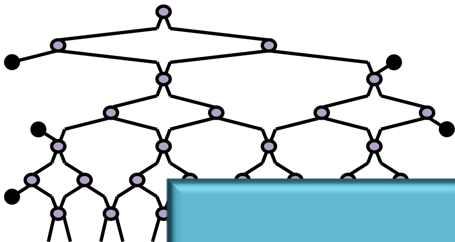
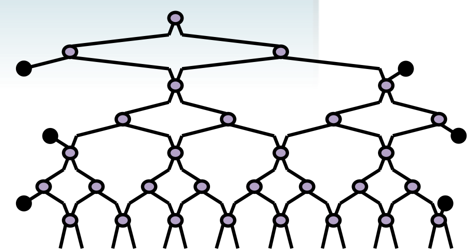
Coogee January 2011

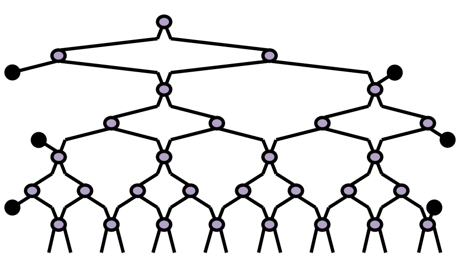


A class of entangling quantum circuits that can be efficiently simulated

Guifre Vidal

collaboration with
Glen Evenbly





THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

Outline

Glen Evenbly

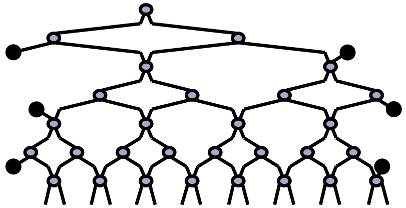
- Introduction

  Quantum circuits, simulatability and entanglement

- MPS and TTN

- MERA

- branching MERA



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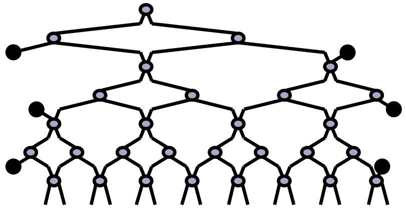
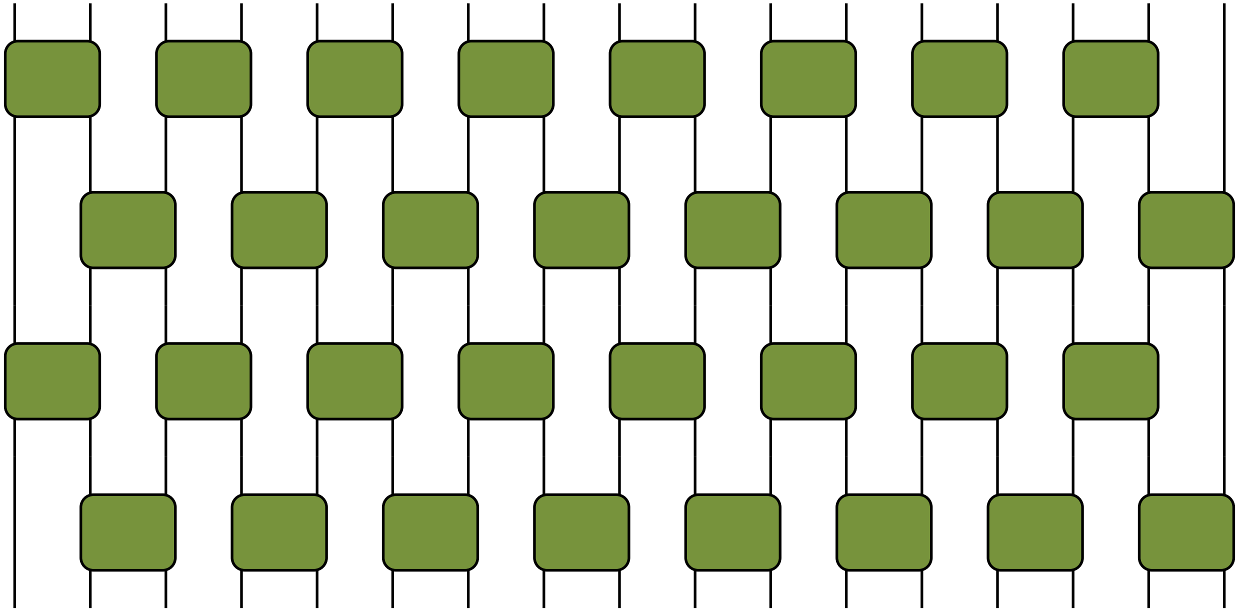
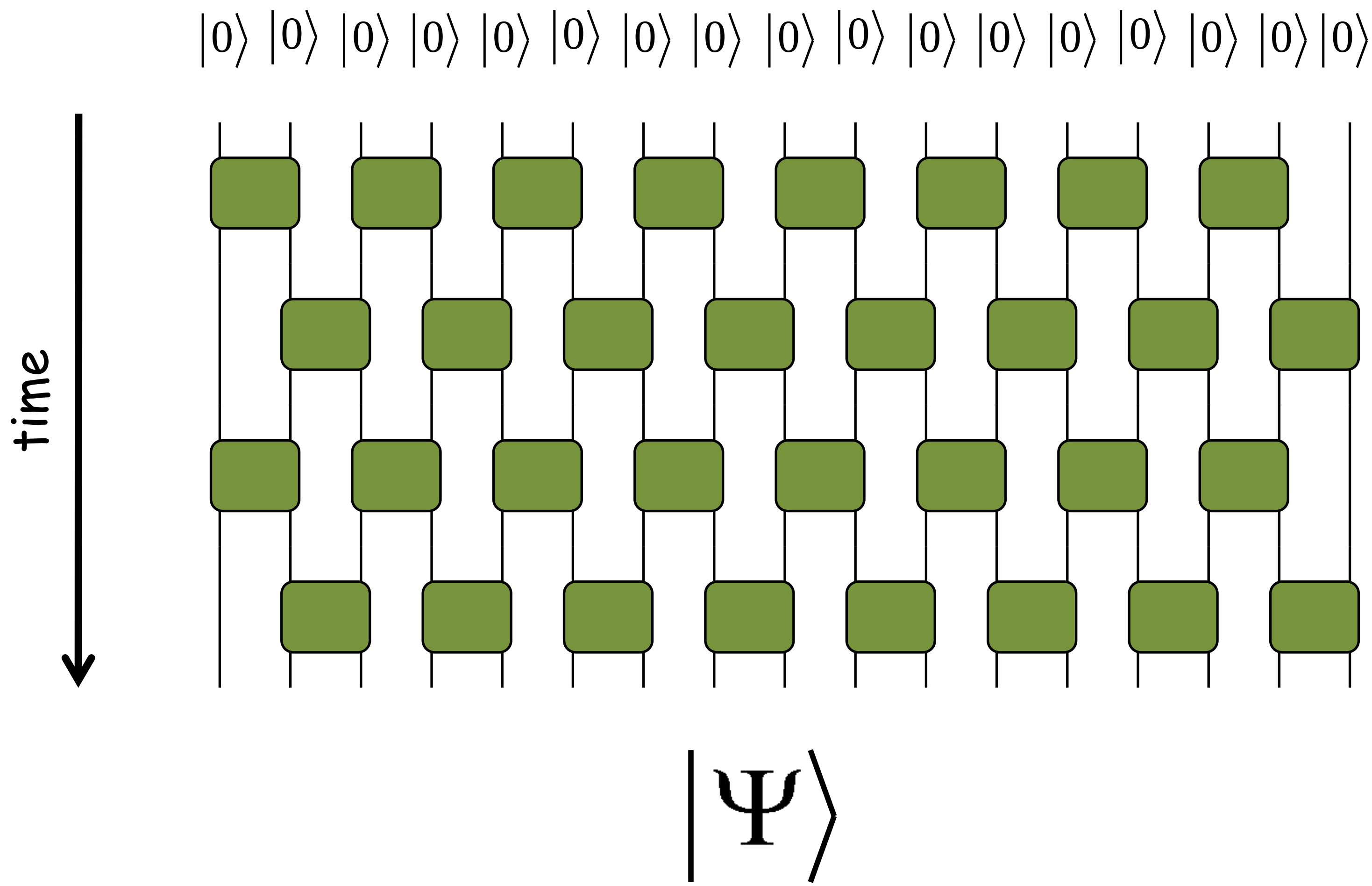


Quantum Circuit



Quantum Circuit

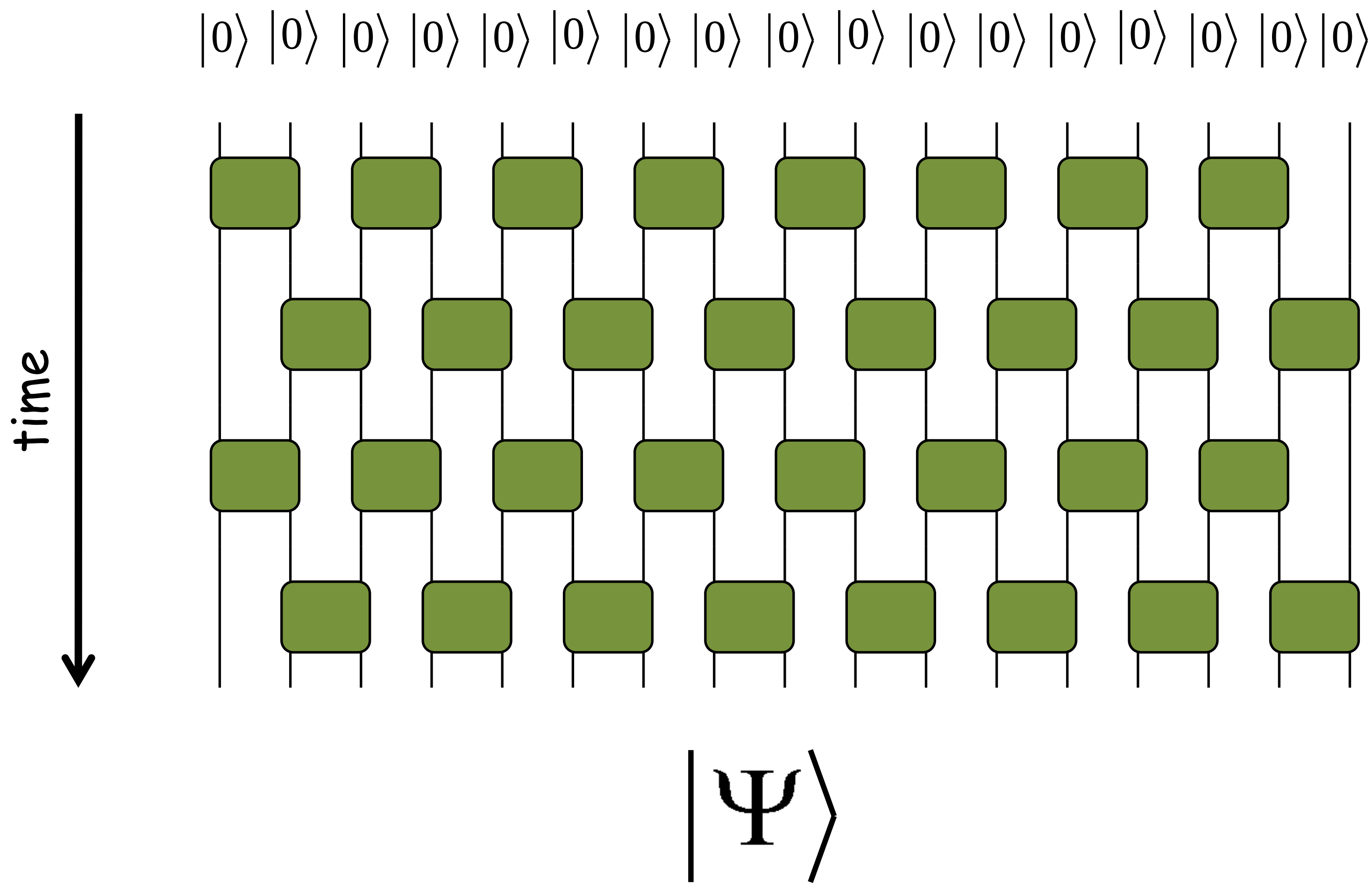
Can be used to *efficiently* encode many-body states:

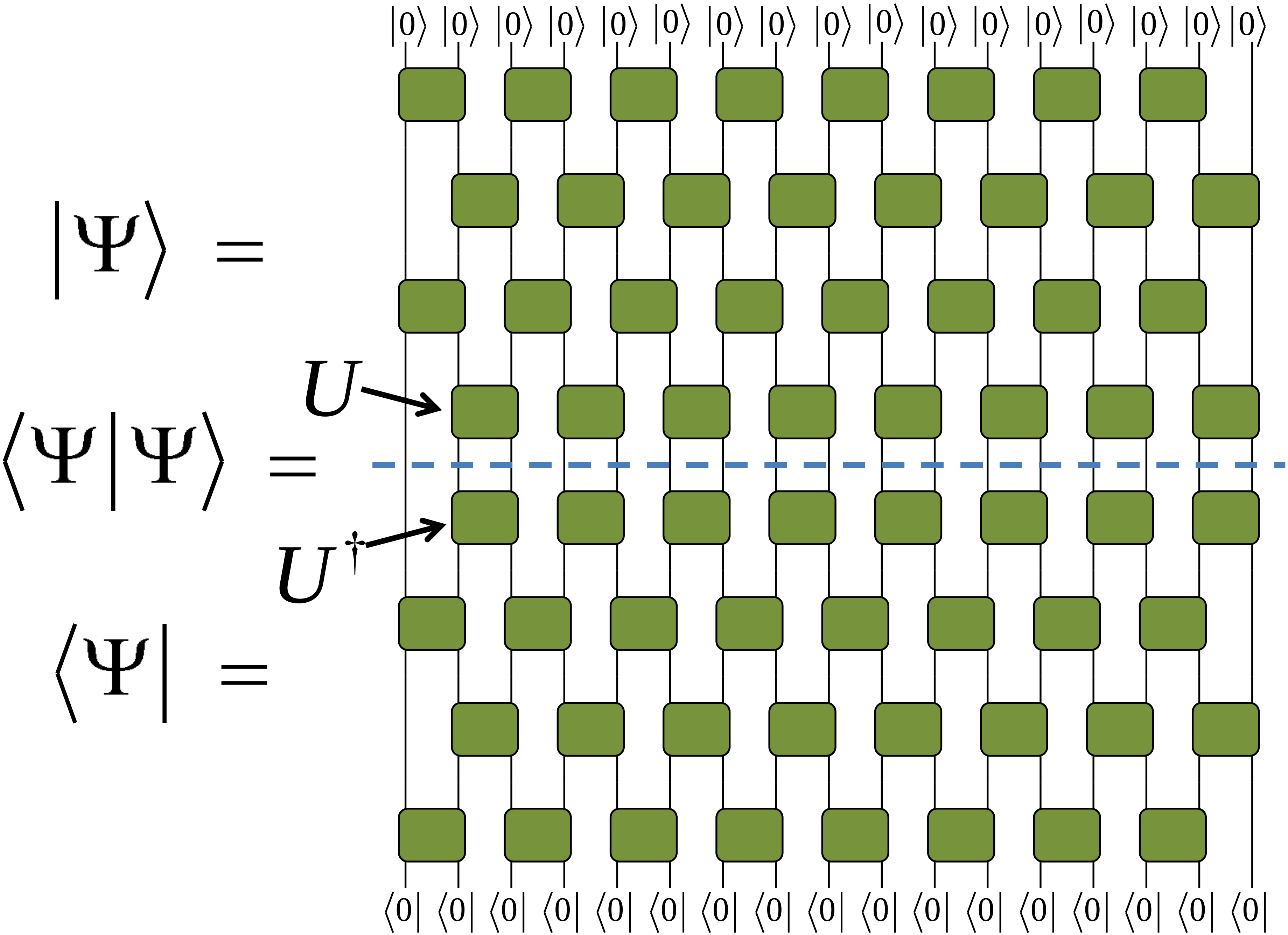


Quantum Circuit as a many-body variational ansatz

Questions:

1) Cost of computing a local reduced density matrix
2) Entropy of a block of contiguous sites





$|\Psi\rangle =$
$\langle\Psi|\Psi\rangle =$
$\langle\Psi| =$
U
$U^\dagger$
$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$
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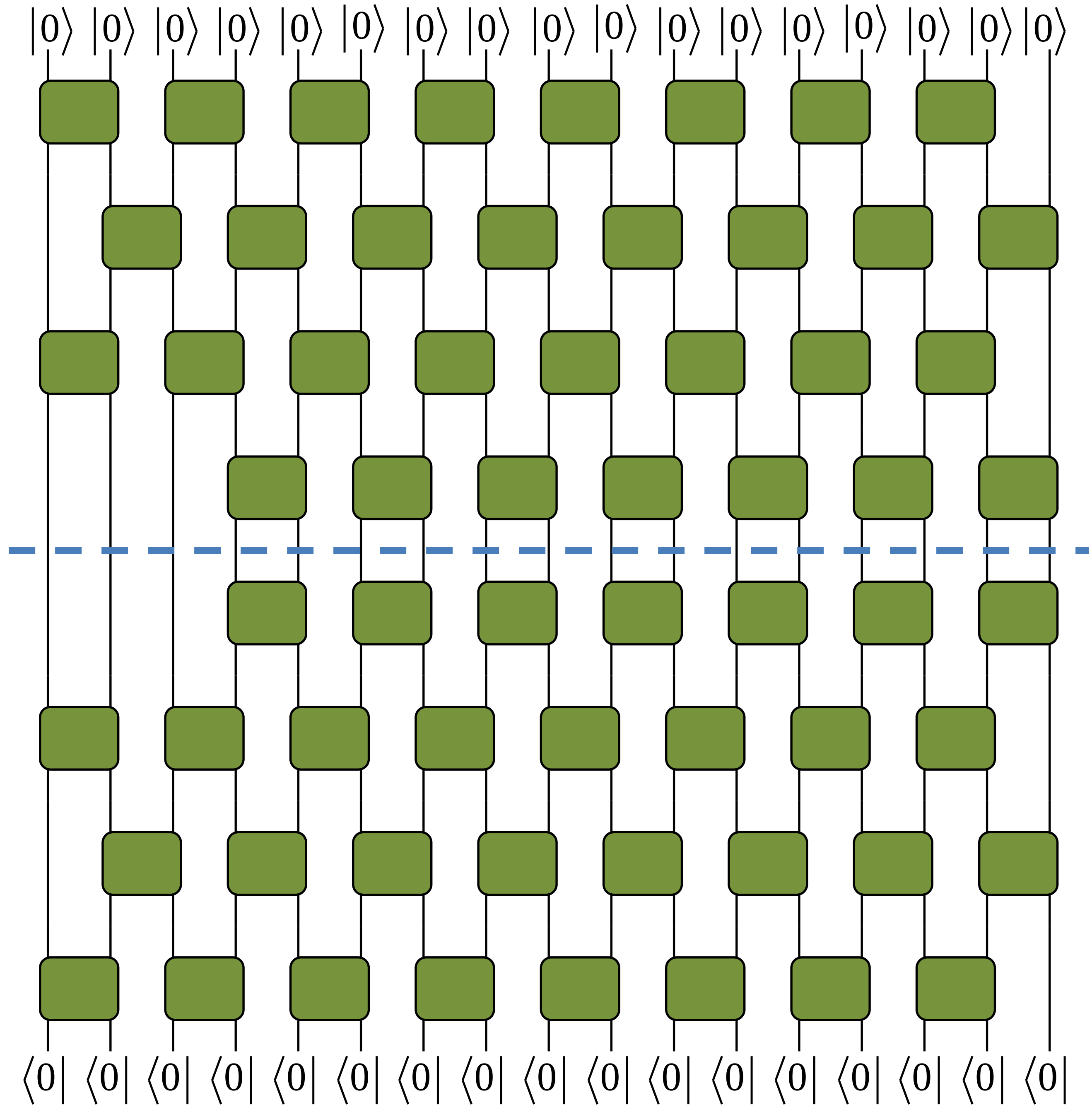
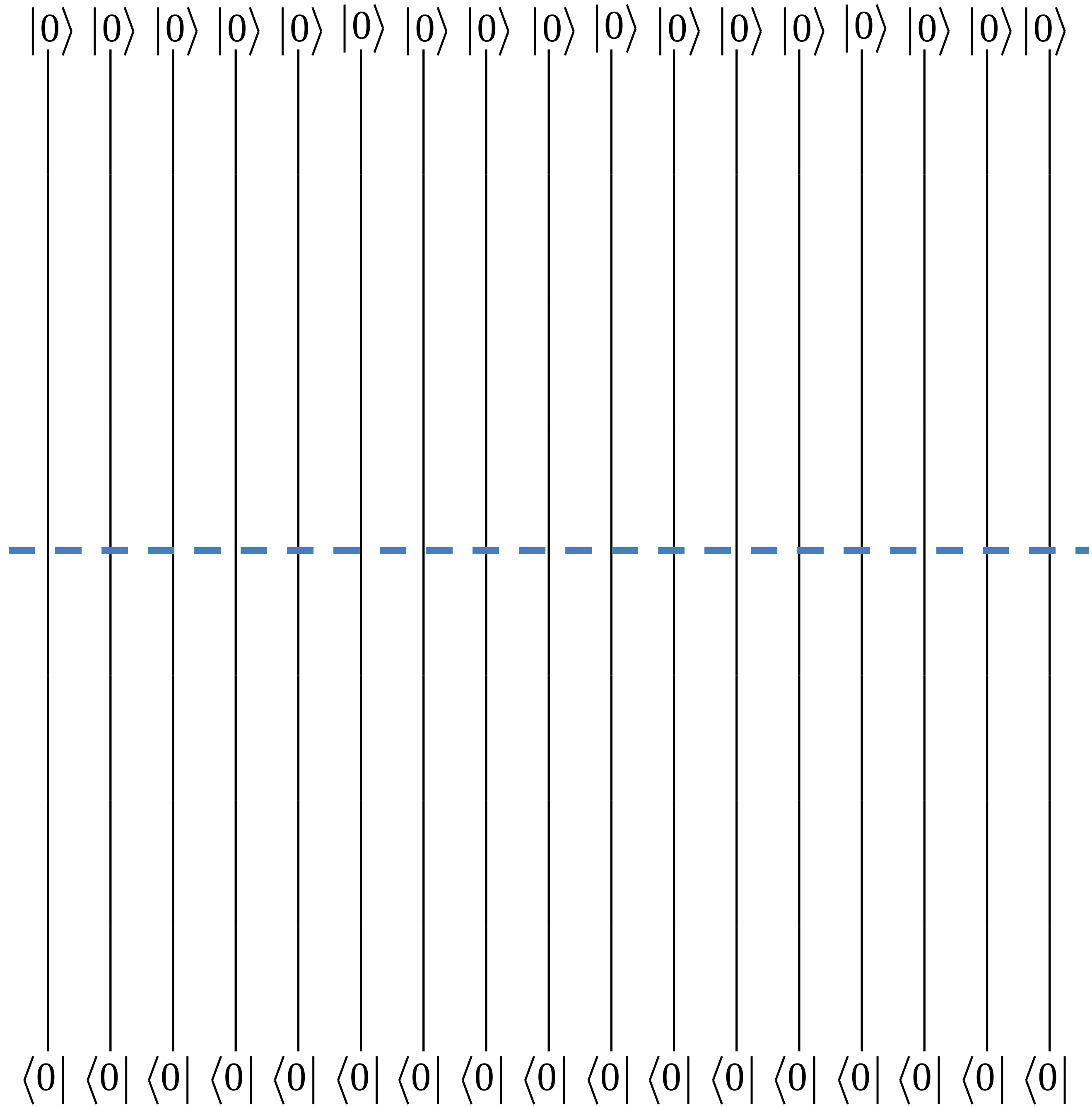
$$\langle \Psi | \Psi \rangle =$$

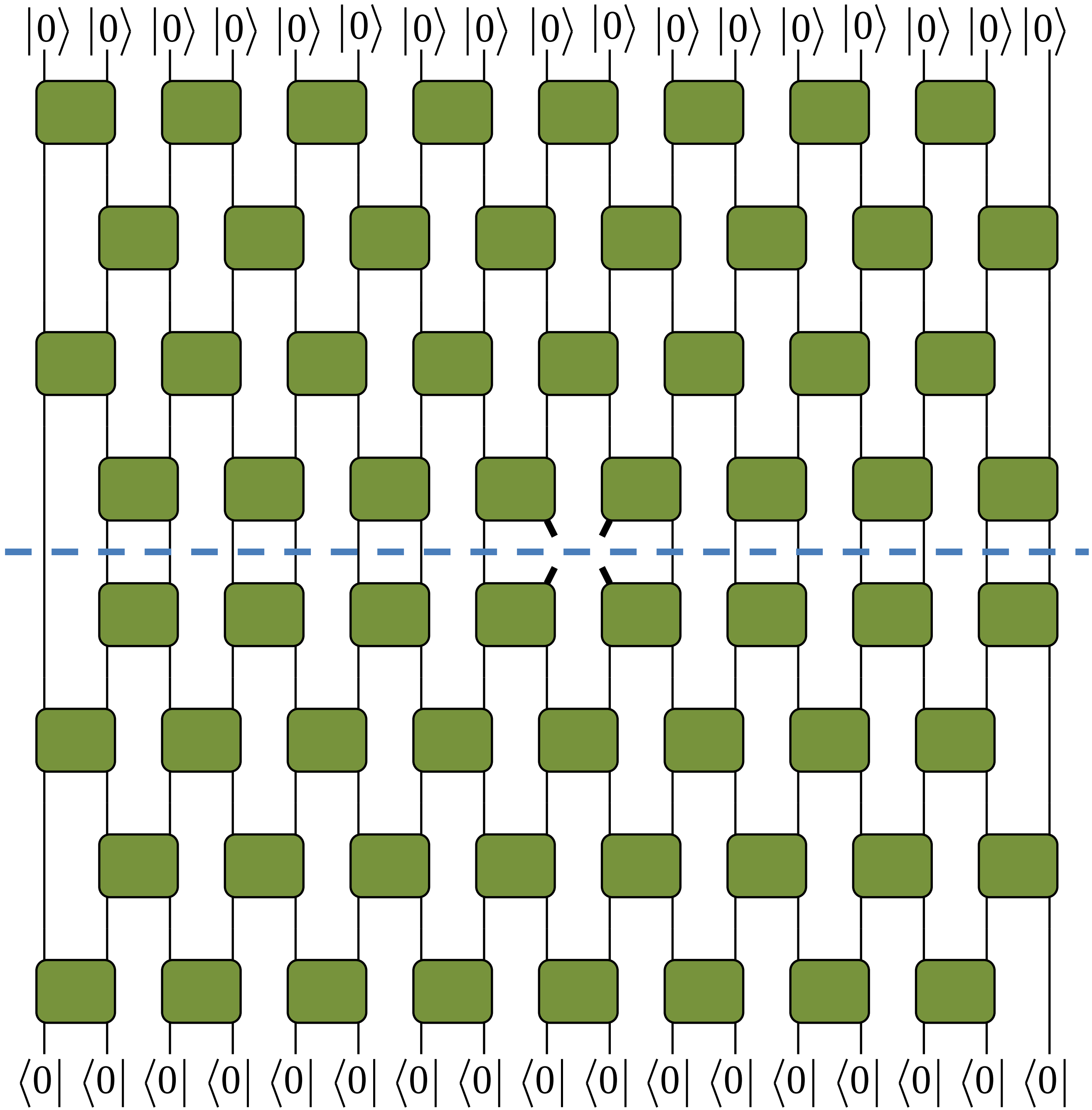


$$\langle \Psi | \Psi \rangle =$$



Cost of computing a local reduced density matrix

$$\rho(A) =$$



Cost of computing a local reduced density matrix

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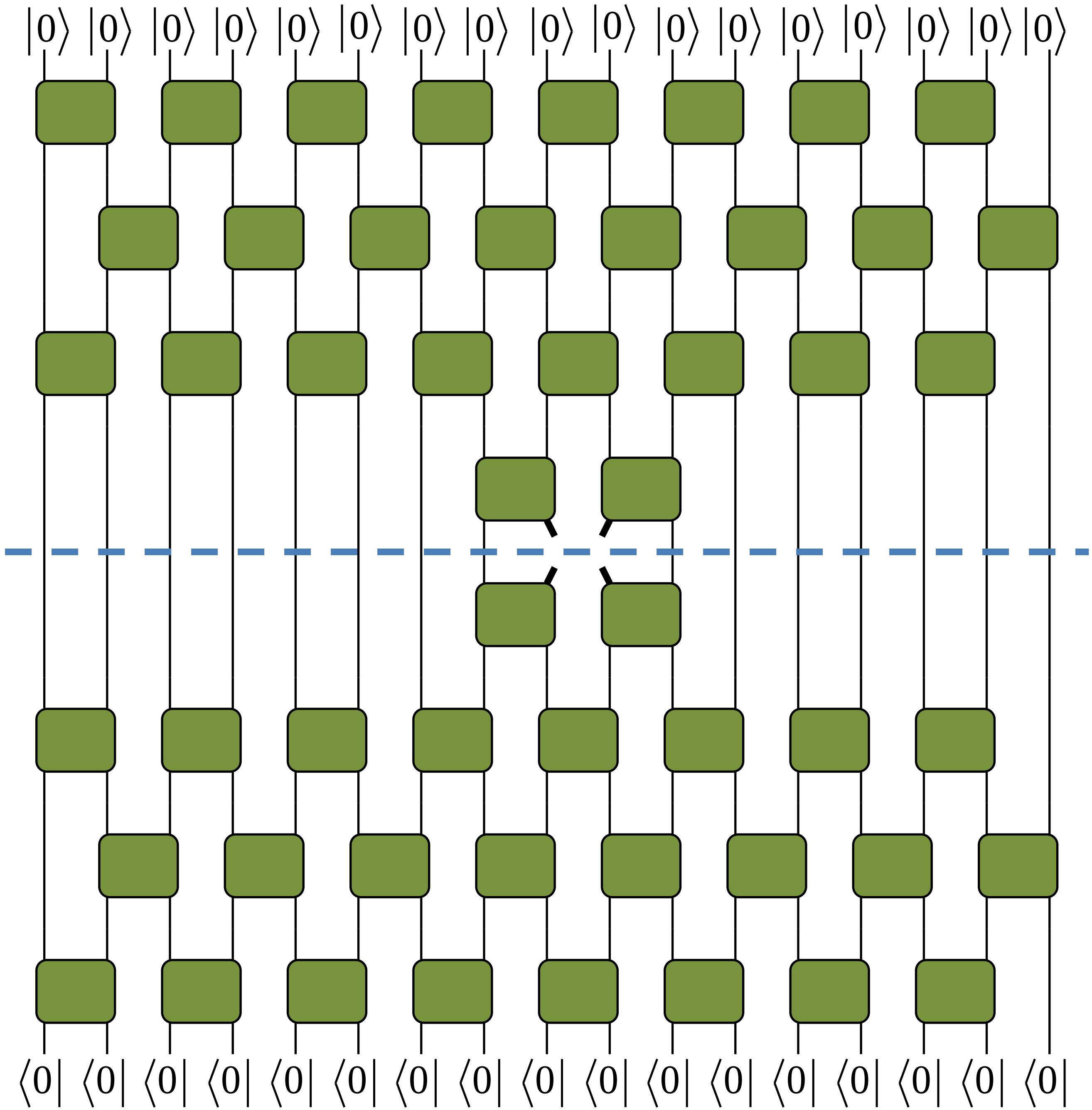

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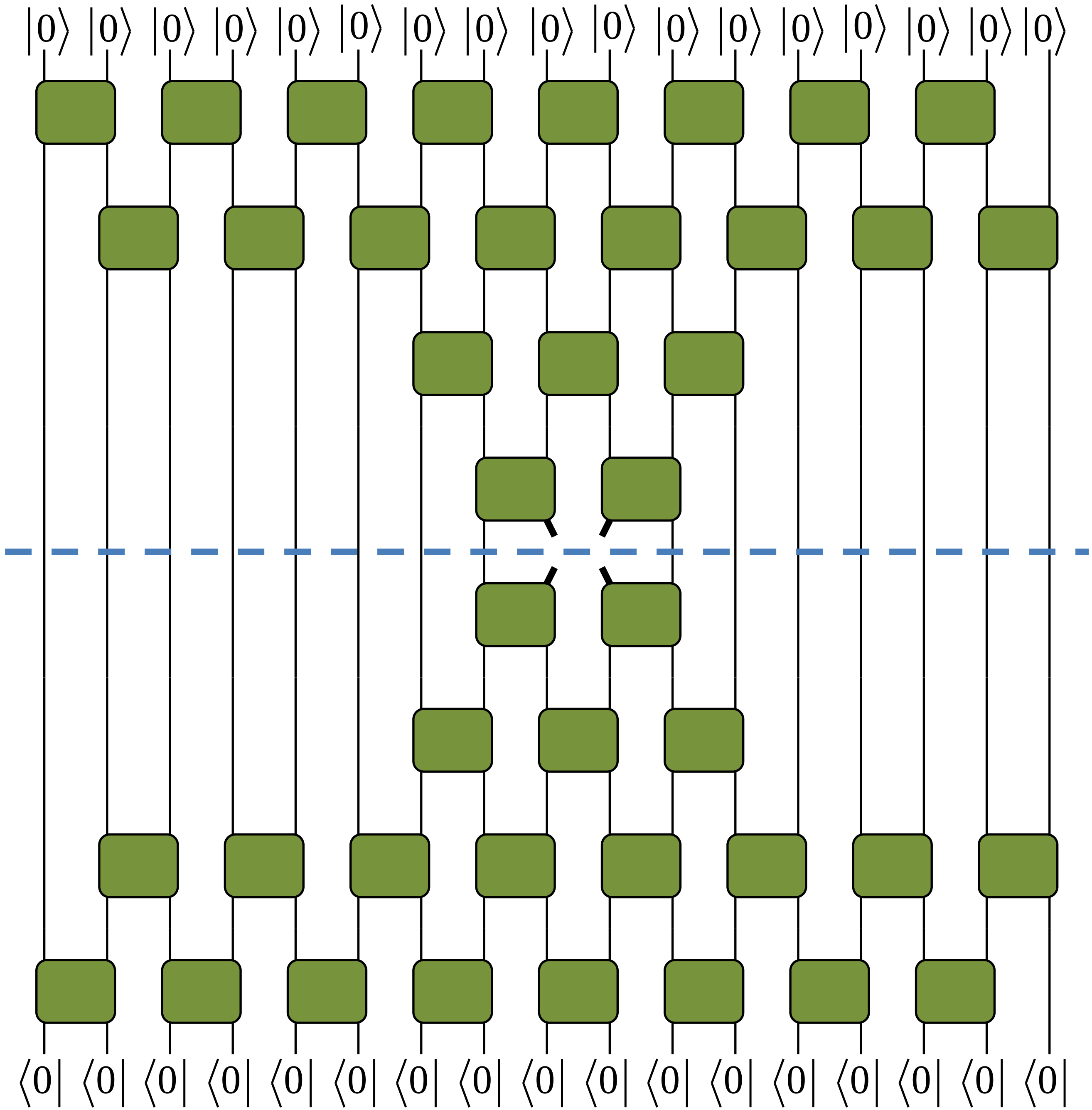

Cost of computing a local reduced density matrix

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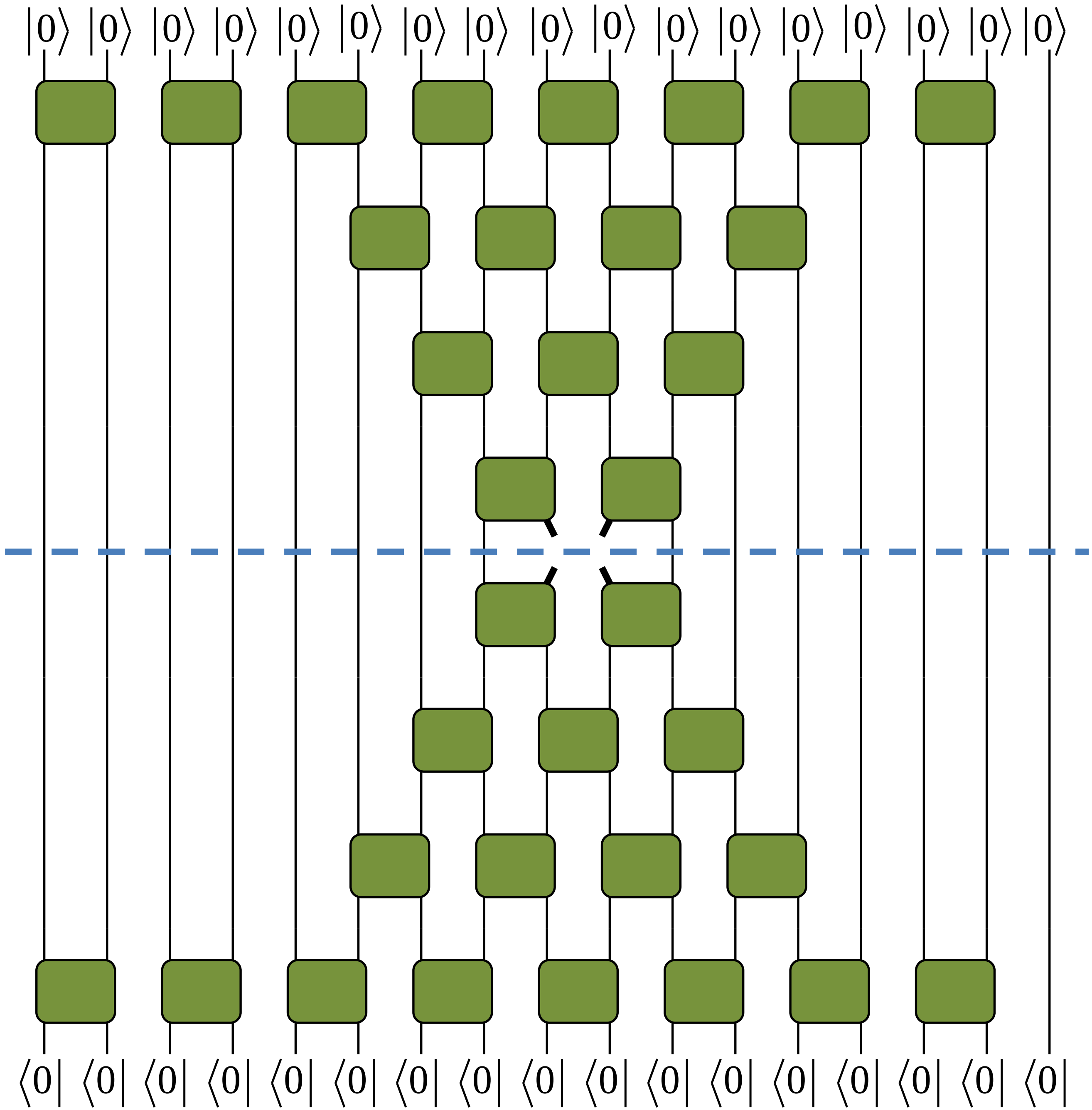
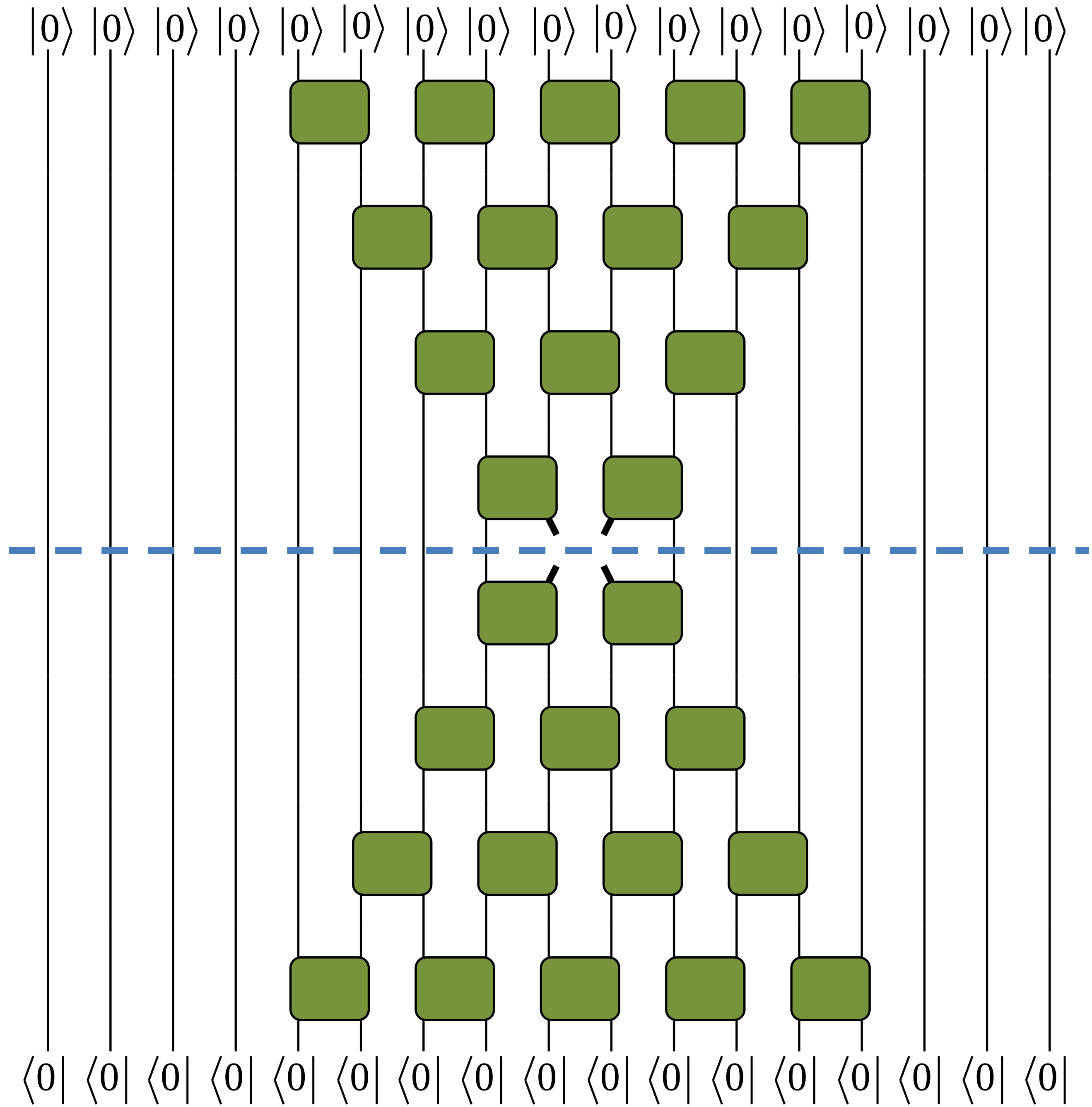


Cost of computing a local reduced density matrix

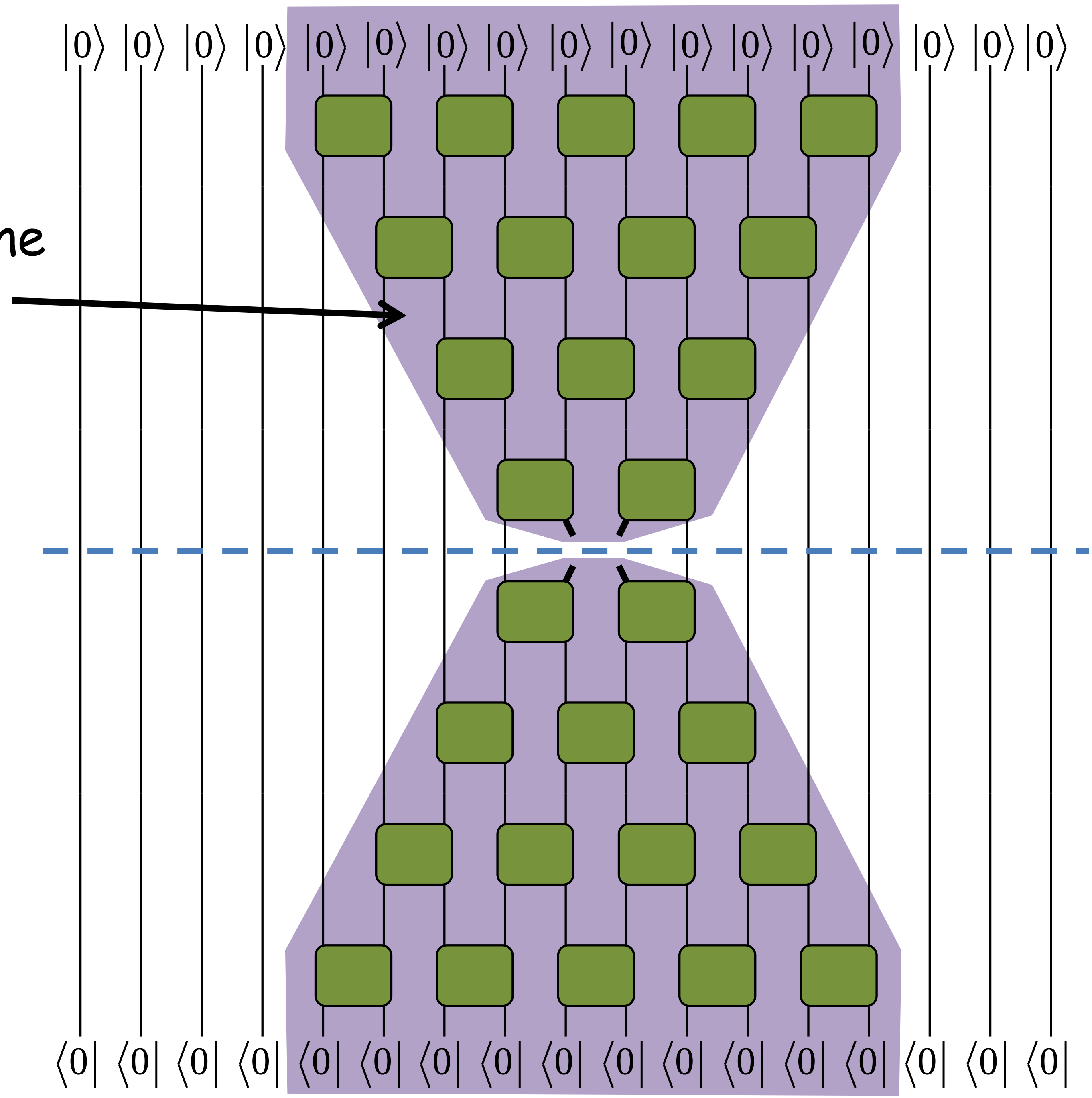
$$\rho(A) =$$

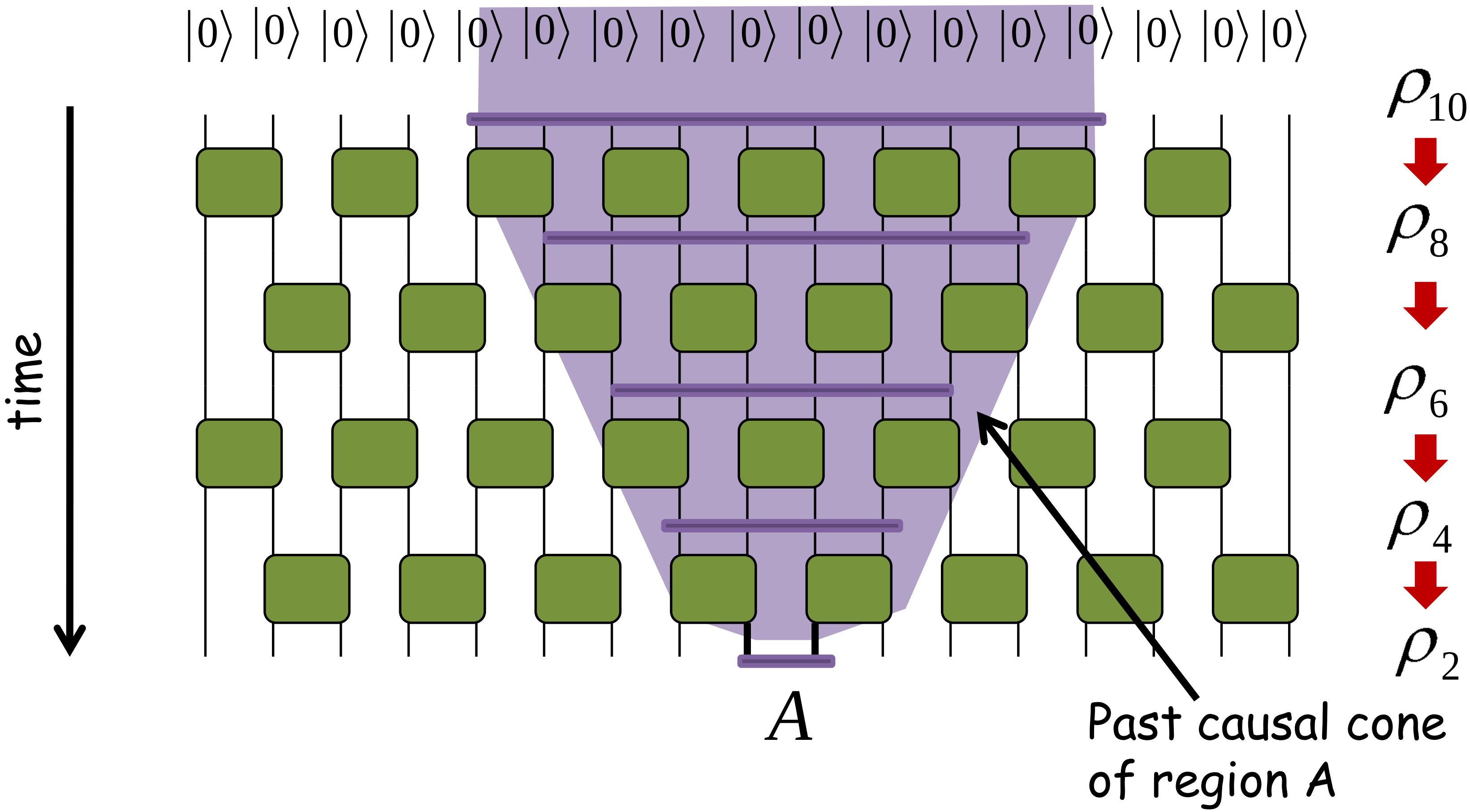


Cost of computing a local reduced density matrix

Past causal cone of region A

$$\rho(A) =$$

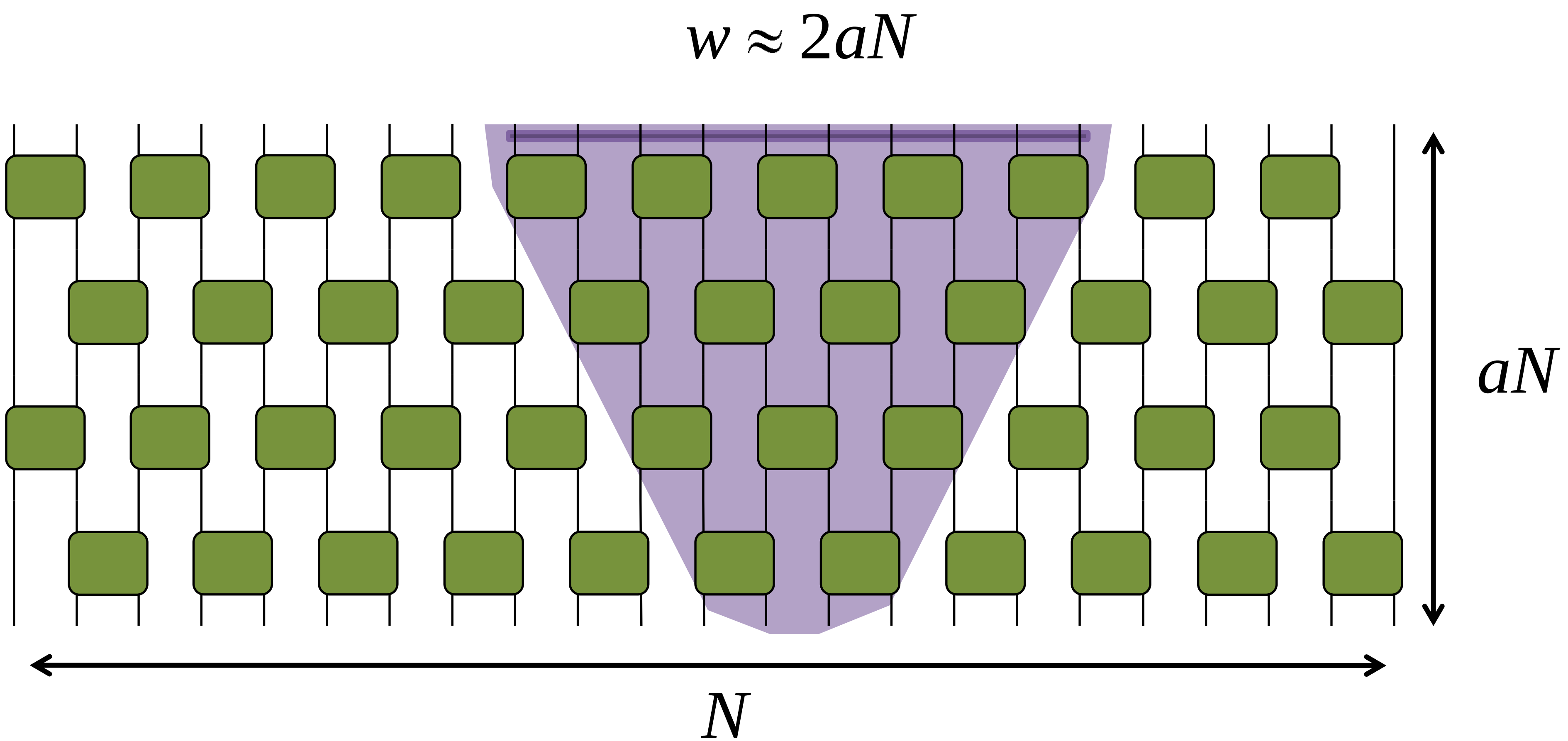




width of causal cone: $w(t)$ $\qquad w \equiv \max_t w(t)$

cost of computing $\rho(A)$: $c \approx \exp(w)$

Example I:

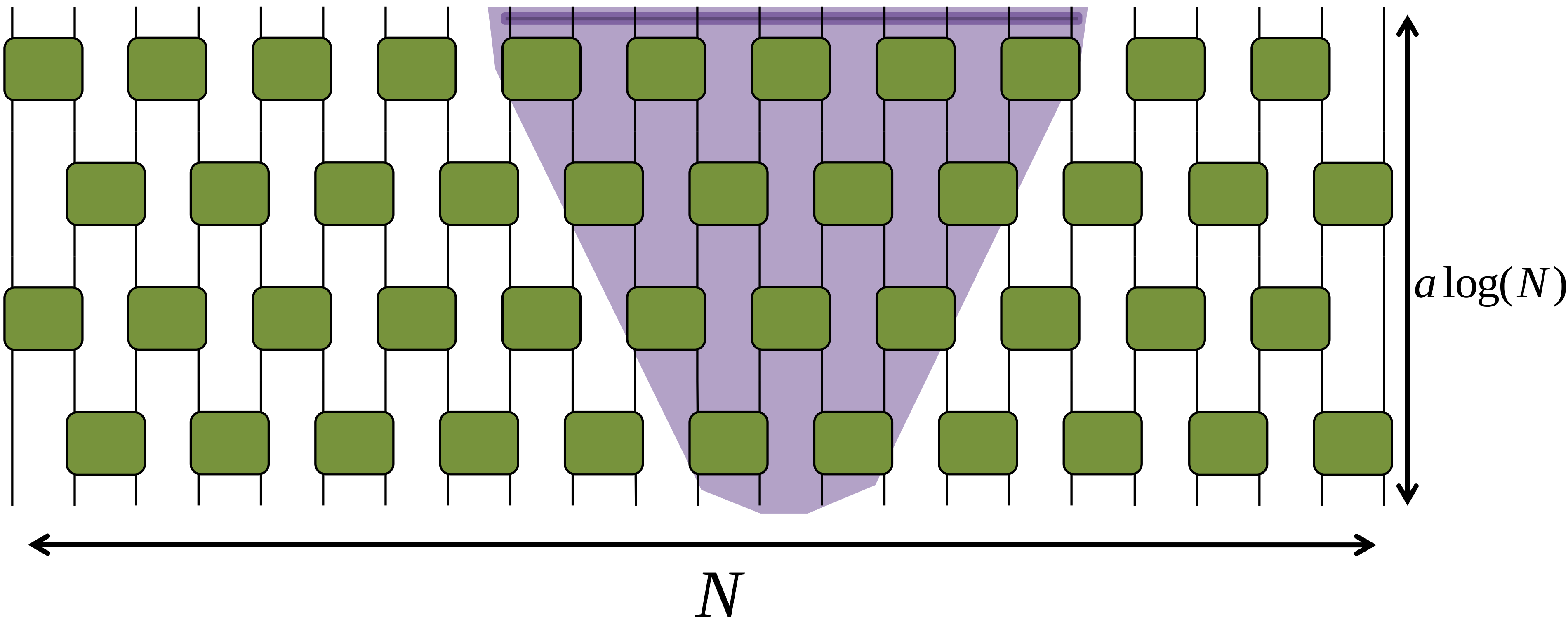


cost of computing $\rho(A)$: $c \approx \exp(2aN)$

inefficient

Example II:

$$w \approx 2a \log(N)$$

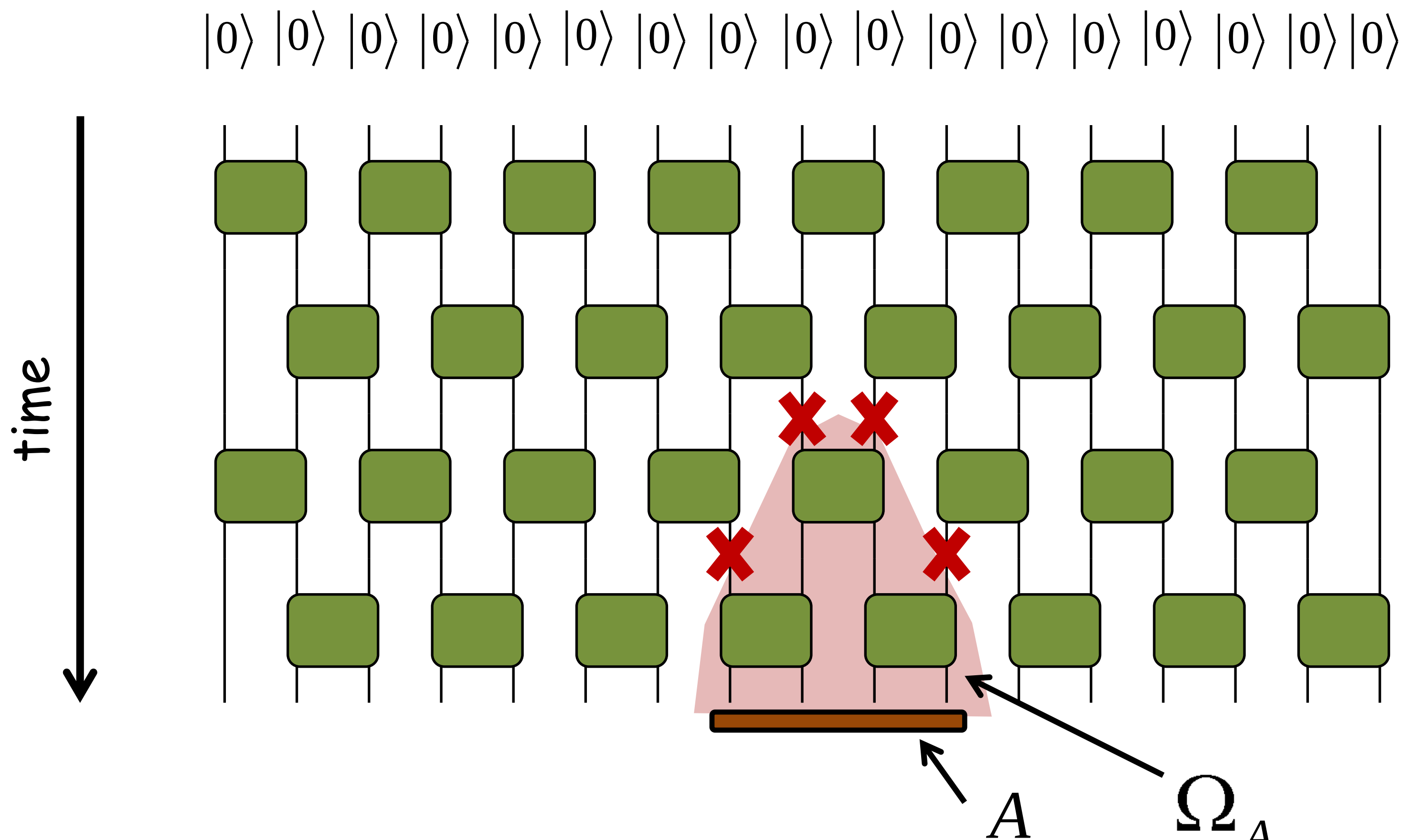


cost of computing $\rho(A)$: $c \approx \exp(2a\log(N)) \approx N^{2a}$

efficient

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



Upper bound on entanglement entropy

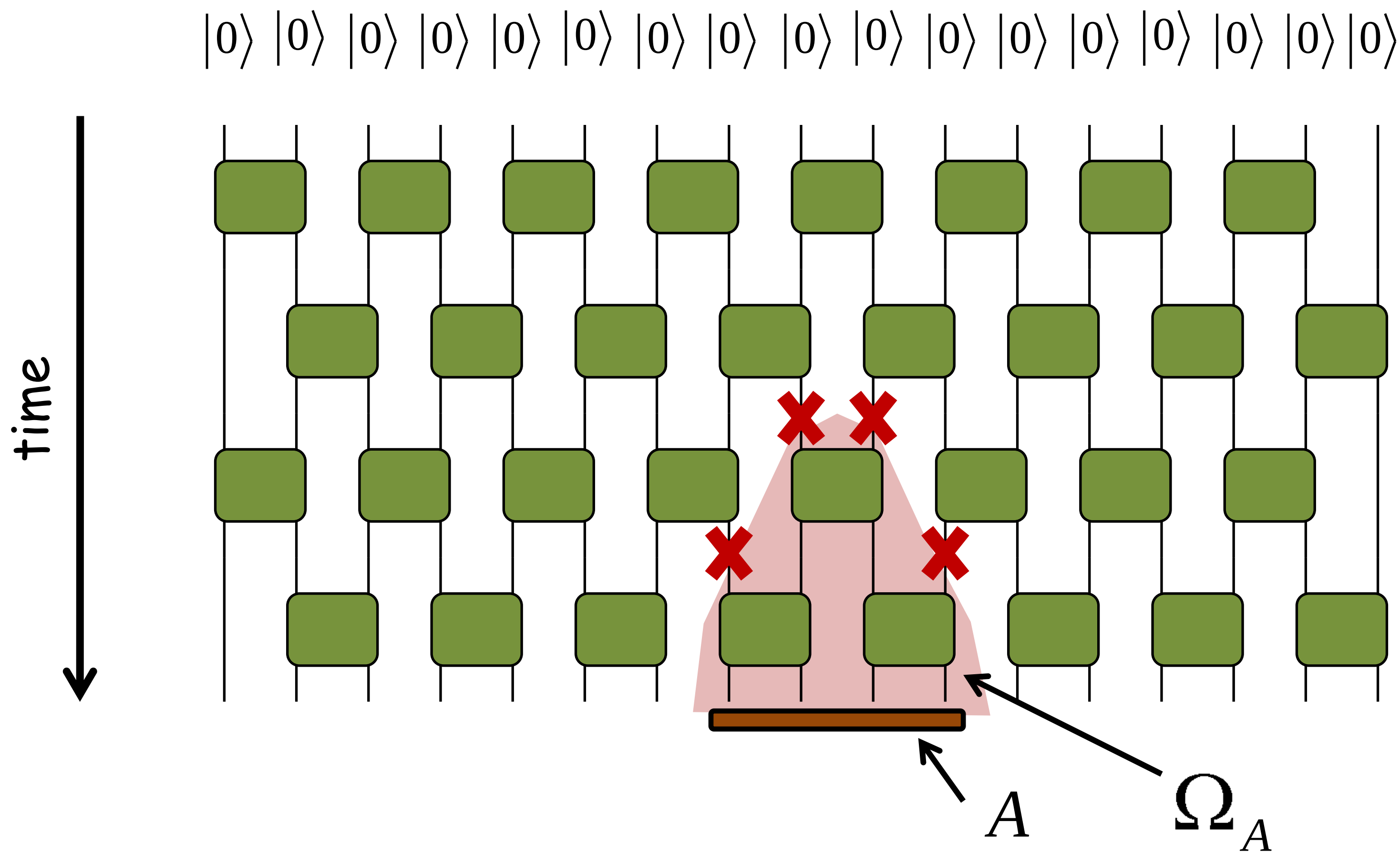
minimal connectivity of region A

# of bond indices

$$n(A) = 4$$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



Upper bound on entanglement entropy

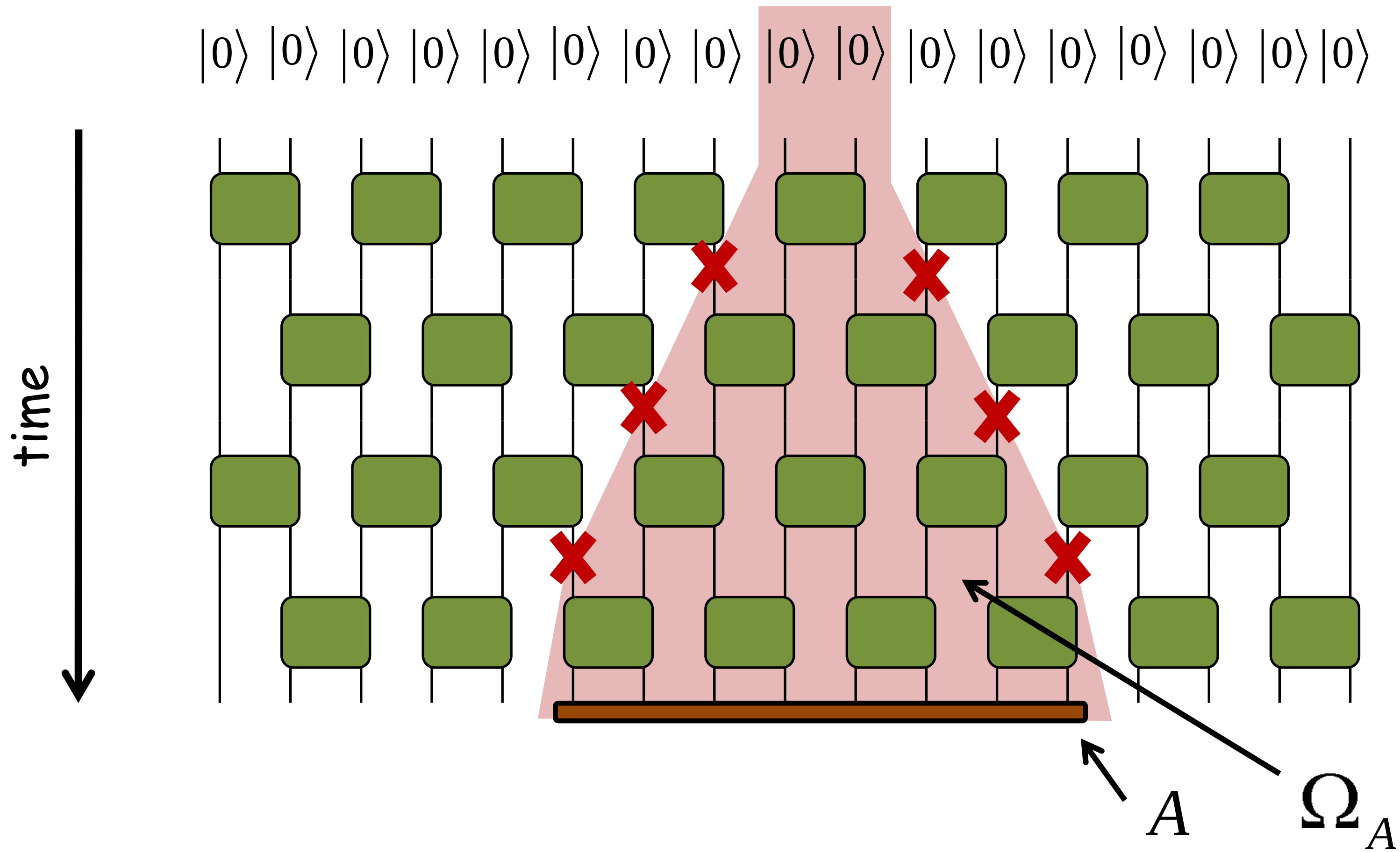
$$S(A) \leq \gamma n(A)$$

# of bond indices

$$n(A) = 4$$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites



Upper bound on entanglement entropy

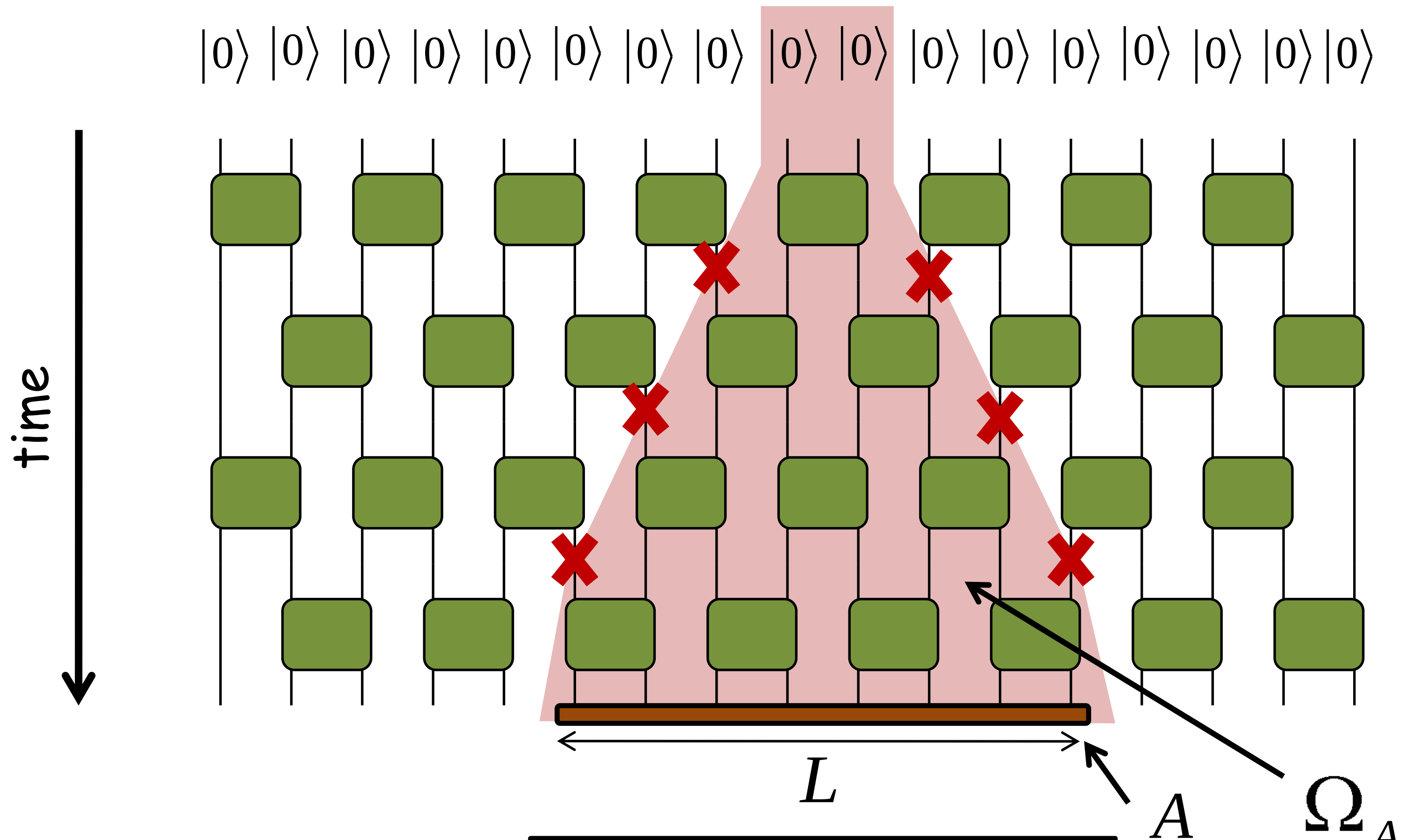
$$S(A) \leq \gamma n(A)$$

# of bond indices

$n(A) = 6$

How entangled is the resulting state?

- Entanglement entropy of a block of contiguous sites

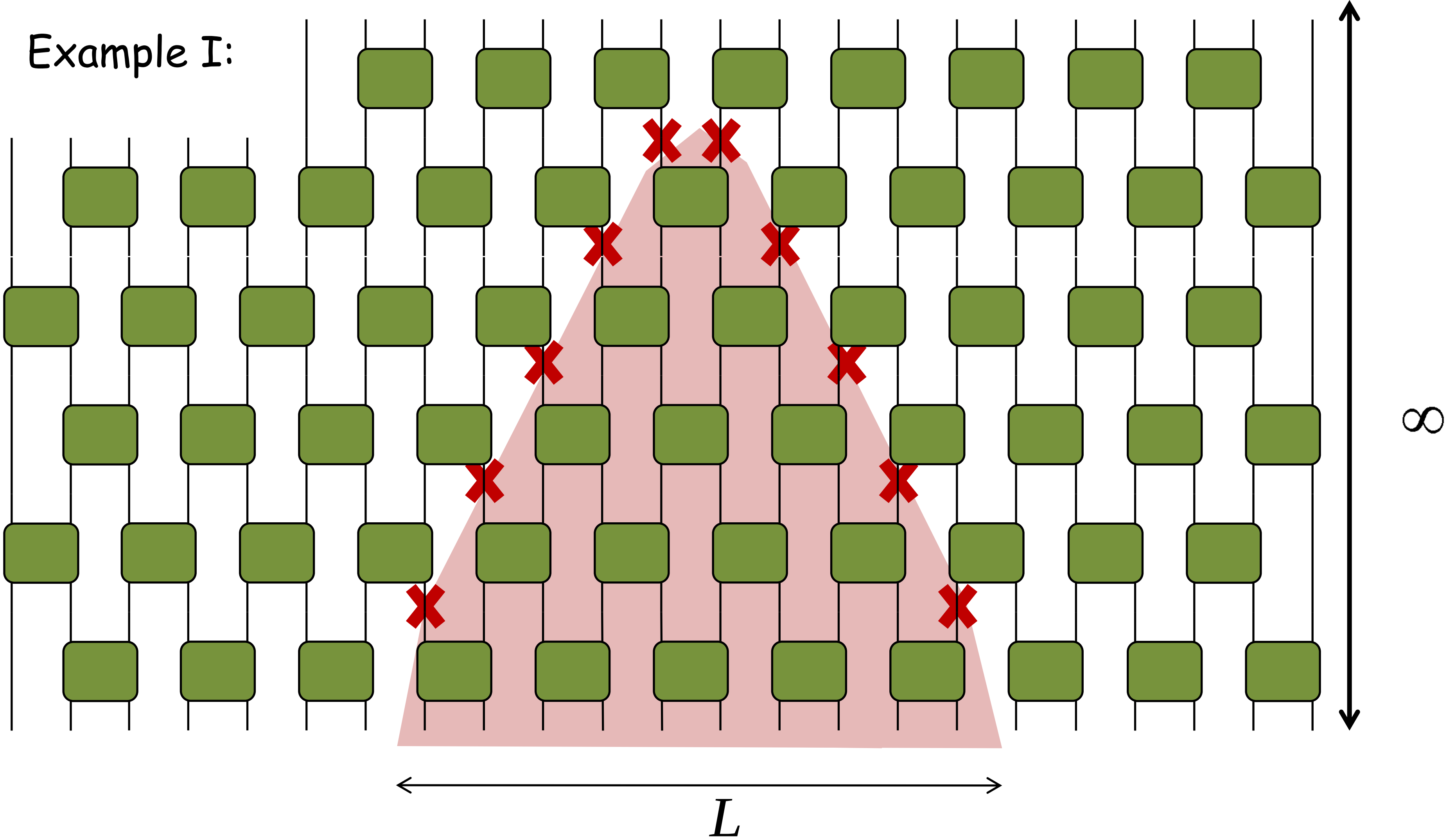


Scaling of entropy with size L of region A

$$S(A) \approx \begin{cases} const \\ \log(L) \\ L \end{cases} \quad ?$$

for simplicity, $N \to \infty$

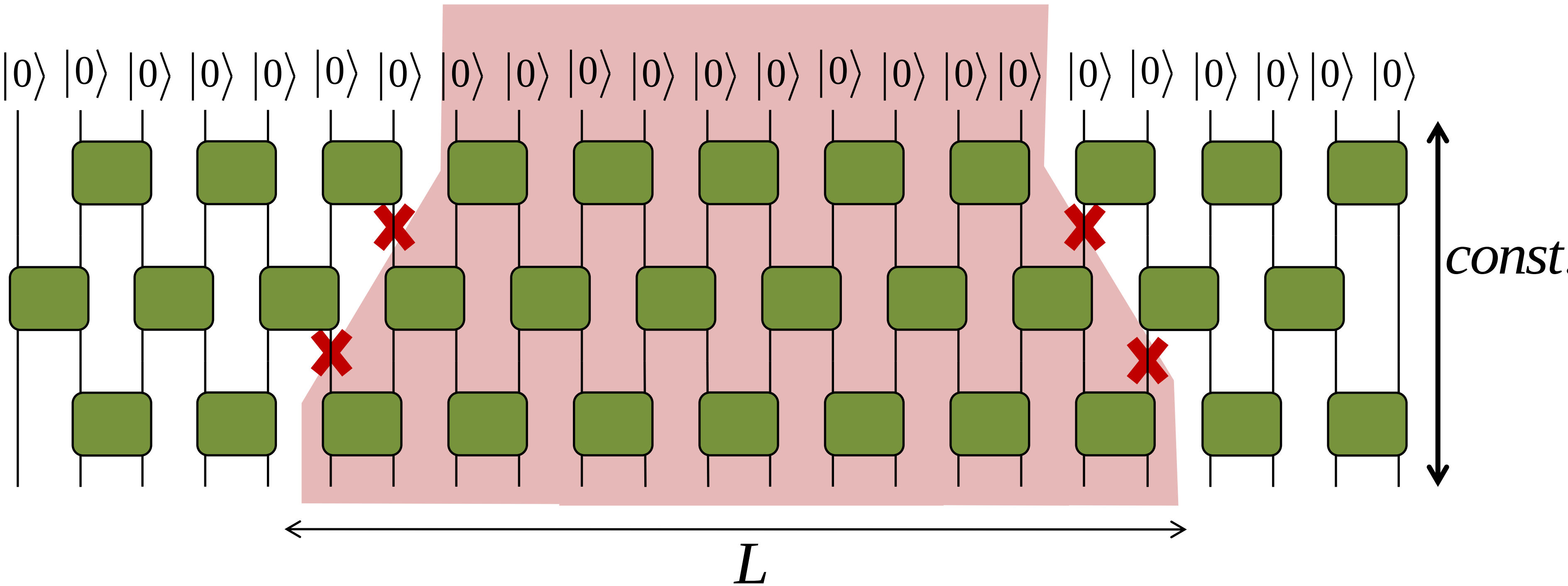
Example I:



$n(A) \approx L$

scaling of entropy: $S(A) \approx L$

Example II:



$$n(A) \approx const$$

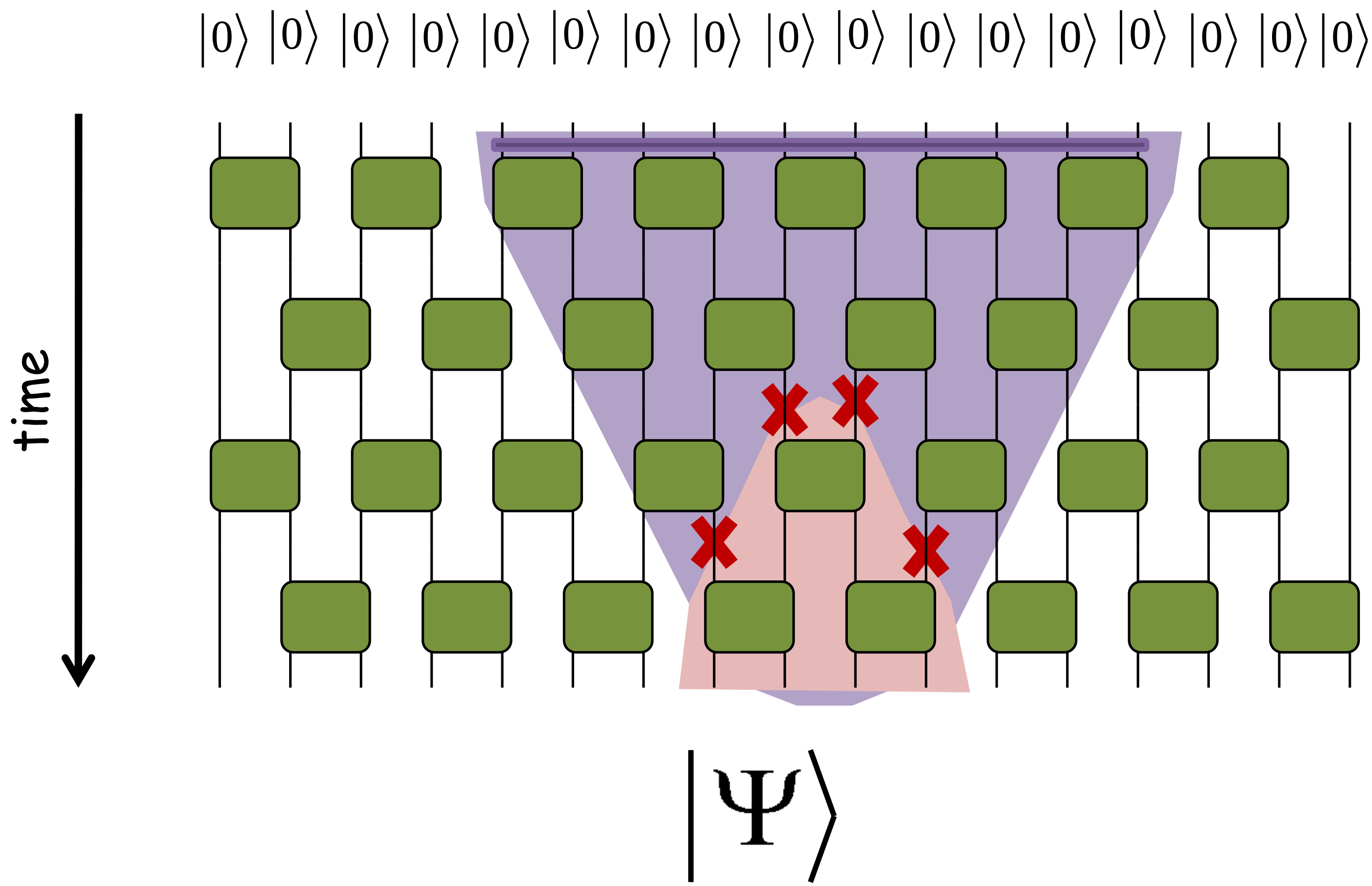
scaling of entropy: $S(A) \approx const$

Summary:

Quantum Circuit as a many-body variational ansatz

Questions:

- Cost of computing a local reduced density matrix
- Entropy of a block of contiguous sites



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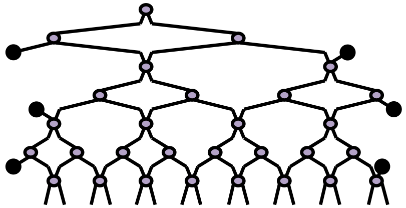
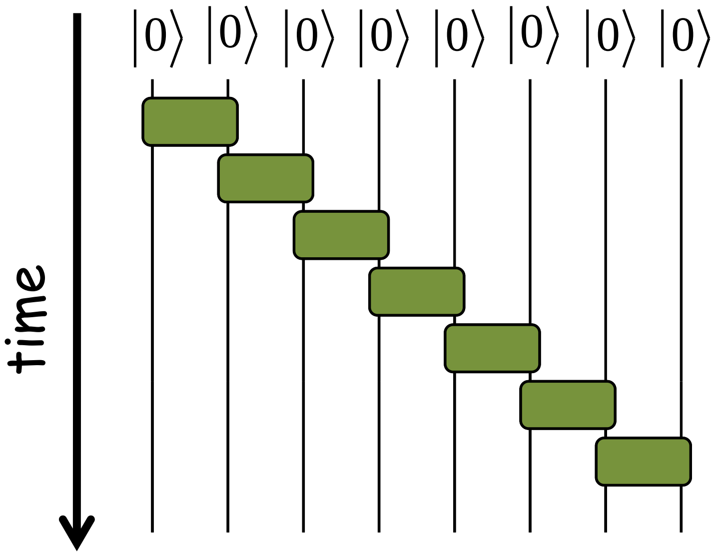
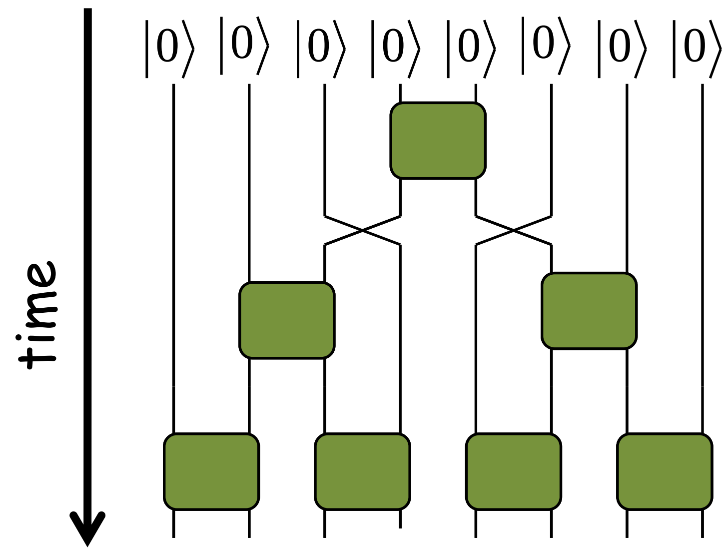


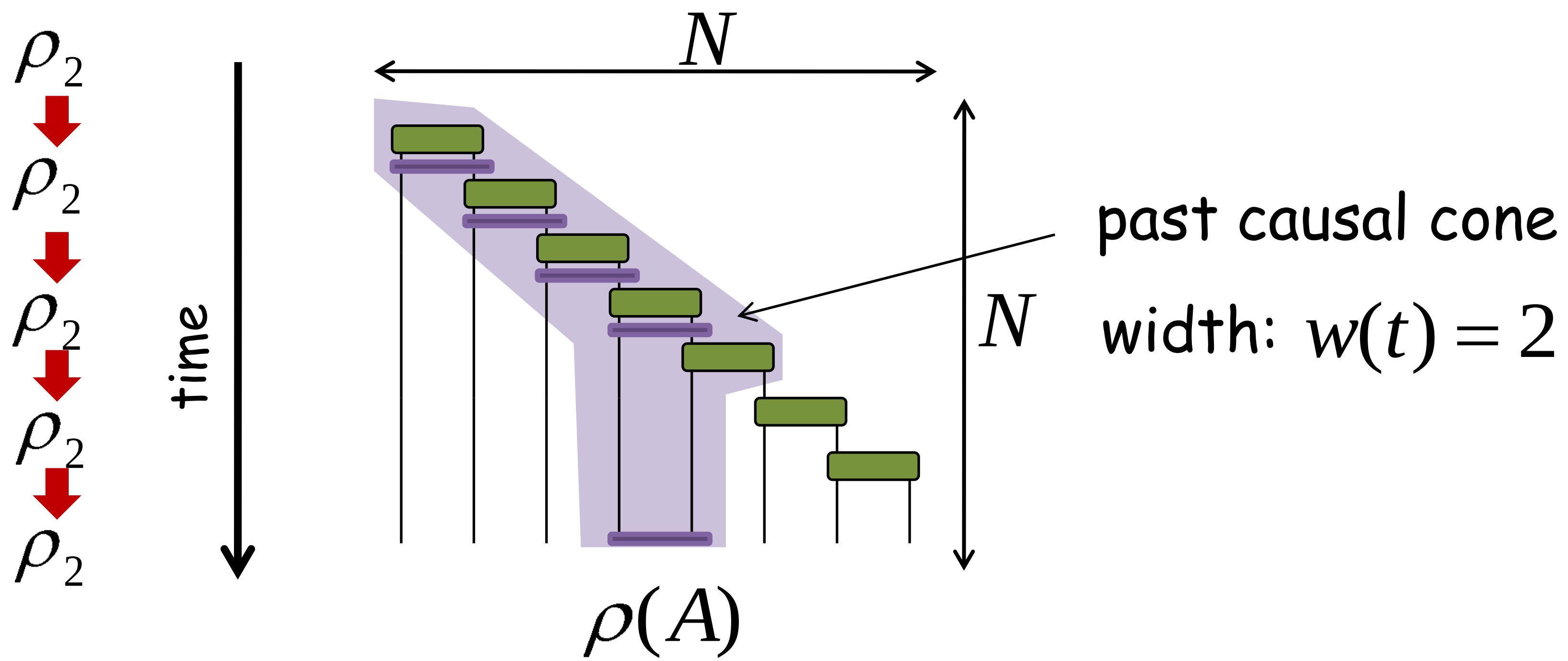


matrix product state
MPS



tree tensor network
TTN

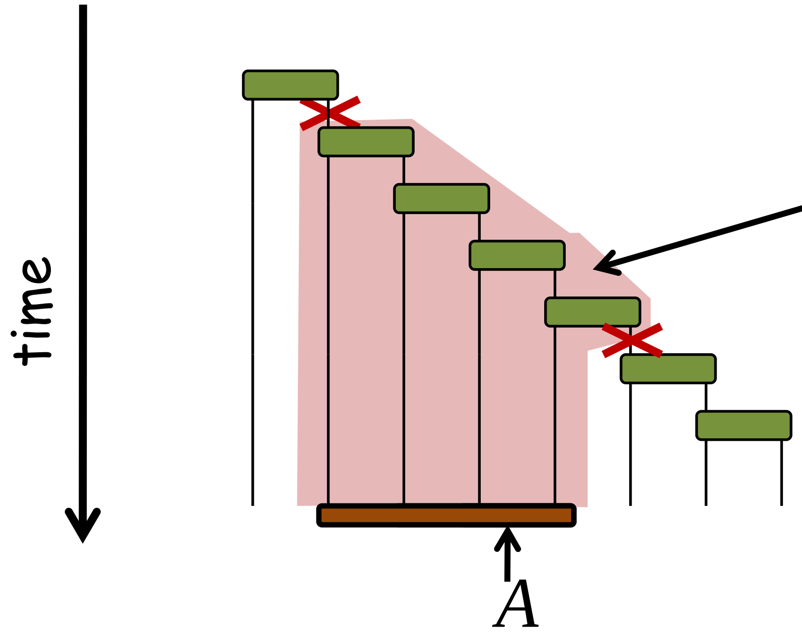
MPS: computational cost



cost of computing $\rho(A)$: $\mathrm{c} \approx \exp(w) = const$

$$\mathrm{c} \approx O(N)$$

MPS: entanglement entropy

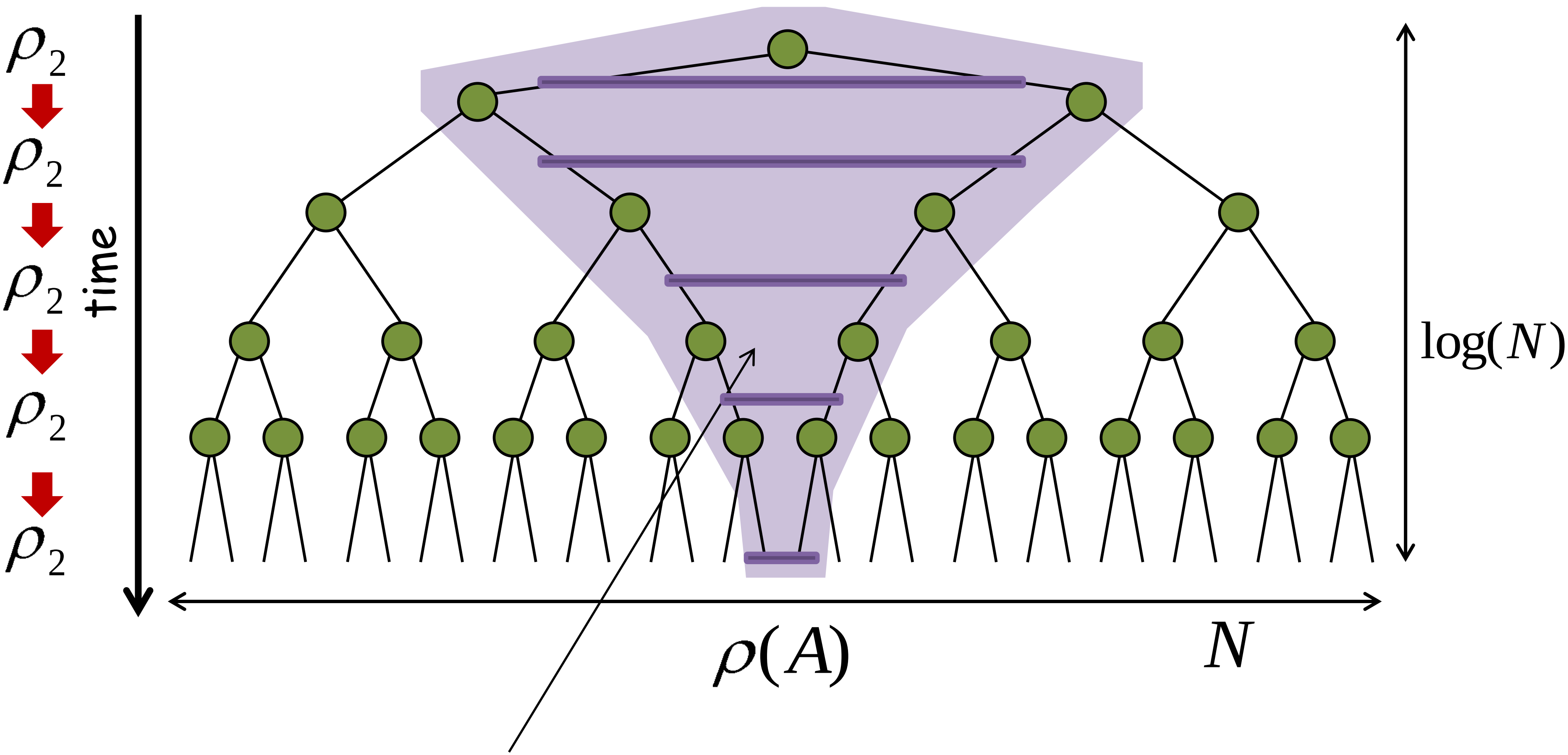


minimally connected region Ω_A

$n(A) \approx 2$

scaling of entropy: $S(A) \approx const$

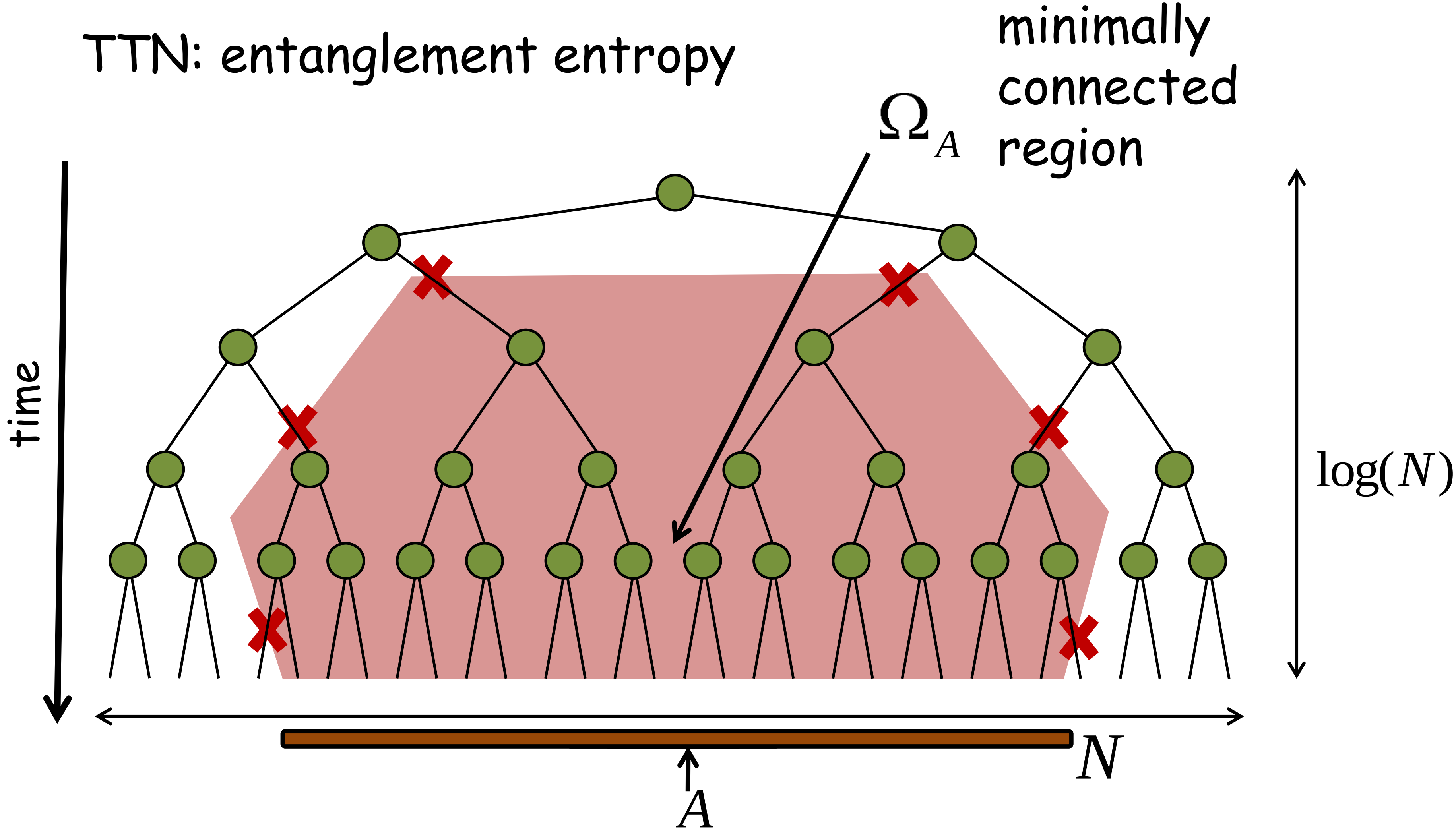
TTN: computational cost



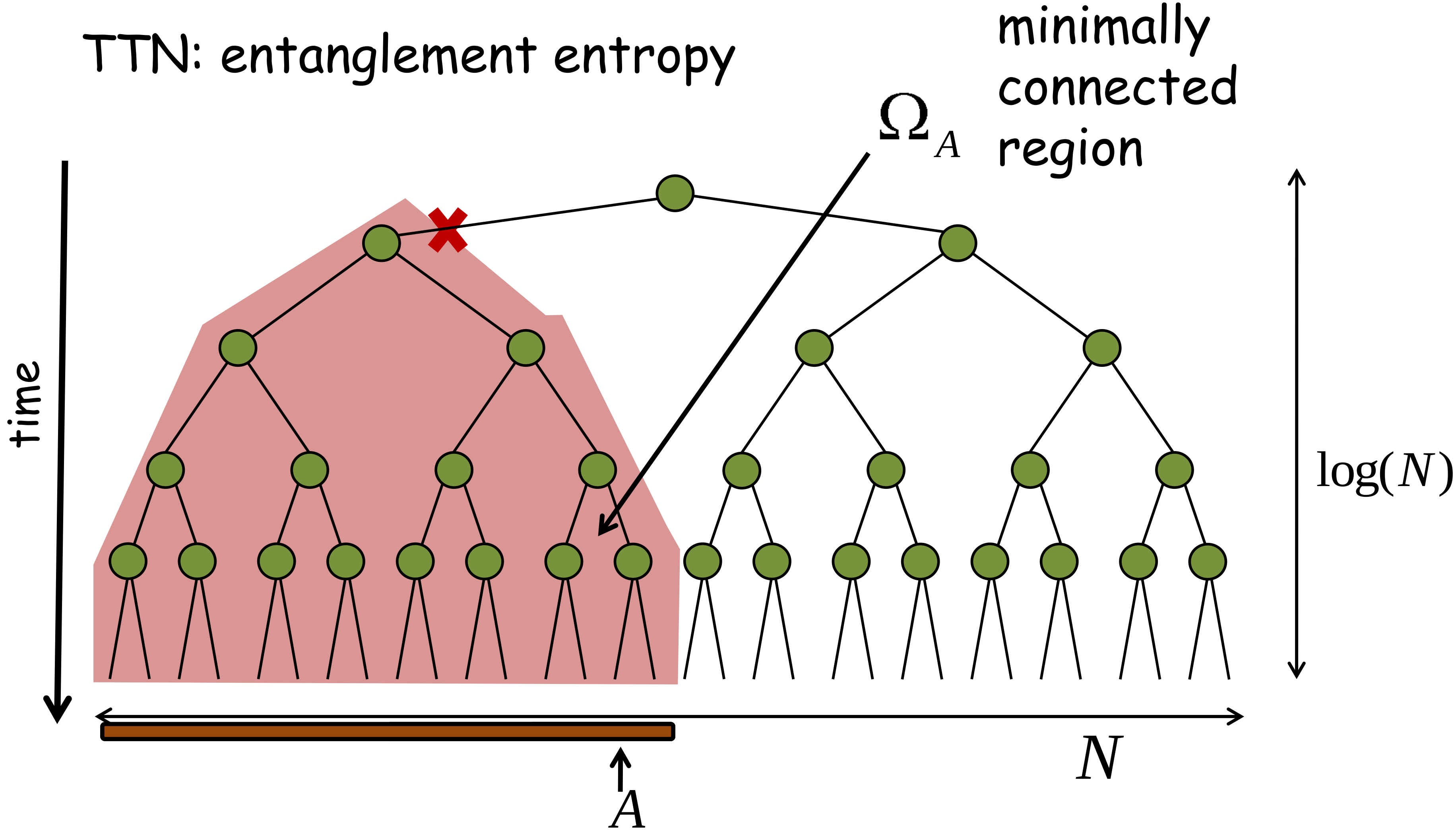
width: $w(t) = 2$

cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $\quad c \approx \log(N)$

TTN: entanglement entropy

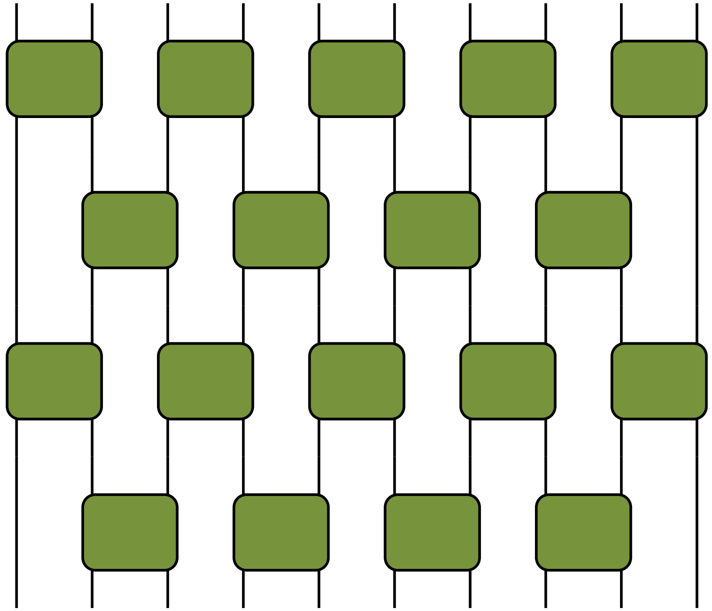


TTN: entanglement entropy

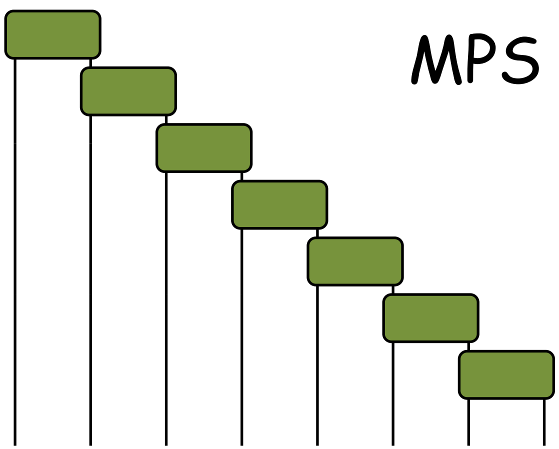


$n(A) \approx 1$

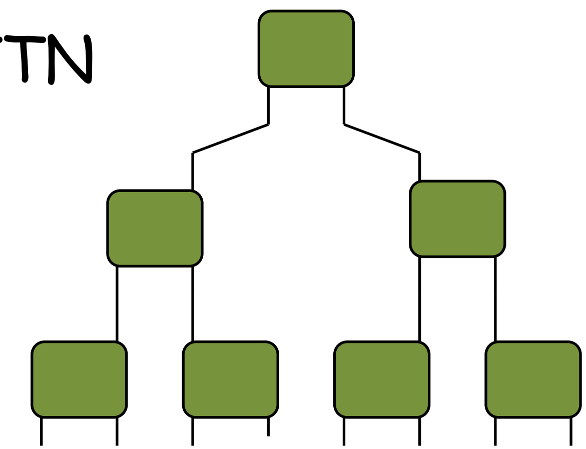
scaling of entropy: $S(A) \approx const$



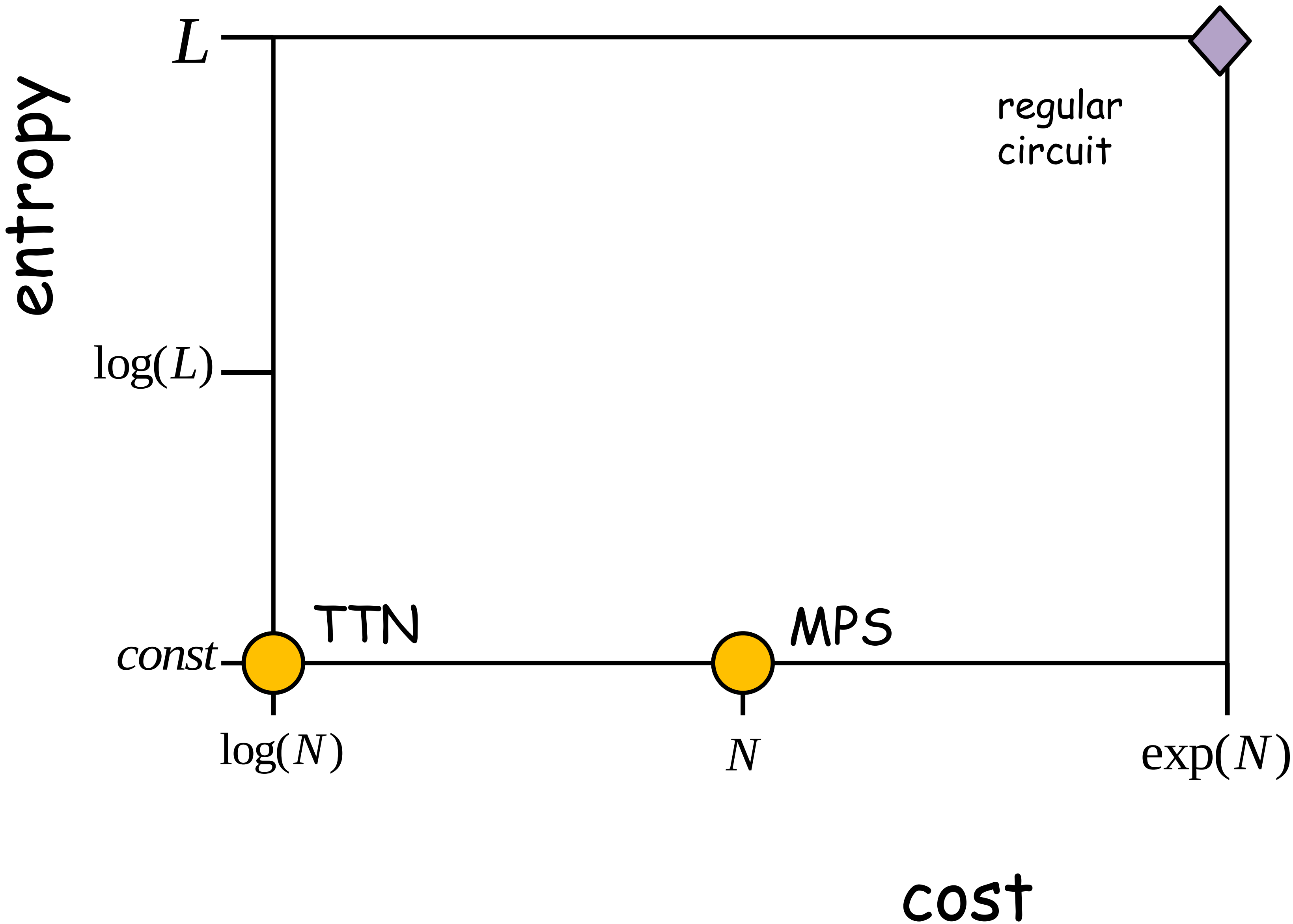
$$c \approx \exp(N)$$
$$S(A) \approx N$$



$$c \approx N$$
$$S(A) \approx const$$



$$c \approx \log(N)$$
$$S(A) \approx const$$



entropy
L
$\log(L)$
$const$
regular
circuit
TTN
MPS
$\log(N)$
N
$\exp(N)$
cost

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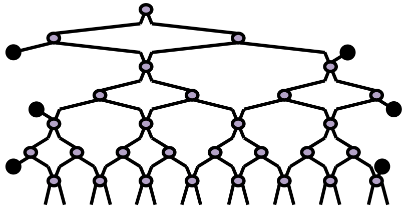
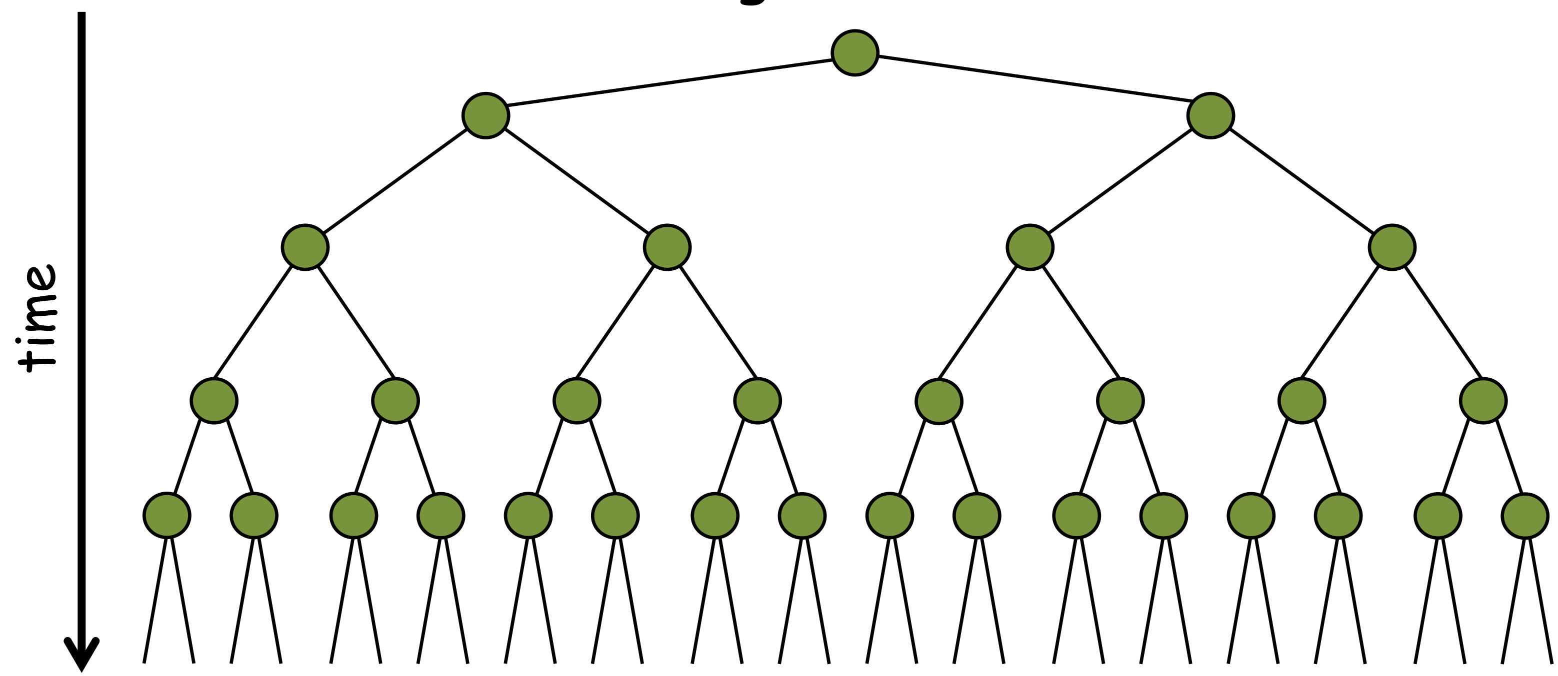
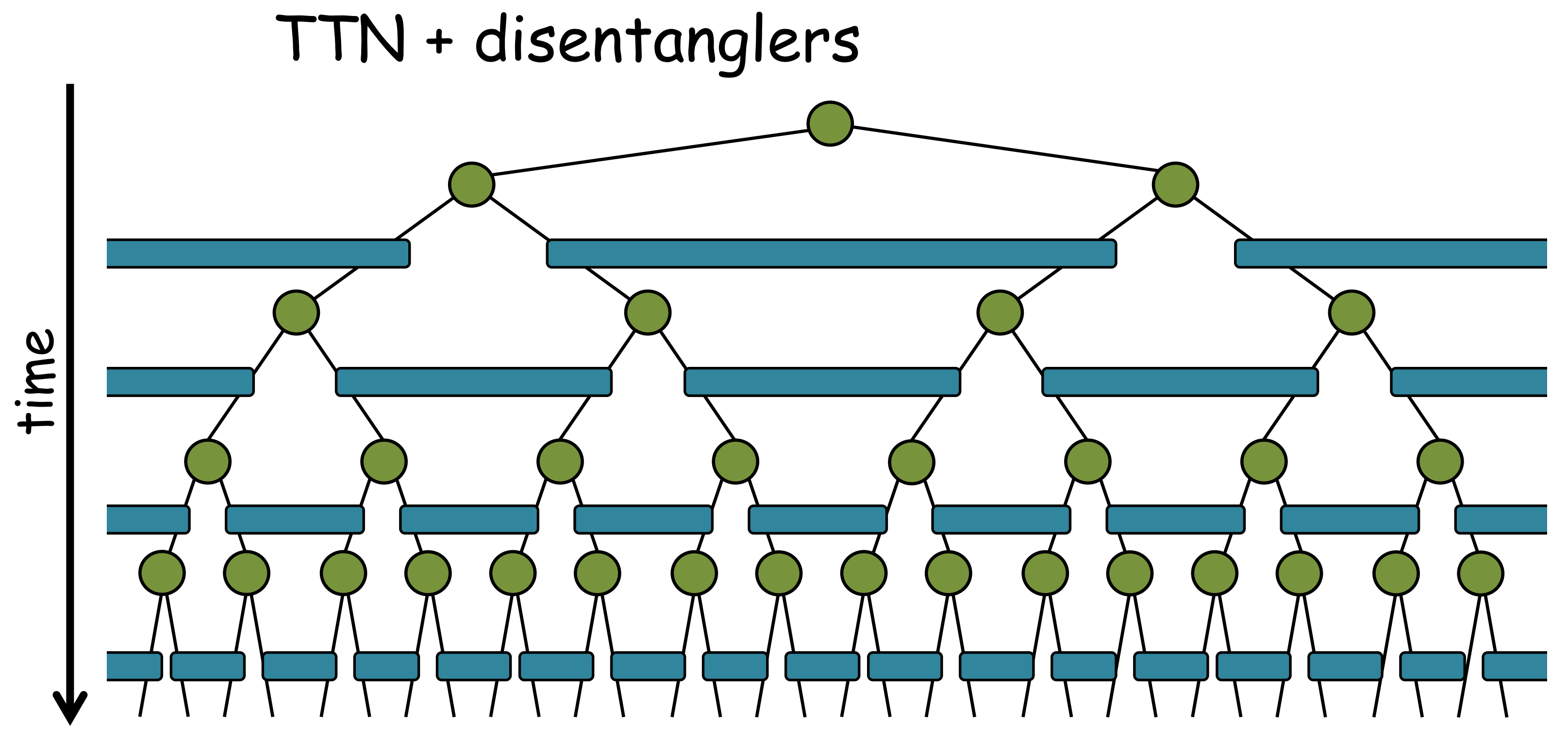


MERA (multi-scale entanglement renormalization ansatz)

TTN + disentanglers

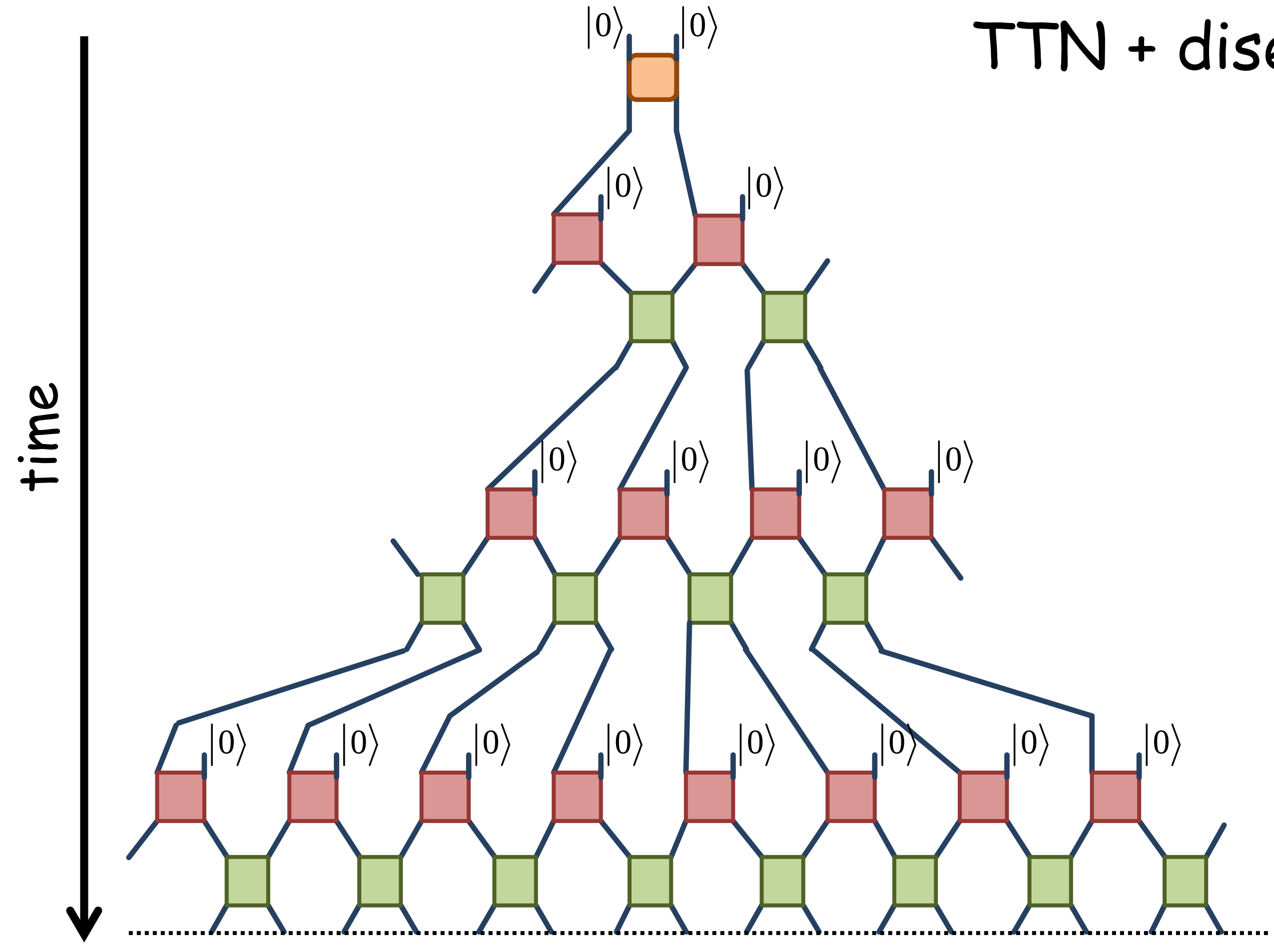


MERA (multi-scale entanglement renormalization ansatz)

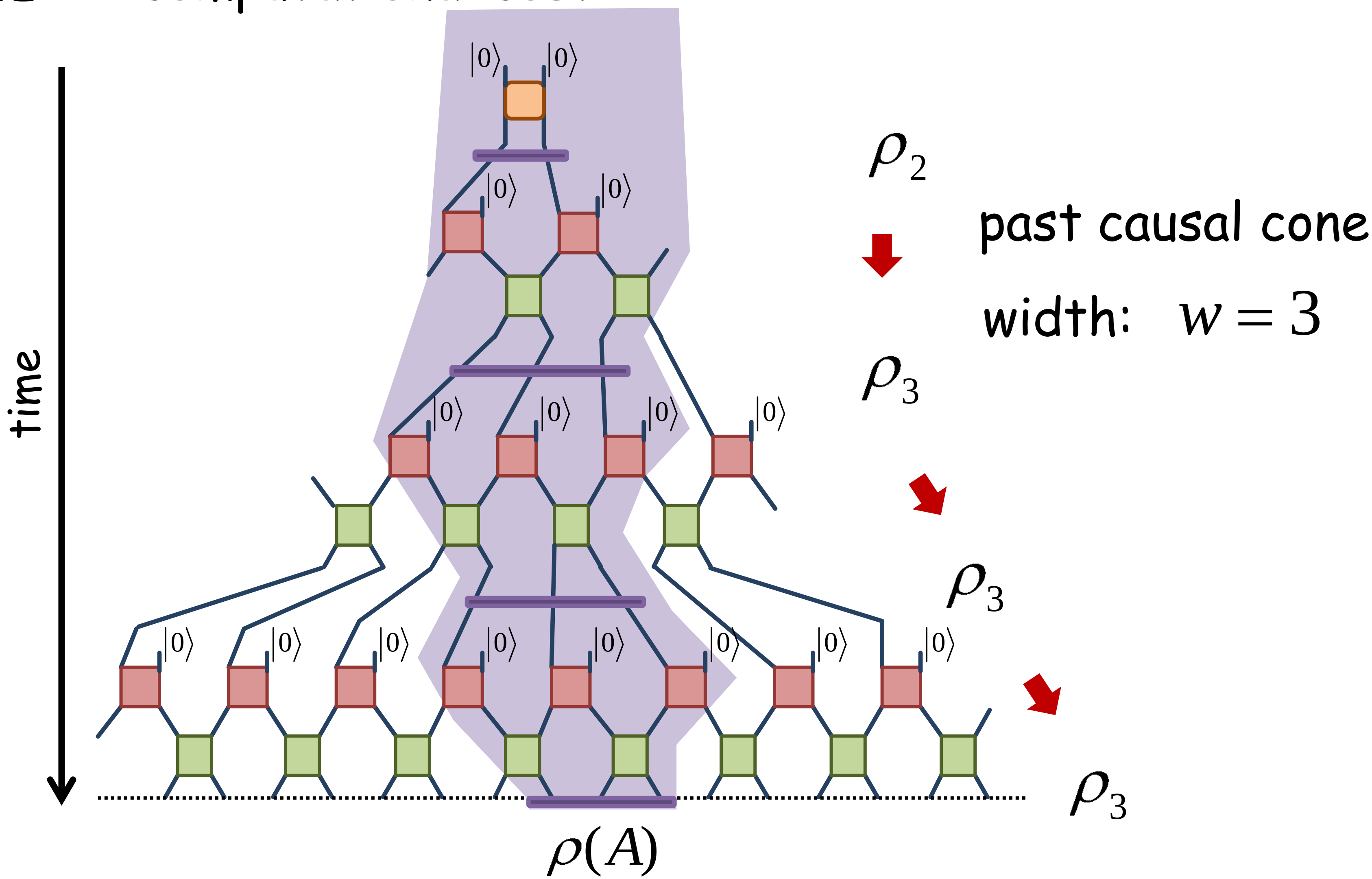


MERA (multi-scale entanglement renormalization ansatz)

TTN + disentanglers

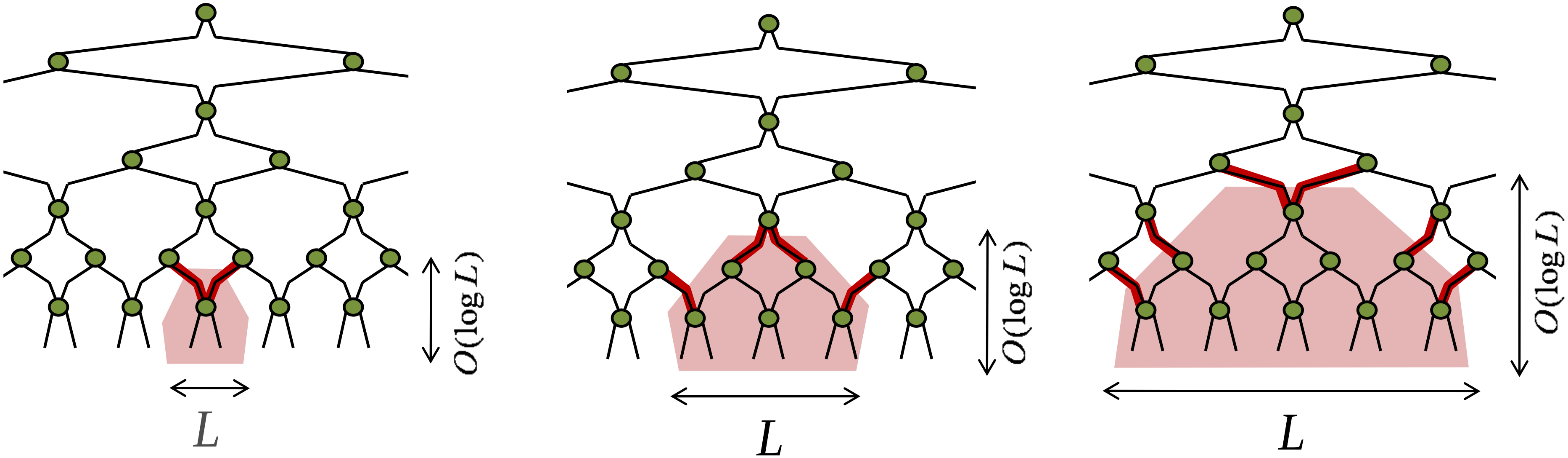


MERA: computational cost



cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $c \approx \log(N)$

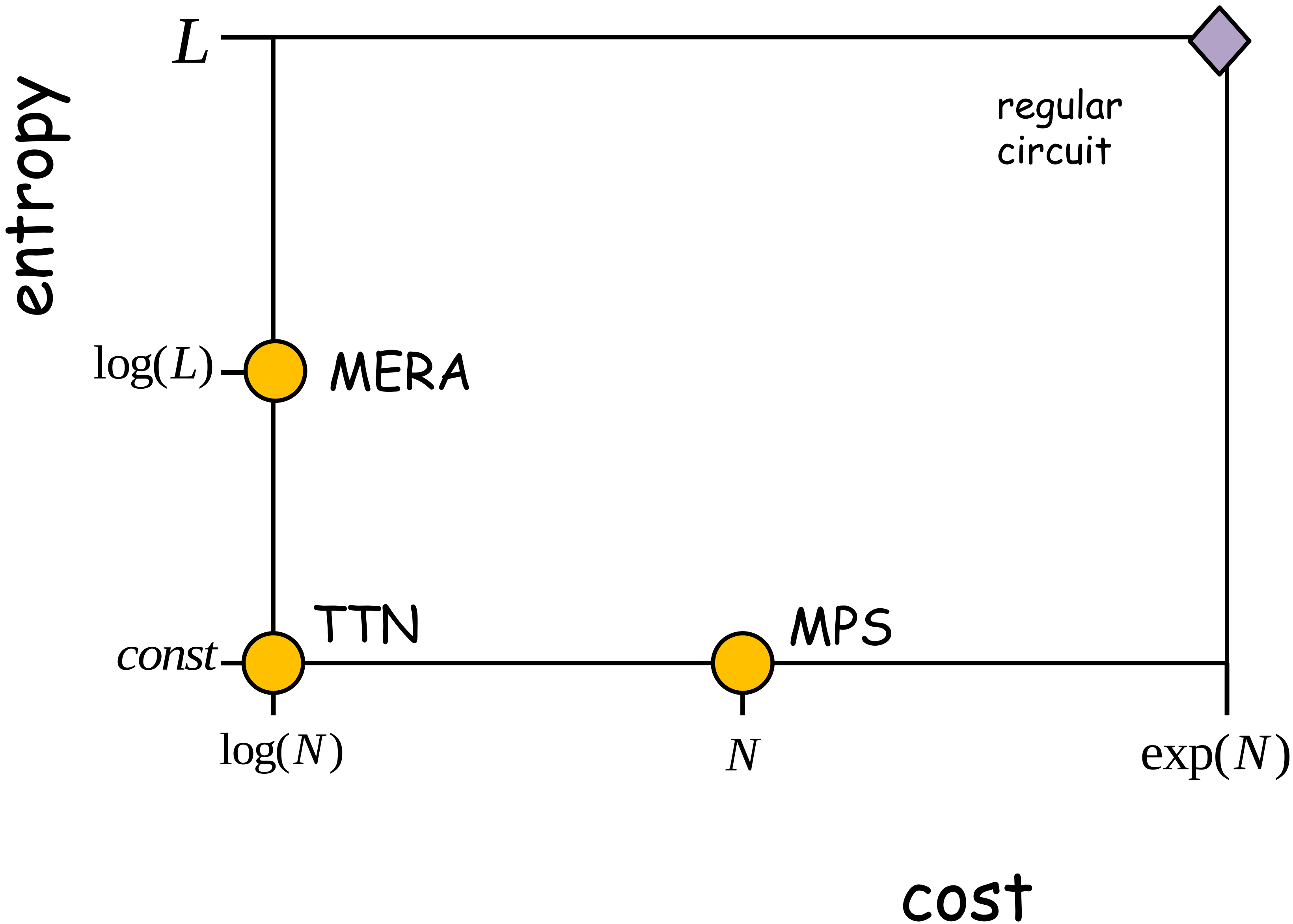
MERA: entanglement entropy



$$n(A) \approx \log(L)$$

scaling of entropy:

$$S(A) \approx \log(L)$$



entropy
L
$\log(L)$
$const$
MERA
TTN
MPS
regular
circuit
$\log(N)$
N
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cost

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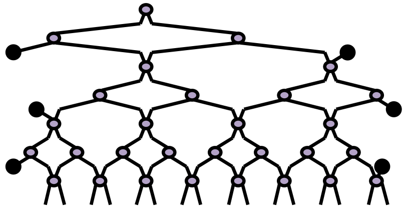
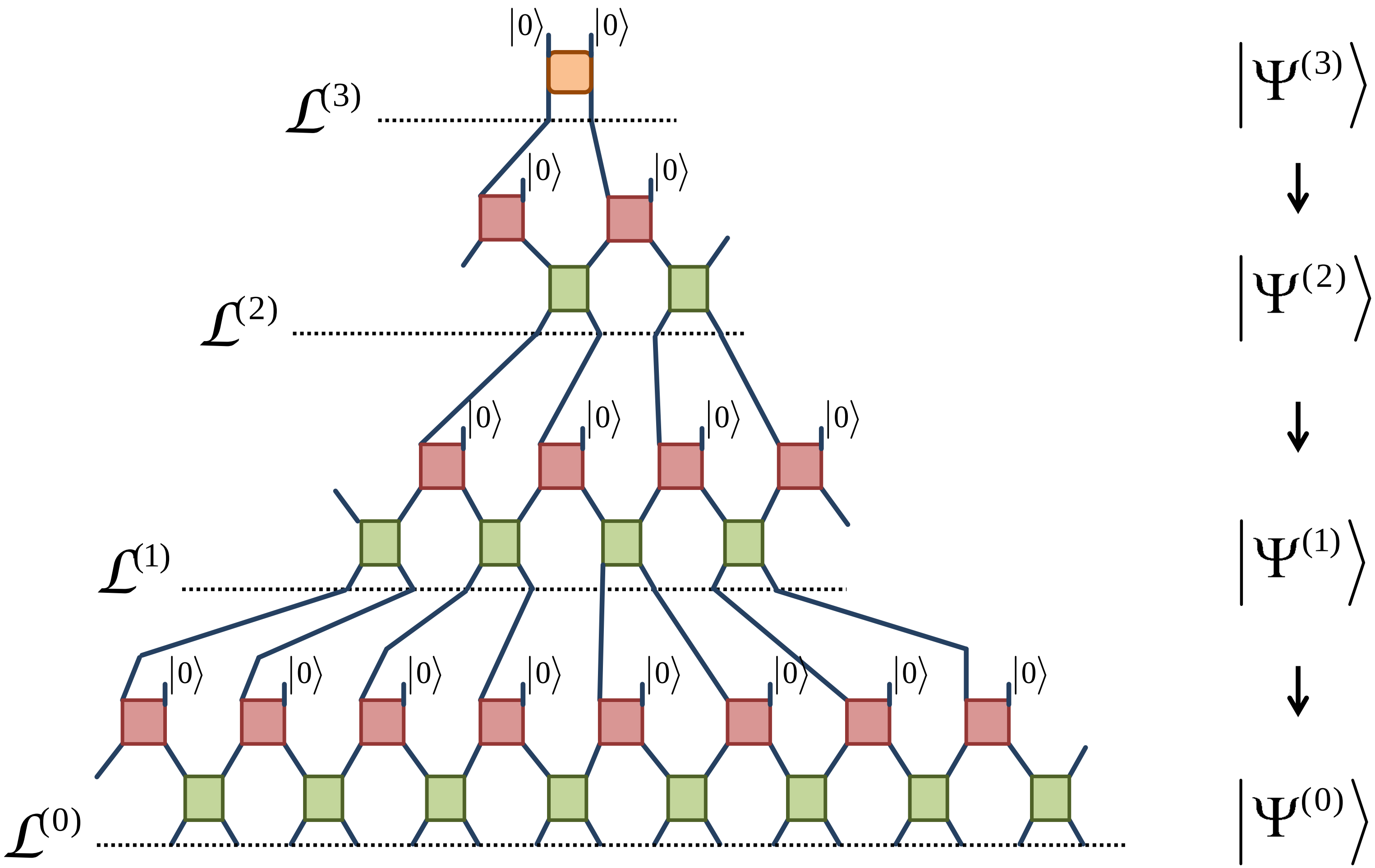
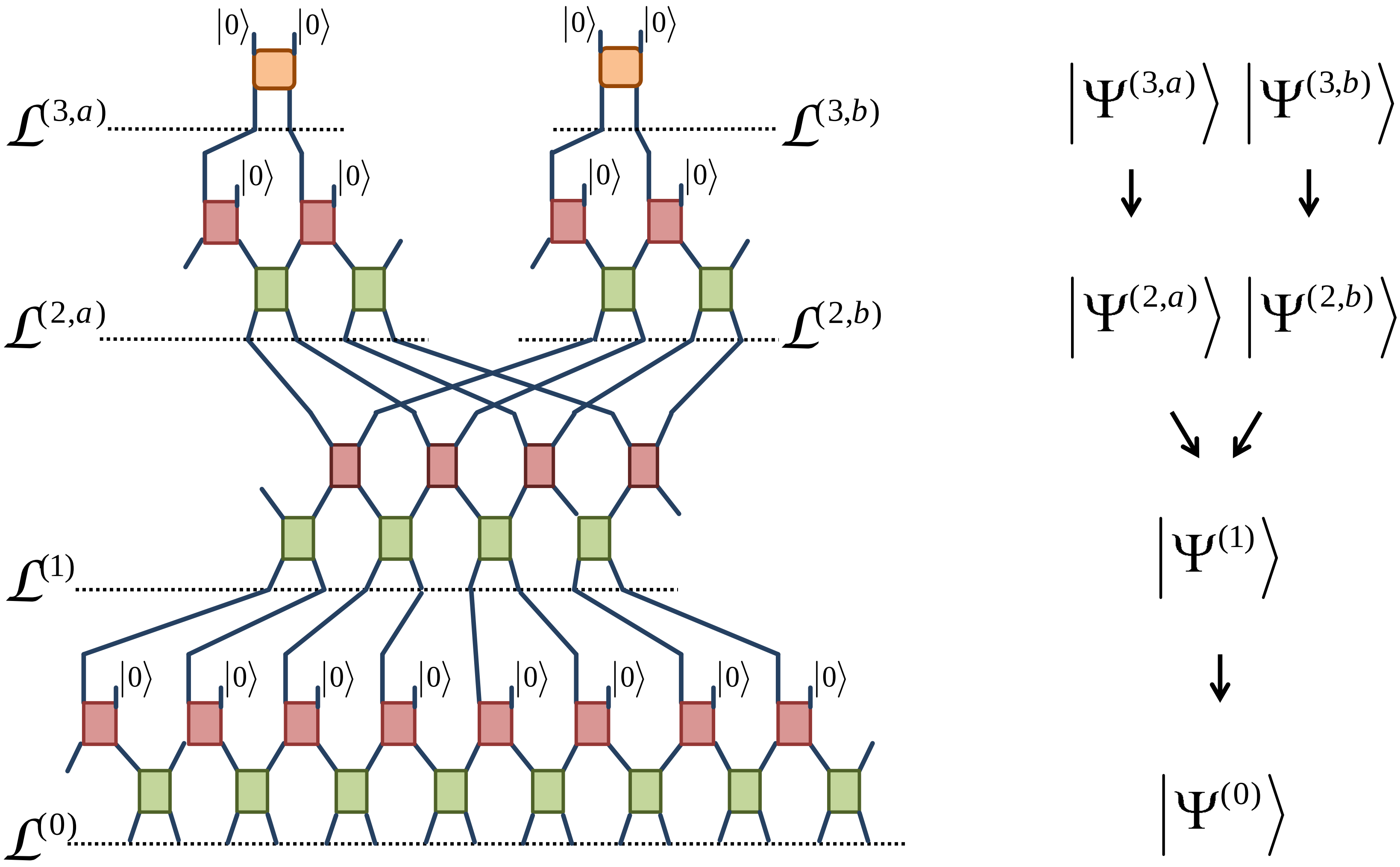


MERA



$|0\rangle$ $|0\rangle$
$\mathcal{L}^{(3)}$
$|0\rangle$ $|0\rangle$
$\mathcal{L}^{(2)}$
$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$
$\mathcal{L}^{(1)}$
$|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$ $|0\rangle$
$\mathcal{L}^{(0)}$
$|\Psi^{(3)}\rangle$
$|\Psi^{(2)}\rangle$
$|\Psi^{(1)}\rangle$
$|\Psi^{(0)}\rangle$

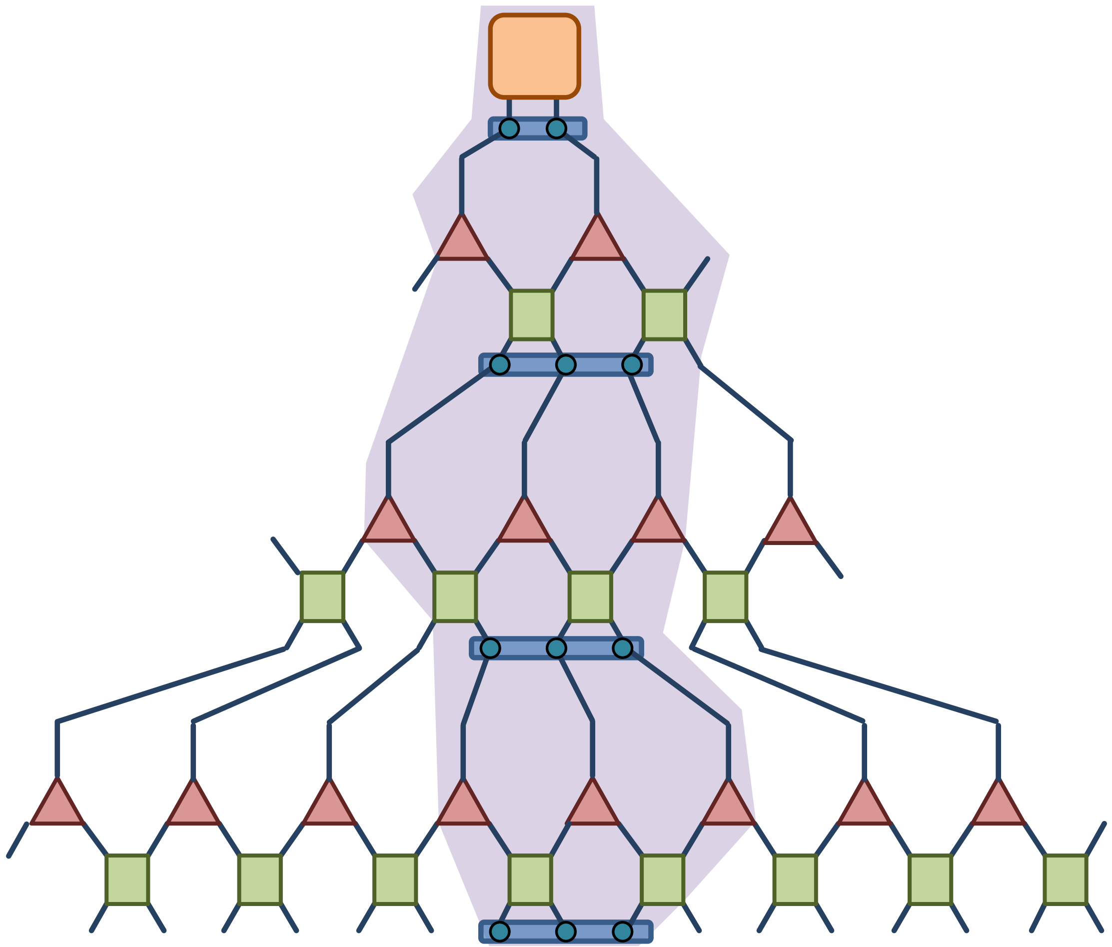
branching MERA

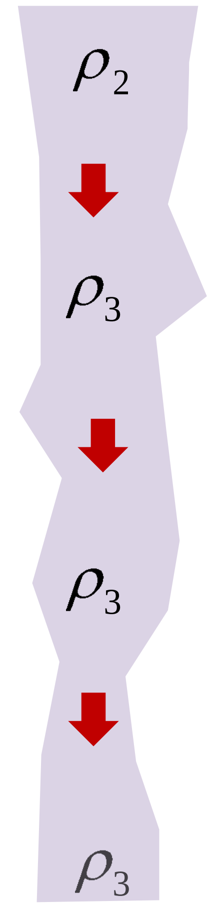


MERA: computational cost

past causal cone

width: $w = 3$



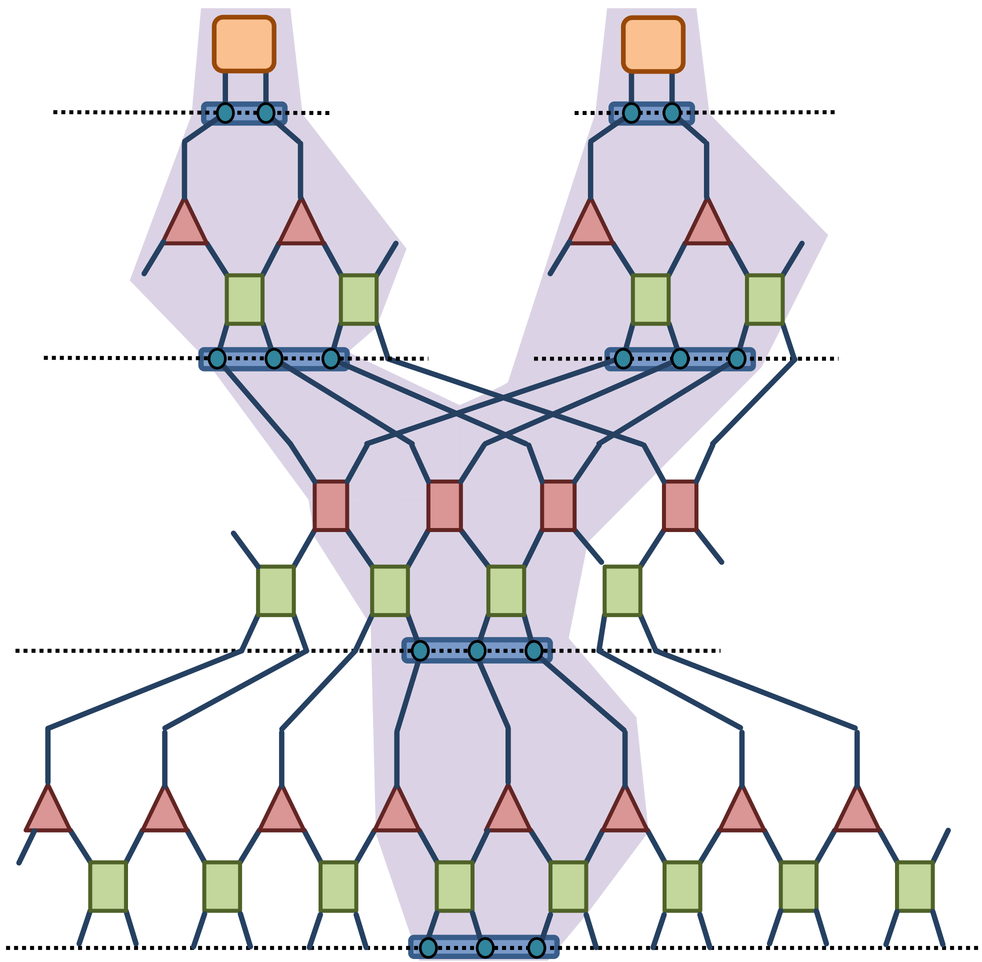


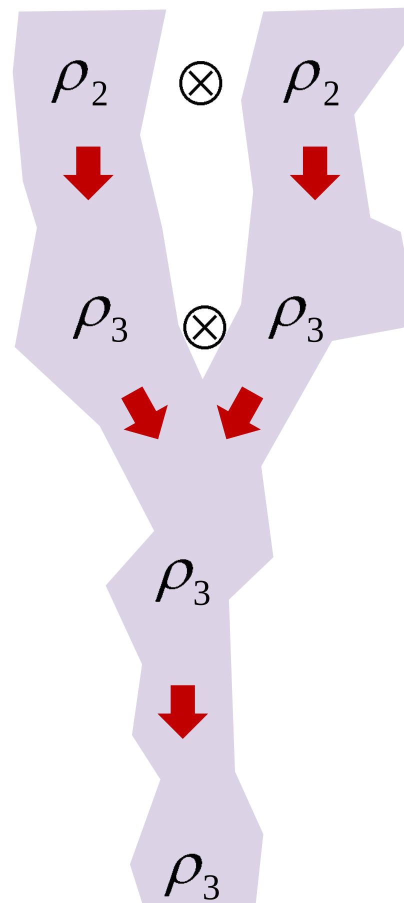
cost of computing $\rho(A)$: $c \approx \exp(w) = const$ $c \approx \log(N)$

branching MERA: computational cost

past causal cone

width: $w' = 2w$

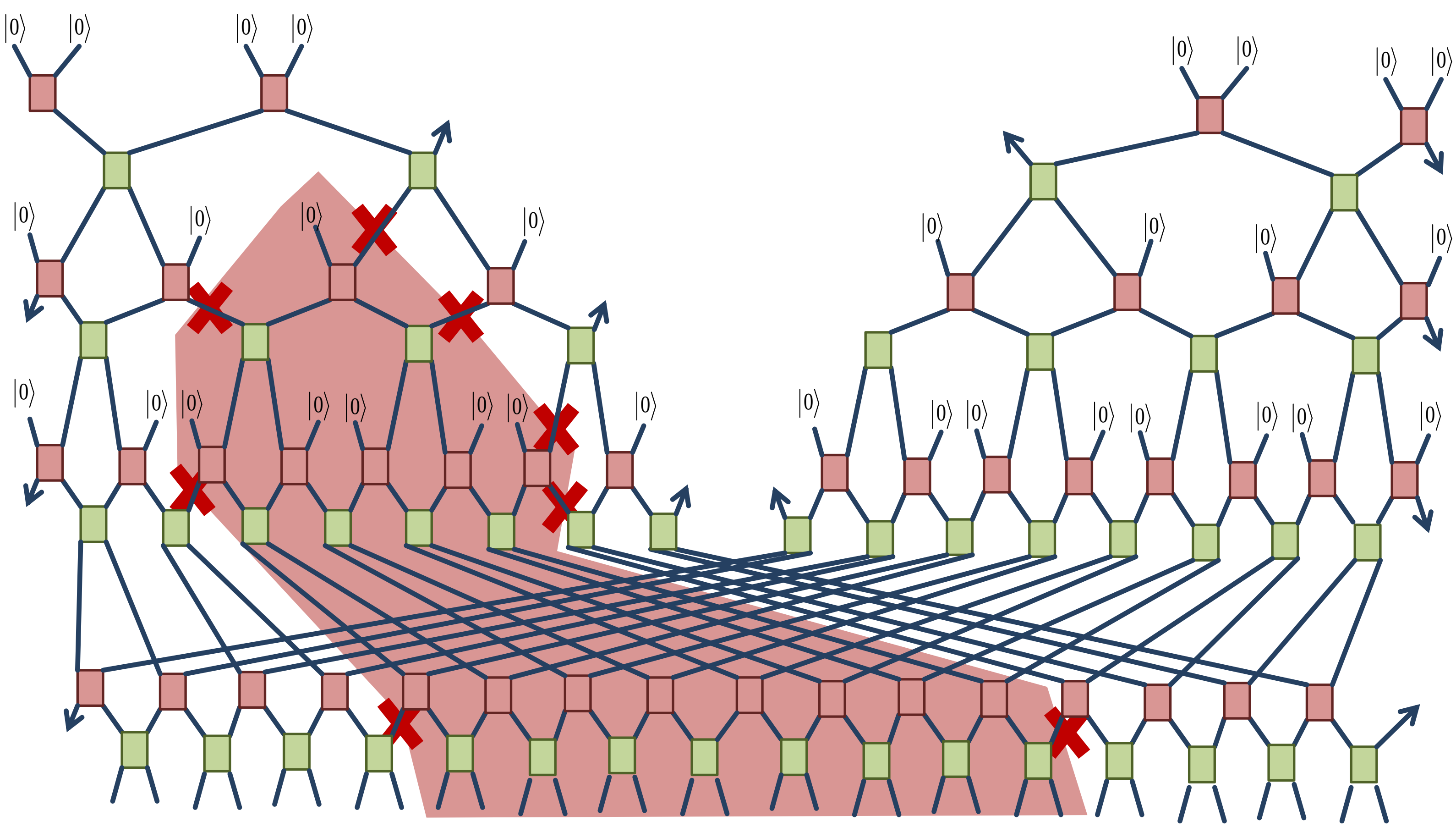




cost of computing $\rho(A)$: $c \approx 2\exp(w)$

$c \approx 2\log(N)$

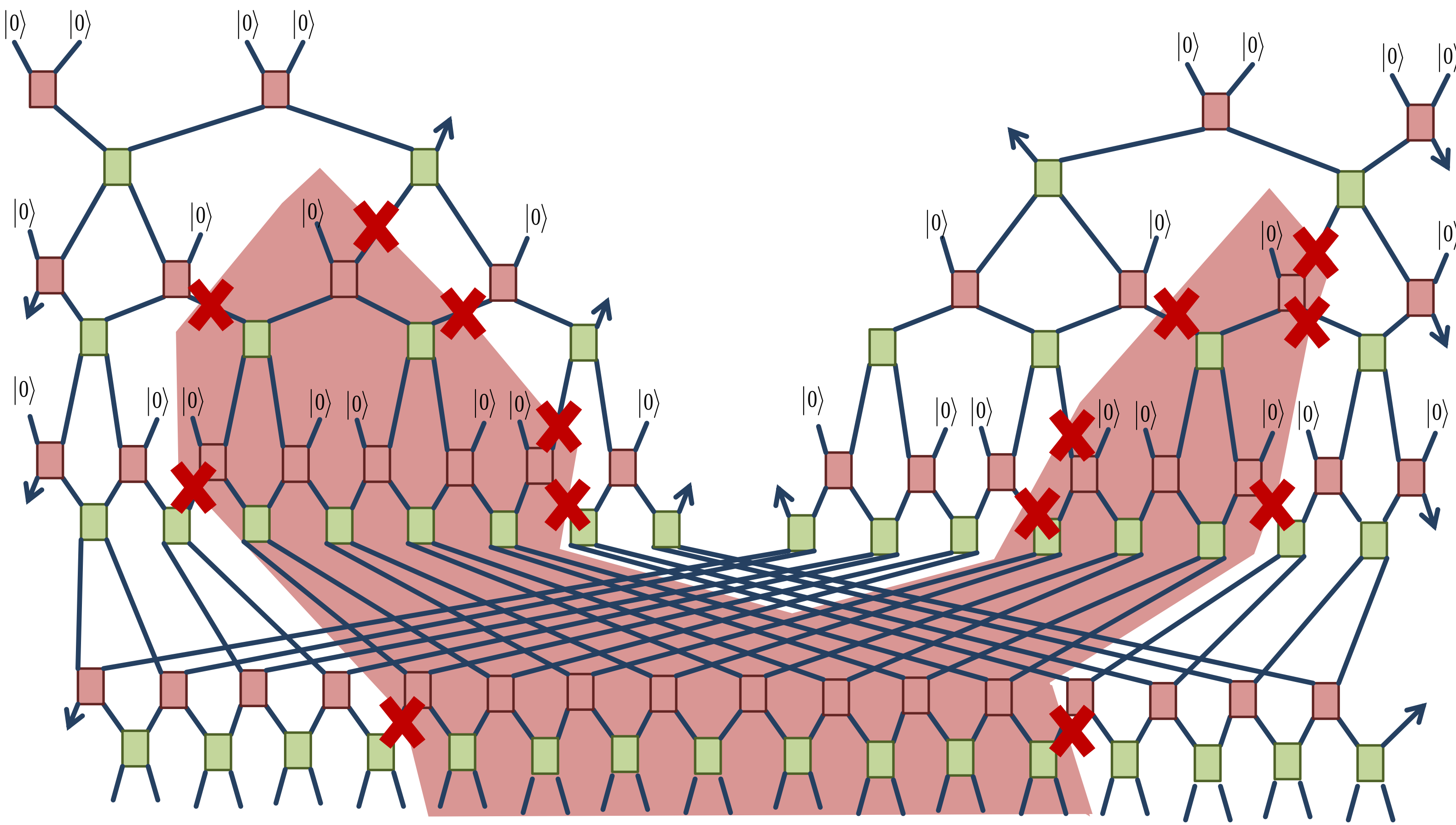
MERA: entanglement entropy



$$n(A) \approx \log(L)$$

scaling of entropy: $S(A) \approx \log(L)$

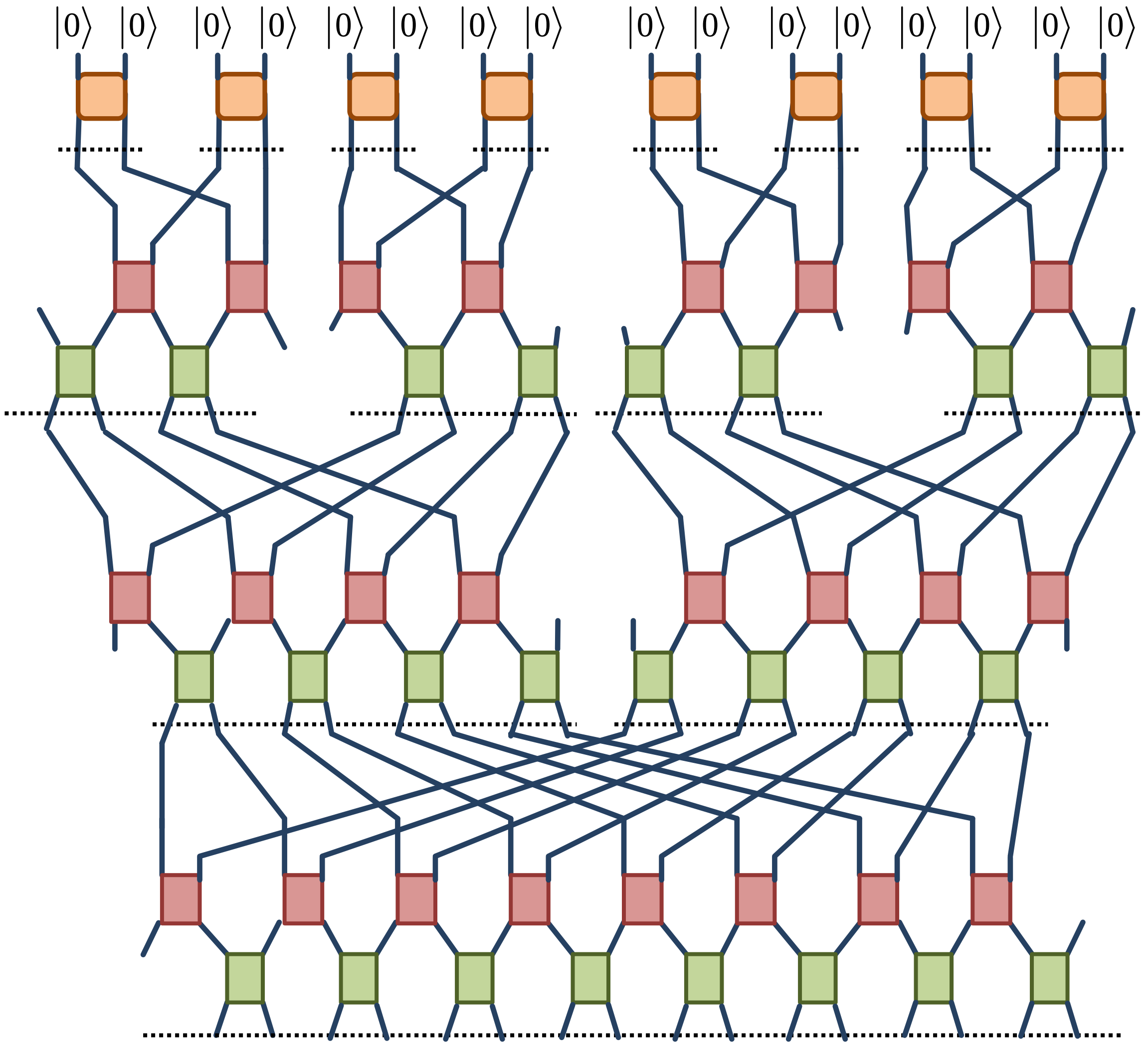
ranching MERA: entanglement entropy



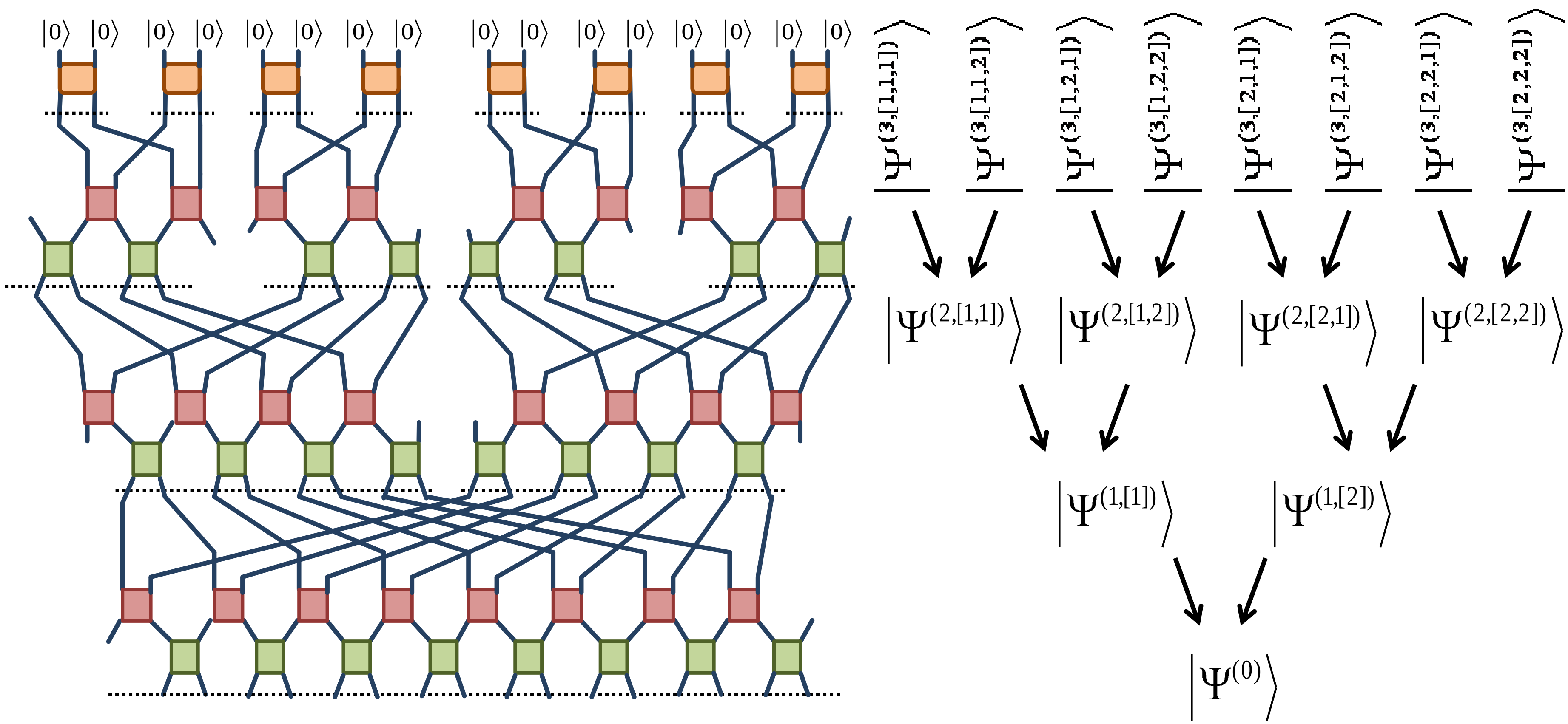
$$n(A) \approx 2\log(L)$$

scaling of entropy: $S(A) \approx 2\log(L)$

branching MERA

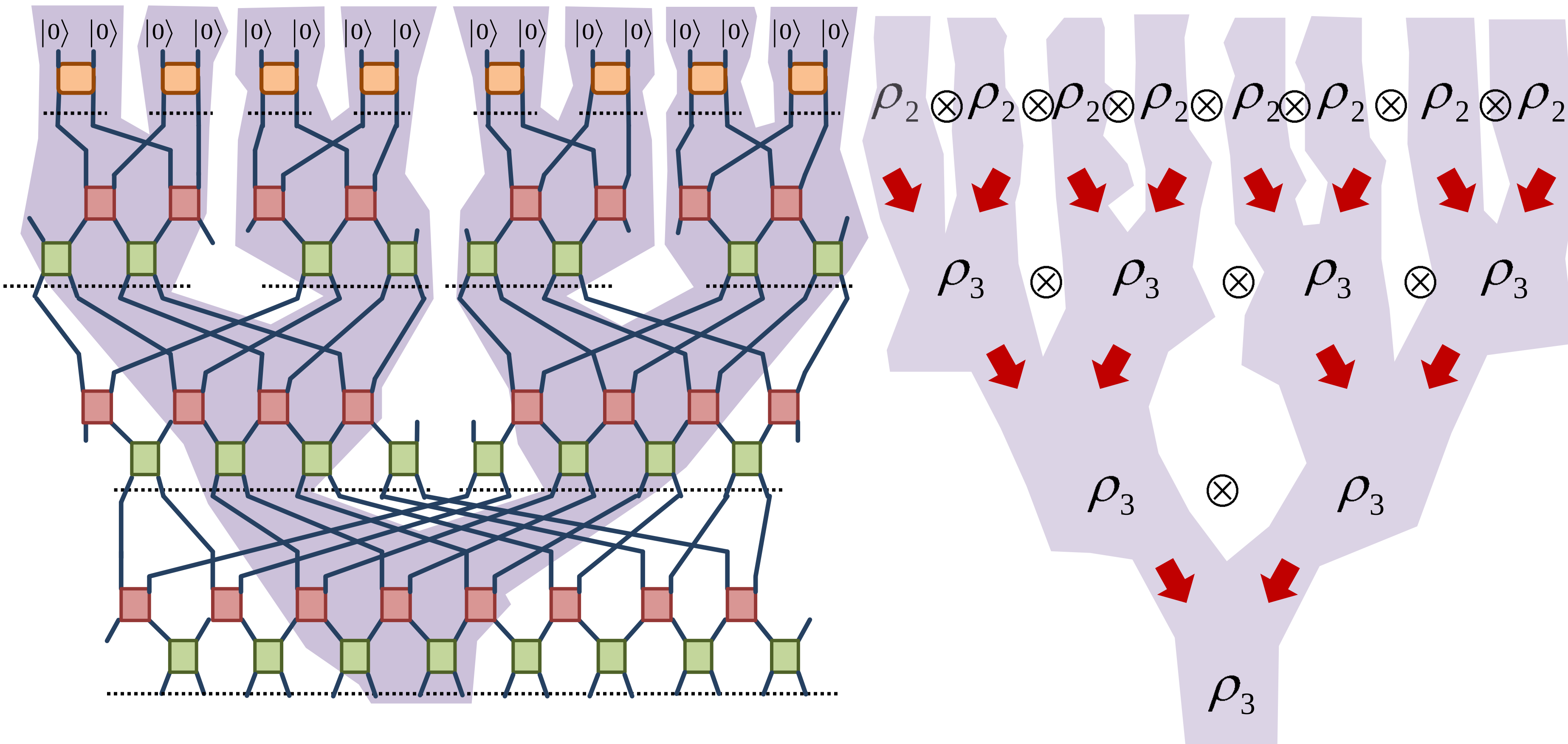


branching MERA



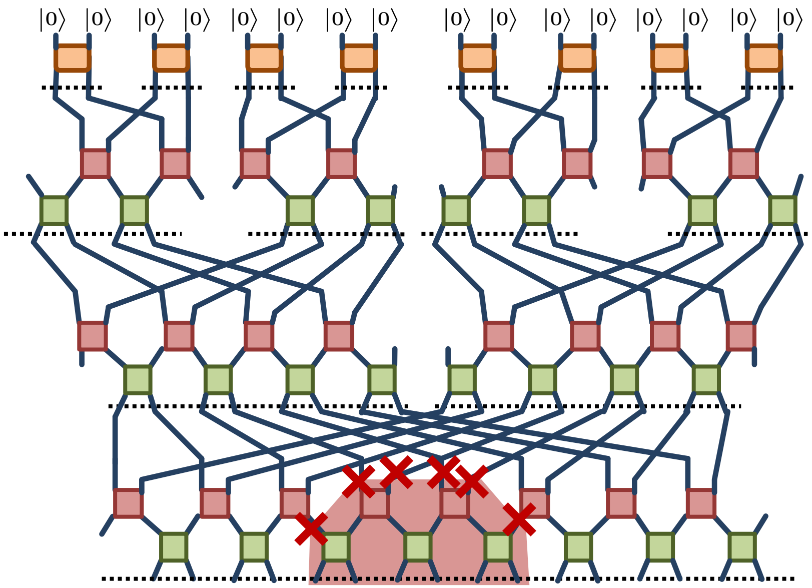
branching MERA: computational cost

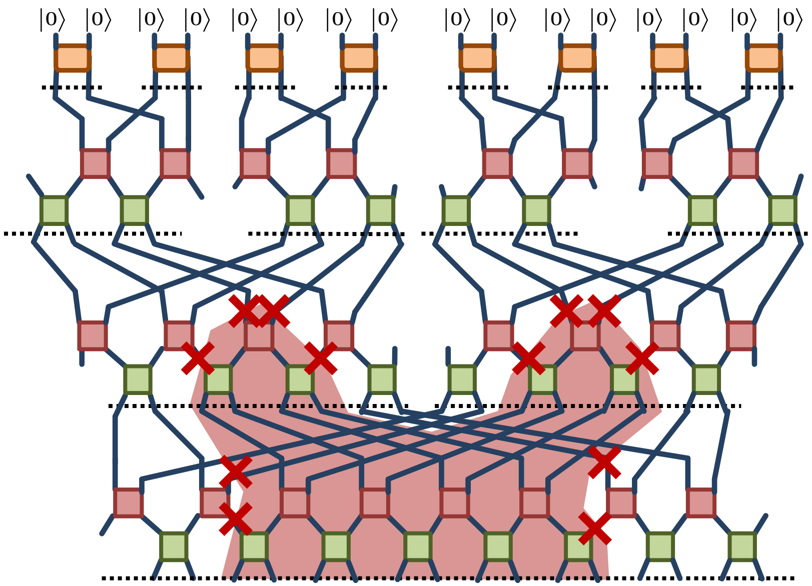
past causal cone width: $w' = qw$



cost of computing $\rho(A)$: $c \approx q\exp(w)$ $\quad c \approx O(N)$

branching MERA: entanglement entropy



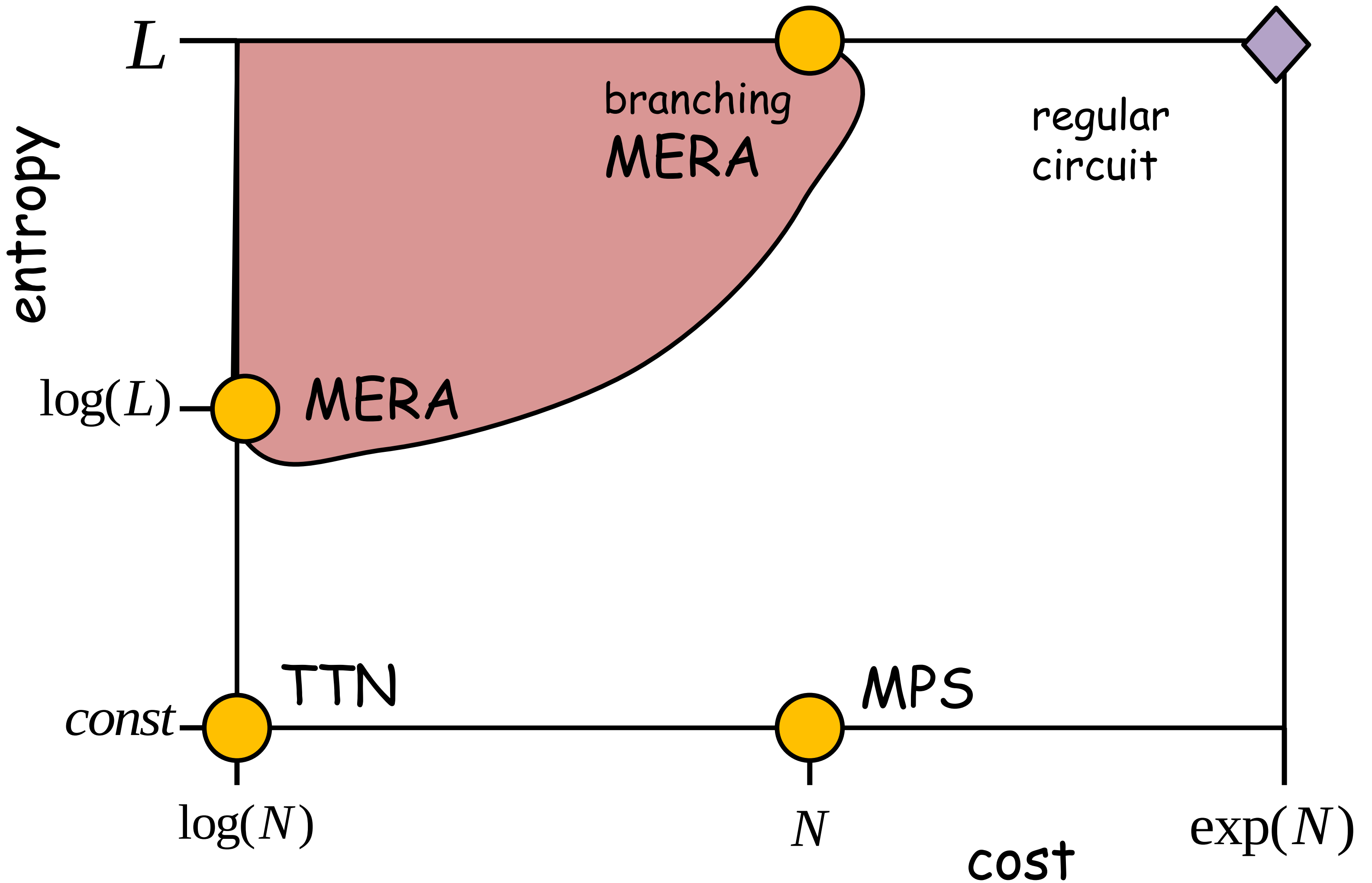


$$n(A) \approx O(L)$$

scaling of entropy: $S(A) \approx L$

Conclusions

- quantum circuits can be used to encode many-body states



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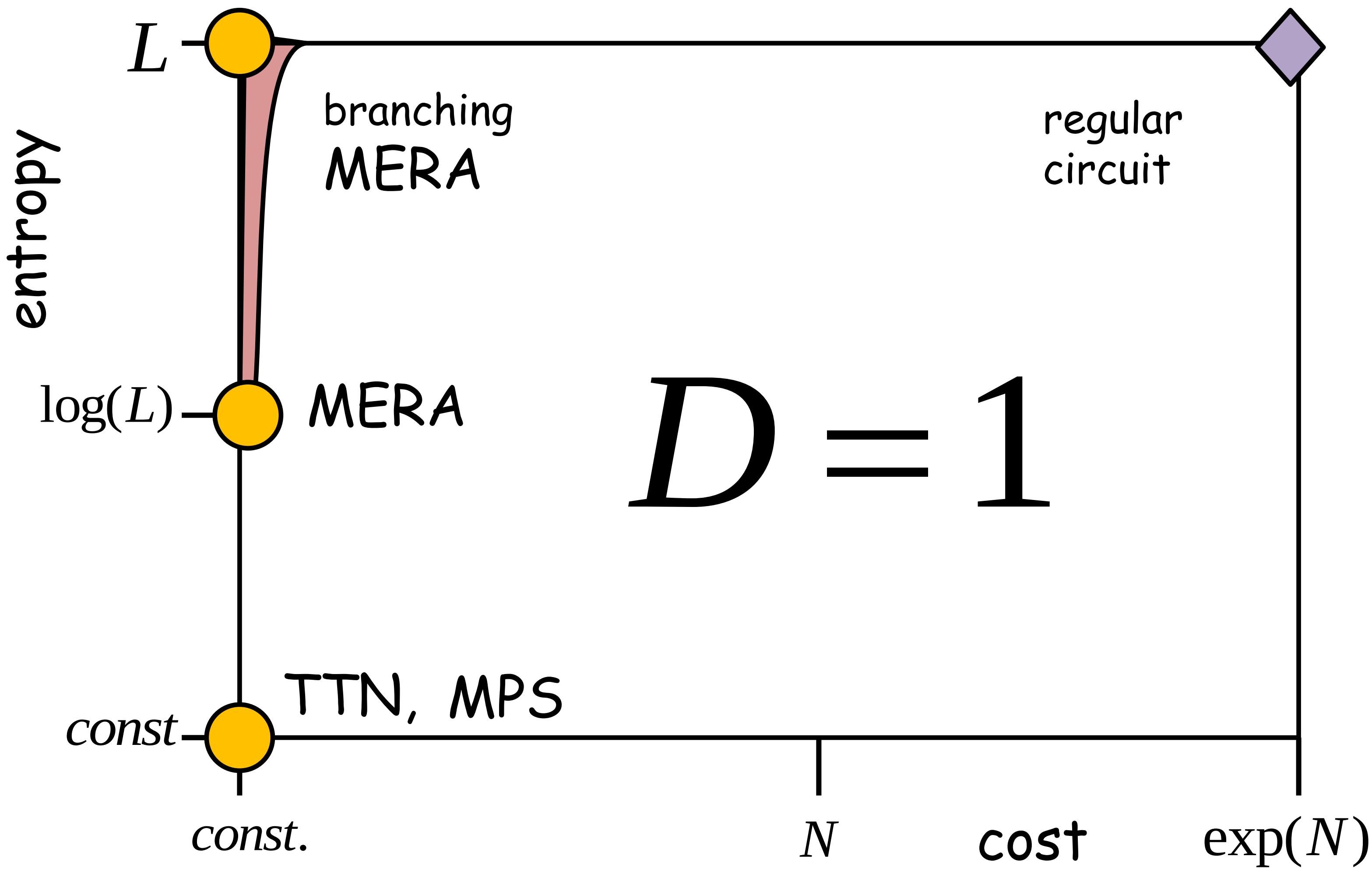
Glen Evenbly

let us add translation (+scale) invariance



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