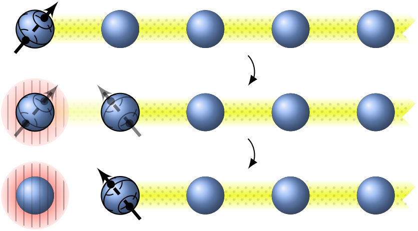
$$H(t) = t\left[(S_1^z)^2 - h_{12}\right] + H_0$$



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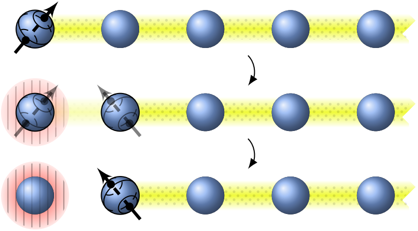
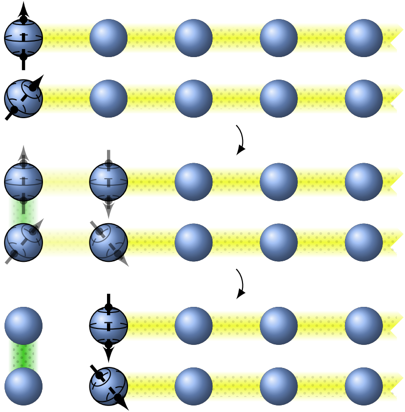


note D_2 symmetry

Two-qubit gate: CPHASE + $\hat{x}$ π rotation

$$H(t) = t\left[W^{AB} - h^A_{12} - h^B_{12}\right] + H^A_0 + H^B_0$$

$$W = \left[(S_1^{\hat{x}})^2 - (S_1^{\hat{y}})^2\right] \otimes S_1^{\hat{z}} + S_1^{\hat{z}} \otimes \left[(S_1^{\hat{x}})^2 - (S_1^{\hat{y}})^2\right]$$



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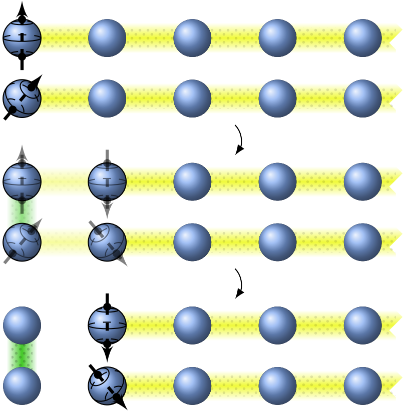
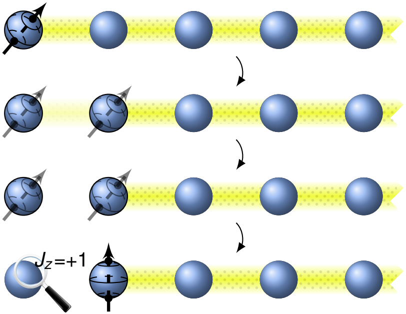
not D_2 symmetric, but doesn't close the gap



Measurement

Turn off coupling, measure J_z

- $+1 \rightarrow |\uparrow\rangle$
- $-1 \rightarrow |\downarrow\rangle$
- $0 \rightarrow R_{\hat{z}}(\pi)$



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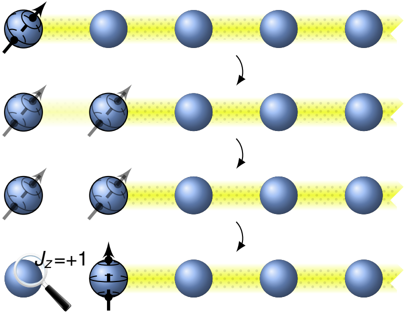
Need full $SO(3)$ symmetry!



Advantages

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- Just operate on the boundary spin (don't consume spins, as in MBQC)
- Only 2-body interactions
- Don't need terribly long chains: edge modes well-localized
- Don't even need chains at all: can terminate with spin-1/2s!
  Or convert everything to spin-1/2.
- Robust to symmetry-preserving disorder in the couplings:
  Only care about total angular momentum
- Gates "immune" to timing errors, intensity fluctuations
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- Indications of *limited* protection against *local* noise @ *low* temperatures
  - Rotating bulk spins doesn't affect the logical state
  - Bigger rotations cost more energy; remove via cooling
  - Rotating boundary spin does affect the logical state
  - Error rates should be suppressed