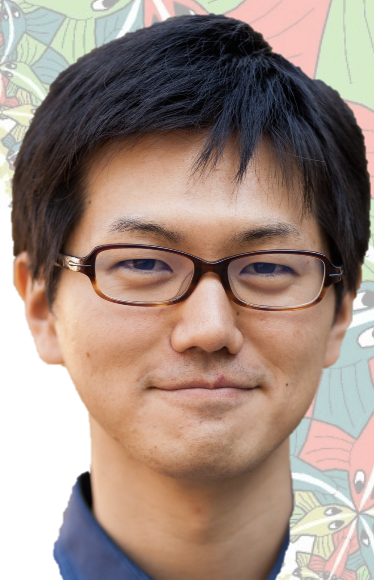
Holographic quantum error correcting codes

(an invitation to study AdS/CFT)

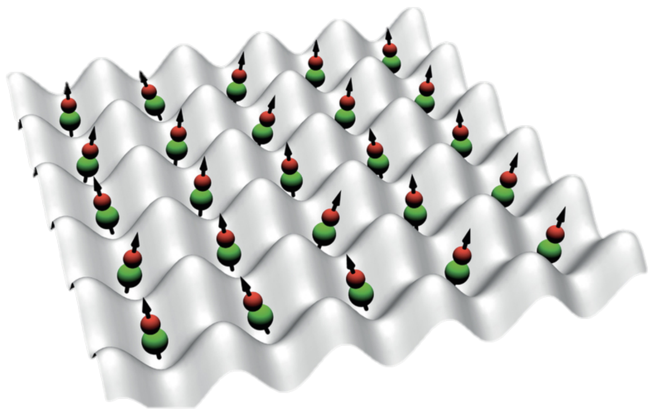
Fernando Pastawski @ Coogee 2015

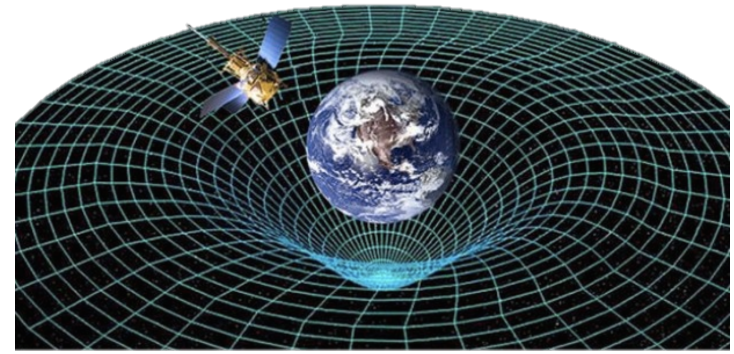
Joint work with: Beni Yoshida, Daniel Harlow & John Preskill



Motivation

- Construct novel (better ?) QECC



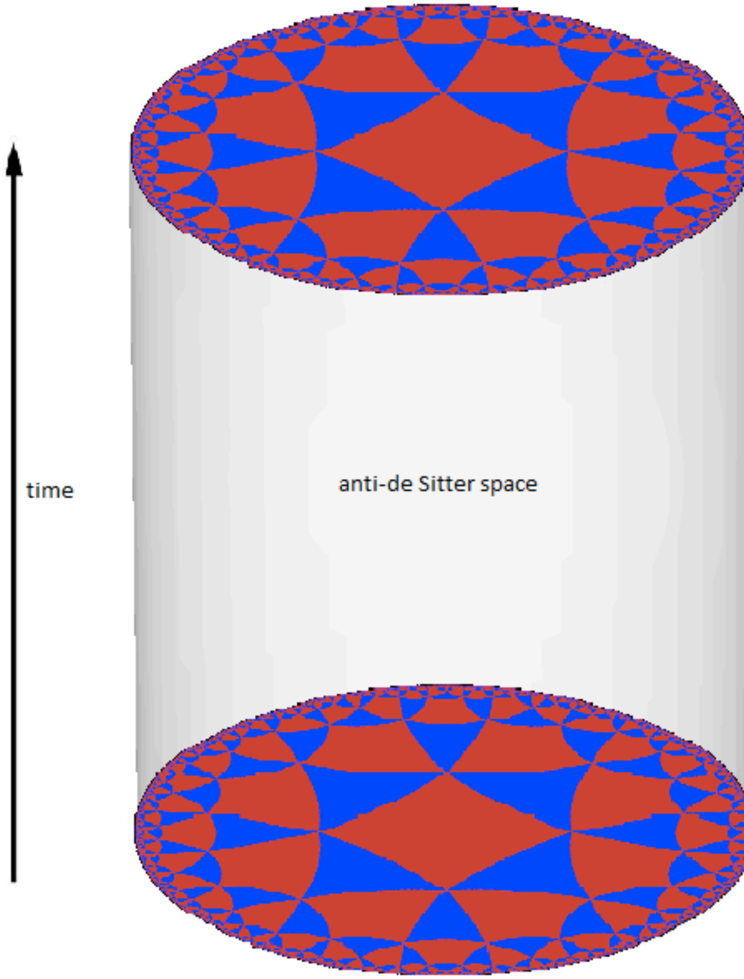


- Help high energy theorists understand appreciate quantum information.
- Help ~~me~~ quantum information theorist understand high energy theory.
- Solve fundamental open problems in physics.

Birds eye view of: AdS/CFT

Dictionary

AdS	CFT
Bulk	Boundary
Classical gravity	Quantum conformal field theory
Distance	Entanglement entropy
Curve length	Streaming information cost
Gravitational dynamics	Entanglement thermodynamics
Bulk fields	CFT operators



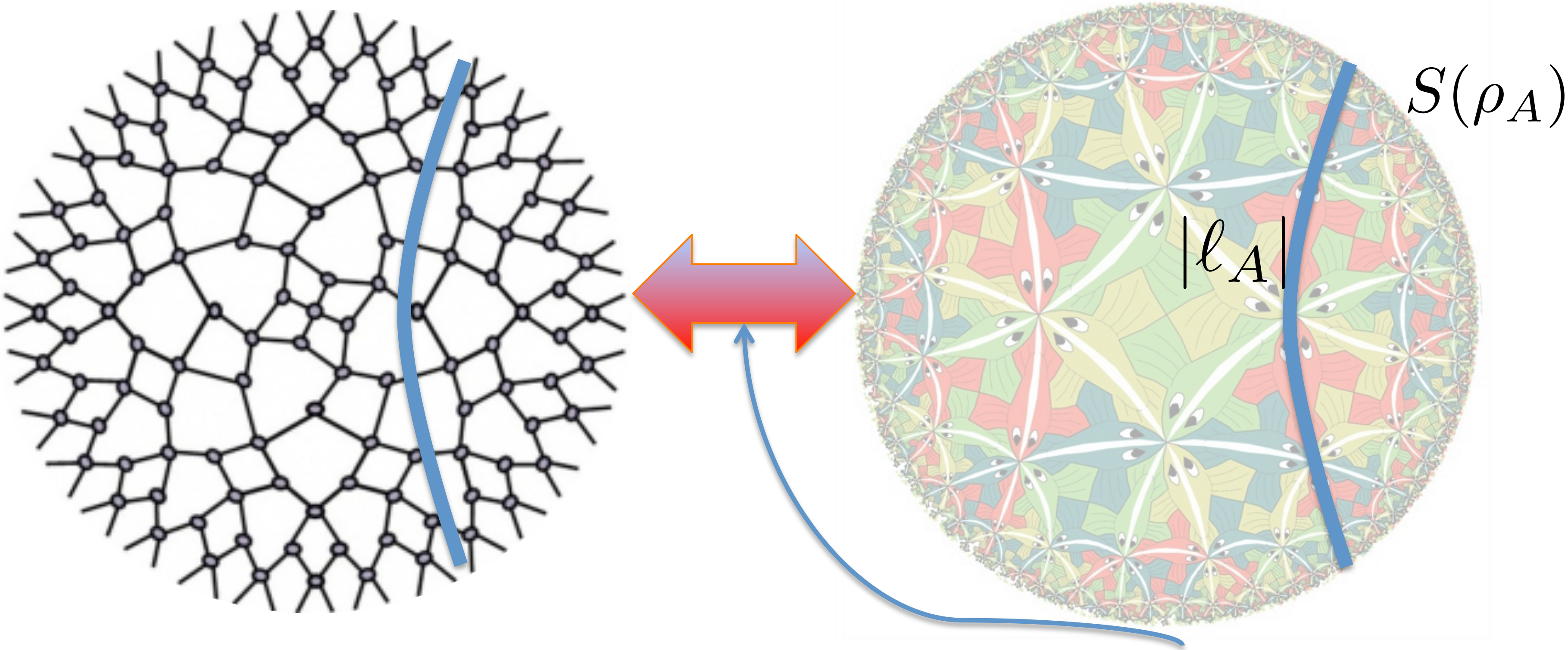
Ryu-Takayanagi

Holographic Derivation of Entanglement Entropy from AdS/CFT (2006)

Distance = entanglement entropy

MERA

AdS metric



Class of Quantum Many-Body States That Can Be Efficiently Simulated Guifre Vidal (2008)

Entanglement renormalization and holography Brian Swingle (2012)

Prepared for submission to JHEP

Bulk Locality and Quantum Error Correction in AdS/CFT

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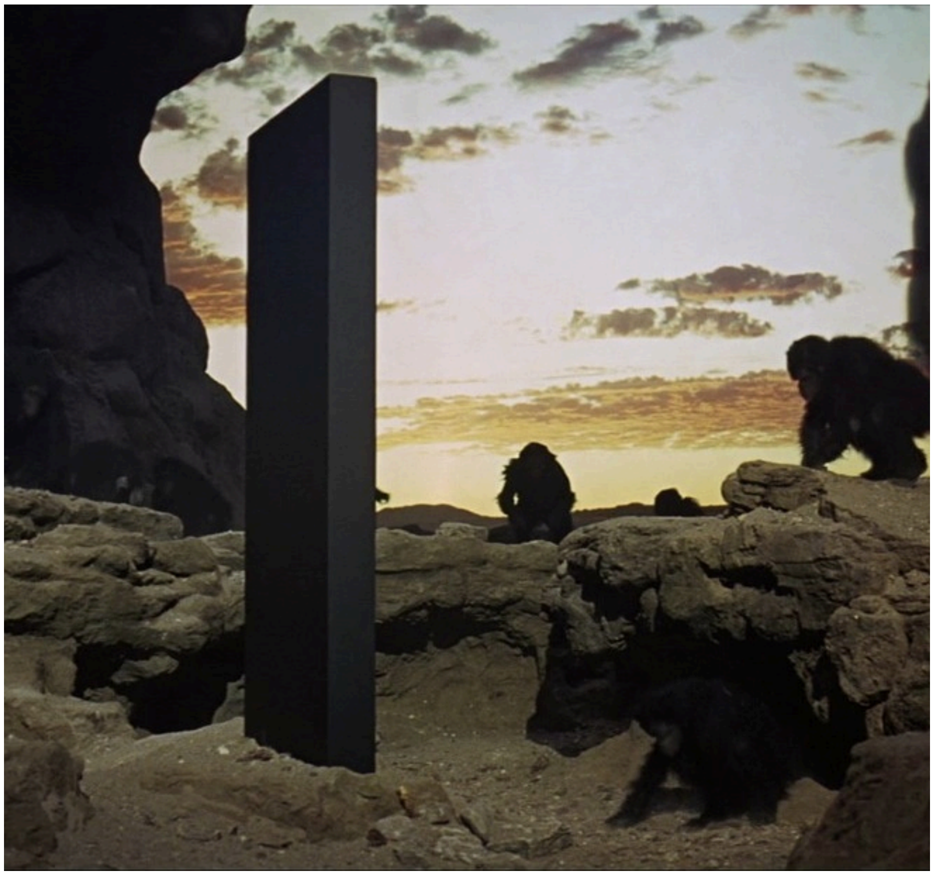
Abstract: We point out a connection between the emergence of bulk locality in AdS/CFT and the theory of quantum error correction. Bulk notions such as Bogoliubov transformations, location in the radial direction, and the holographic entropy bound

Post Shor Quantum Information

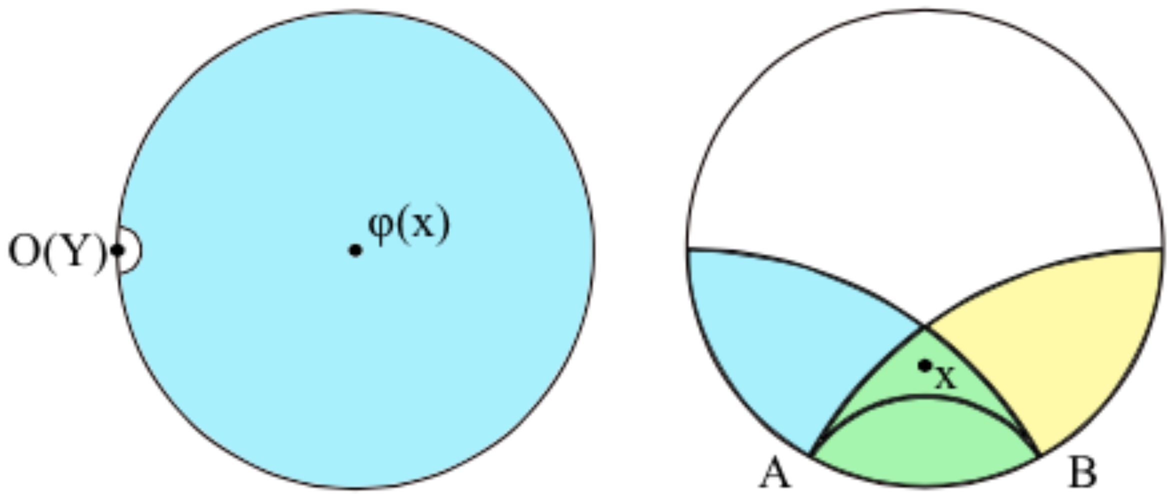


Entanglement entropy (1964)

Quantum Error Correcting Codes (1995)

AdS-Rindler reconstruction

$$\lim_{r\to\infty} r^{\Delta}\phi(r,x) = \mathcal{O}(x)$$

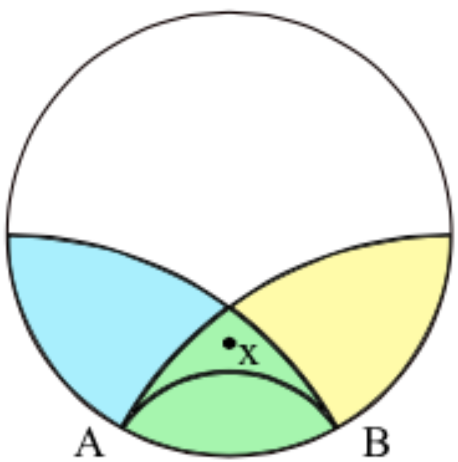


Multiple AdS-Rindler reconstructions on CFT

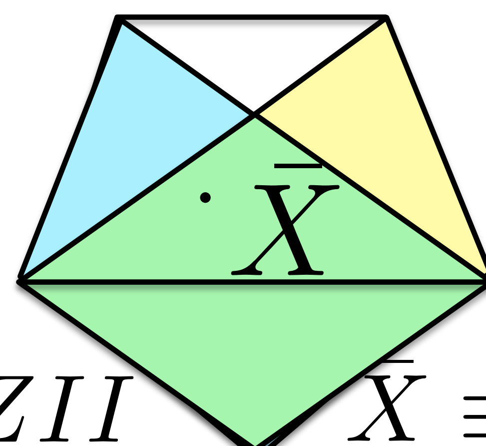
AdS-Rindler reconstruction

AdS/CFT	QECC
Bulk operators	Logical operators
CFT operator(s)	Physical operator(s)
AdS-Rindler reconstruction	Cleaning lemma

AdS -> CFT



$$|\psi\rangle \rightarrow [[5,1,3]](|\psi\rangle)$$



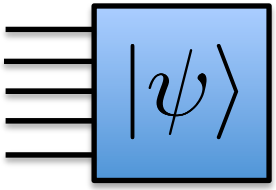
$$\bar{X} \equiv ZXZII \qquad \bar{X} \equiv IZXZI$$

Holographic QECC

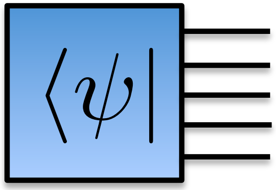
- They have a MERA like structure
- They can realize stabilizer QECC
- They respect bulk locality for physical representations of logical operators.
- Generalize concatenated codes.
- Simple interpretation of cleaning.
- They allow for flexibility in the
  - Lattice realization (shape + curvature)
  - Arrangement of logical inputs.

Tensors

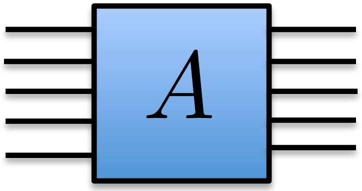
- Ket



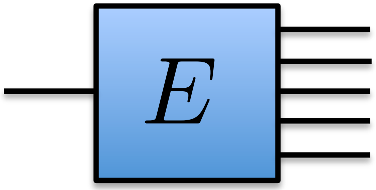
- Bra



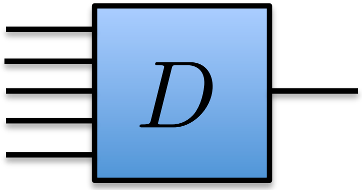
- Operator



- Encoder



- Decoder



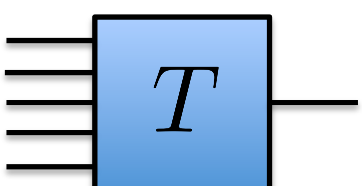
Stabilizer codes = stabilizer states

$$|\psi\rangle \to [[5,1,3]](|\psi\rangle) \qquad |\psi\rangle \to T|\psi\rangle$$

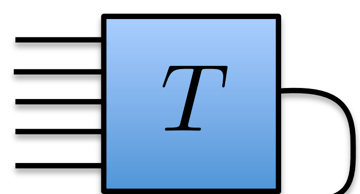
$$P \in \mathcal{S} \Rightarrow T|\psi\rangle = PT|\psi\rangle$$

$$\mathcal{S} = \left\langle \begin{array}{lll} XZZXI, & IXZZX, & XIXZZ \\ ZXIXZ & & \end{array} \right\rangle$$

$$P \text{ implements } L \;\Rightarrow\; T|\psi\rangle = P^{\dagger}TL|\psi\rangle$$



$[[5,1,3]]$



$[[6,0,4]]$

Maximally entangled = Unitary

- [[n,0,d]] means any (d-1) shares are maximally entangled with the rest.
- [[2n,0,n+1]] means maximally entangled along any balanced bipartition!
- Always proportional to a k particle unitary!

Not that special

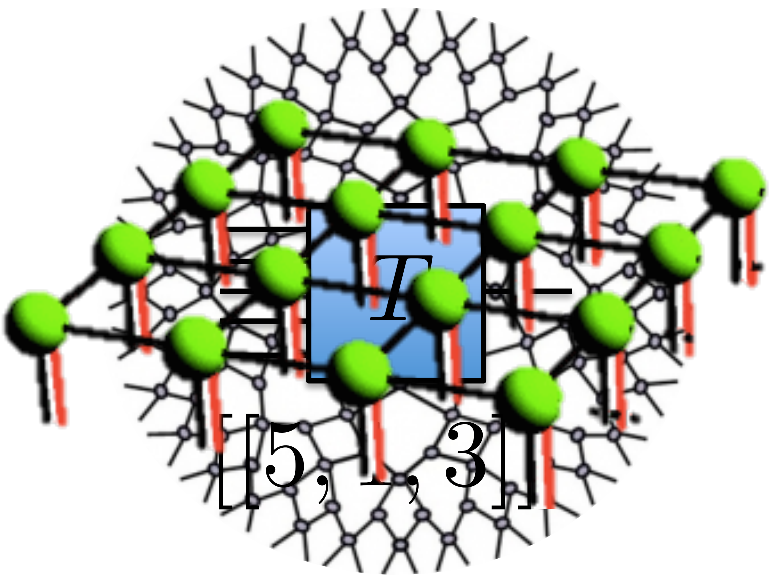
- Canonical typicality:

  Average entanglement for high dimensional states is close to maximal.

Most states represent tensors which are approximately like a unitary.

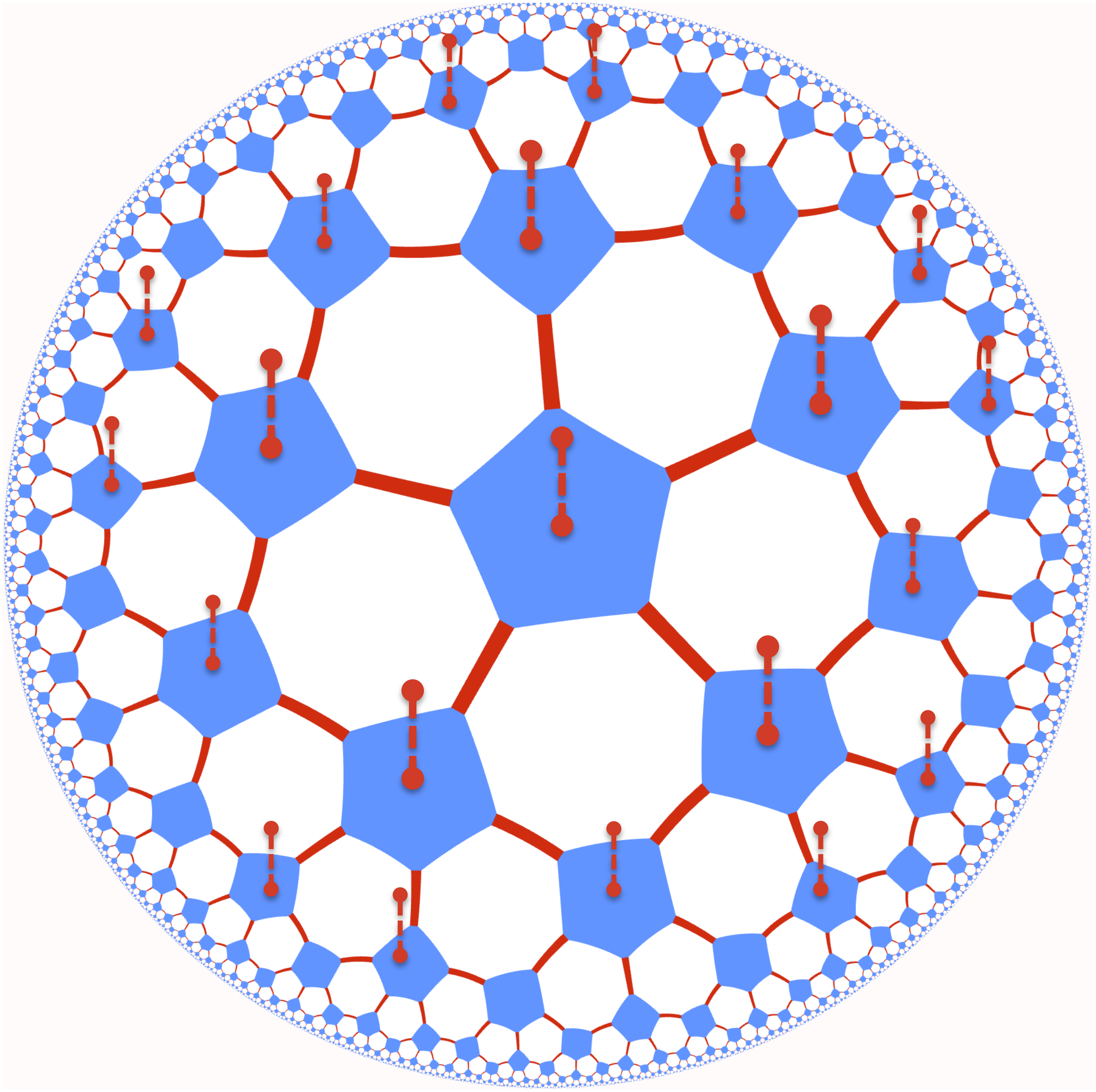
Stabilizer states provide exact realization in low dimension.

The mixer

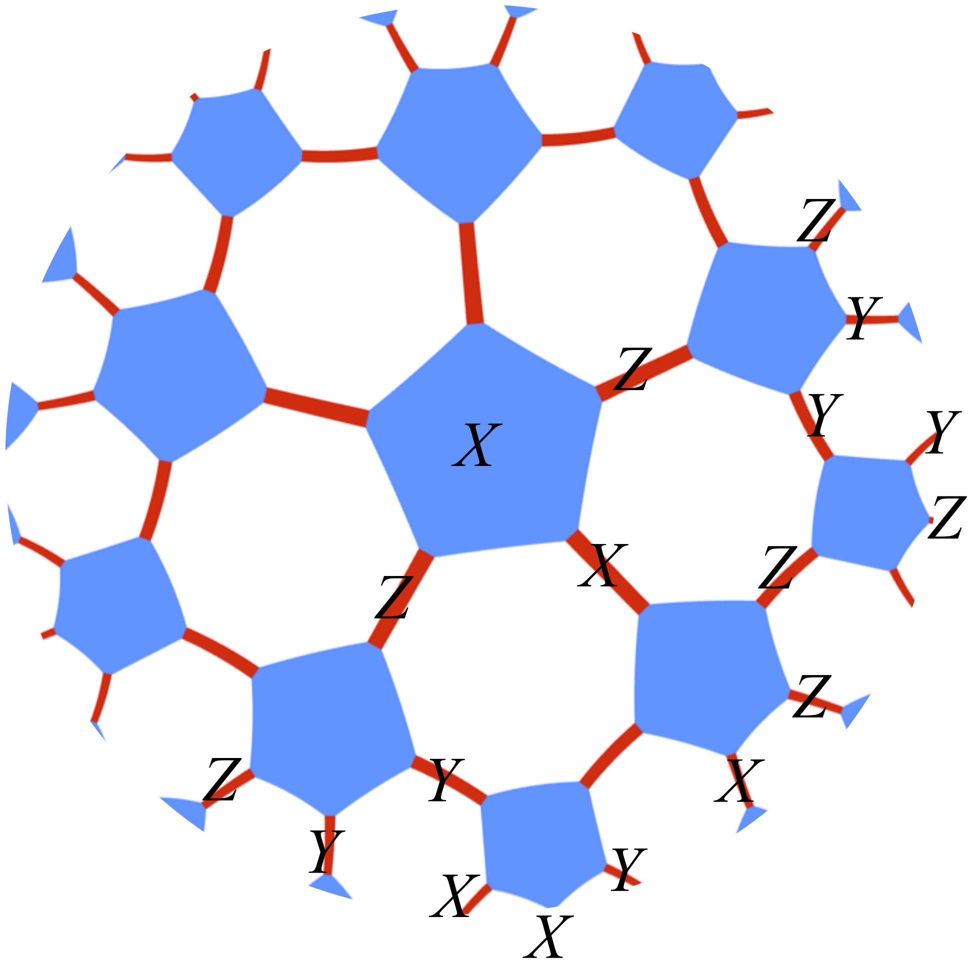


Holographic QECC

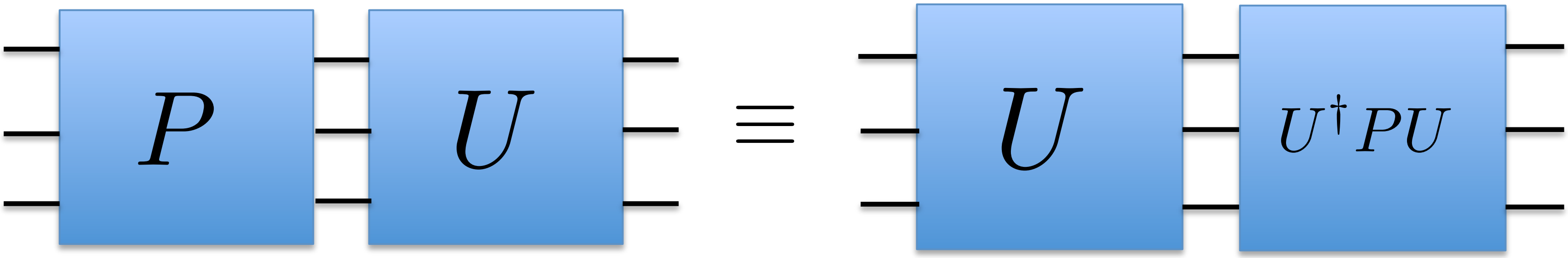
Holographic QECC



Holographic QECC
(Pauli pushing)

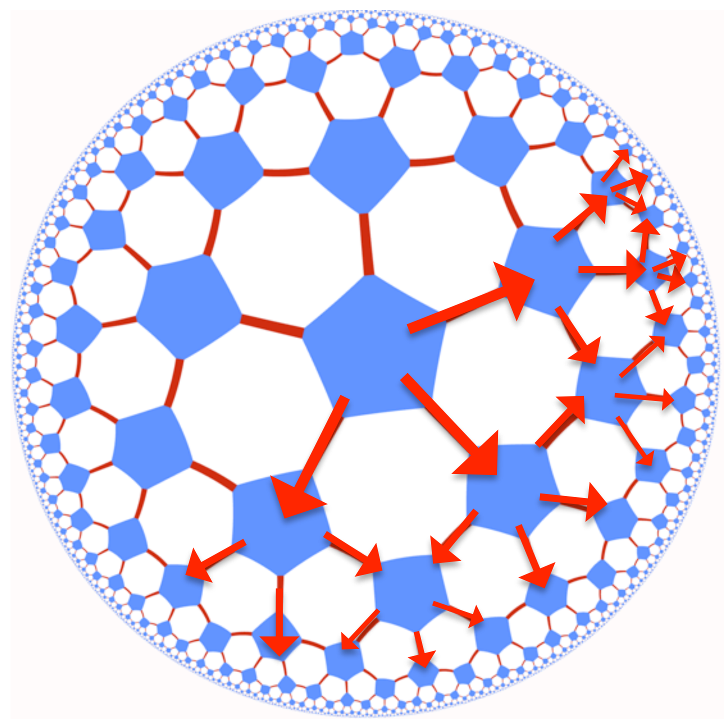


We can always do this!

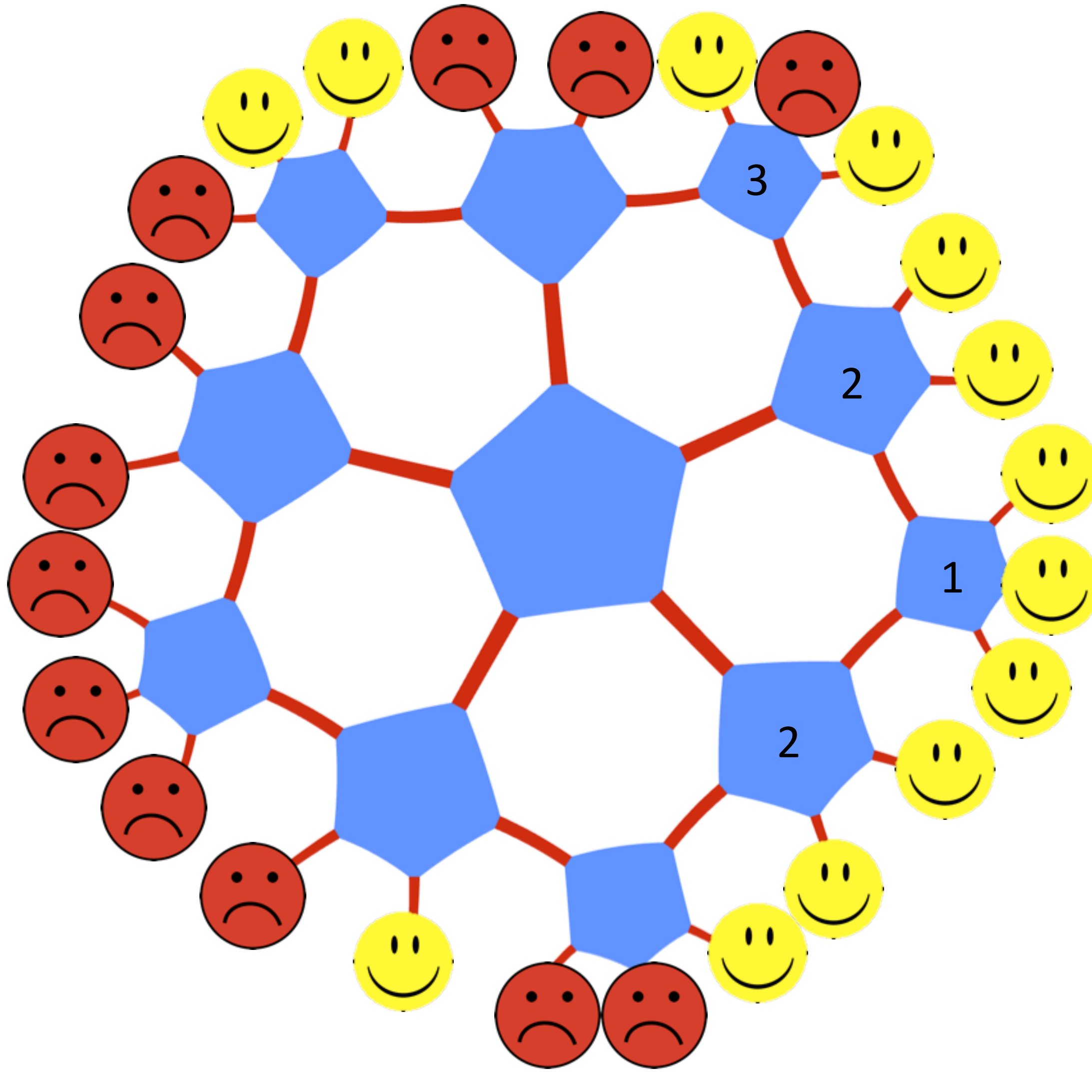


Stabilizer states push like Cliffords!

Hyperbolic lattice
=>
More out than in.



Erasure recovery

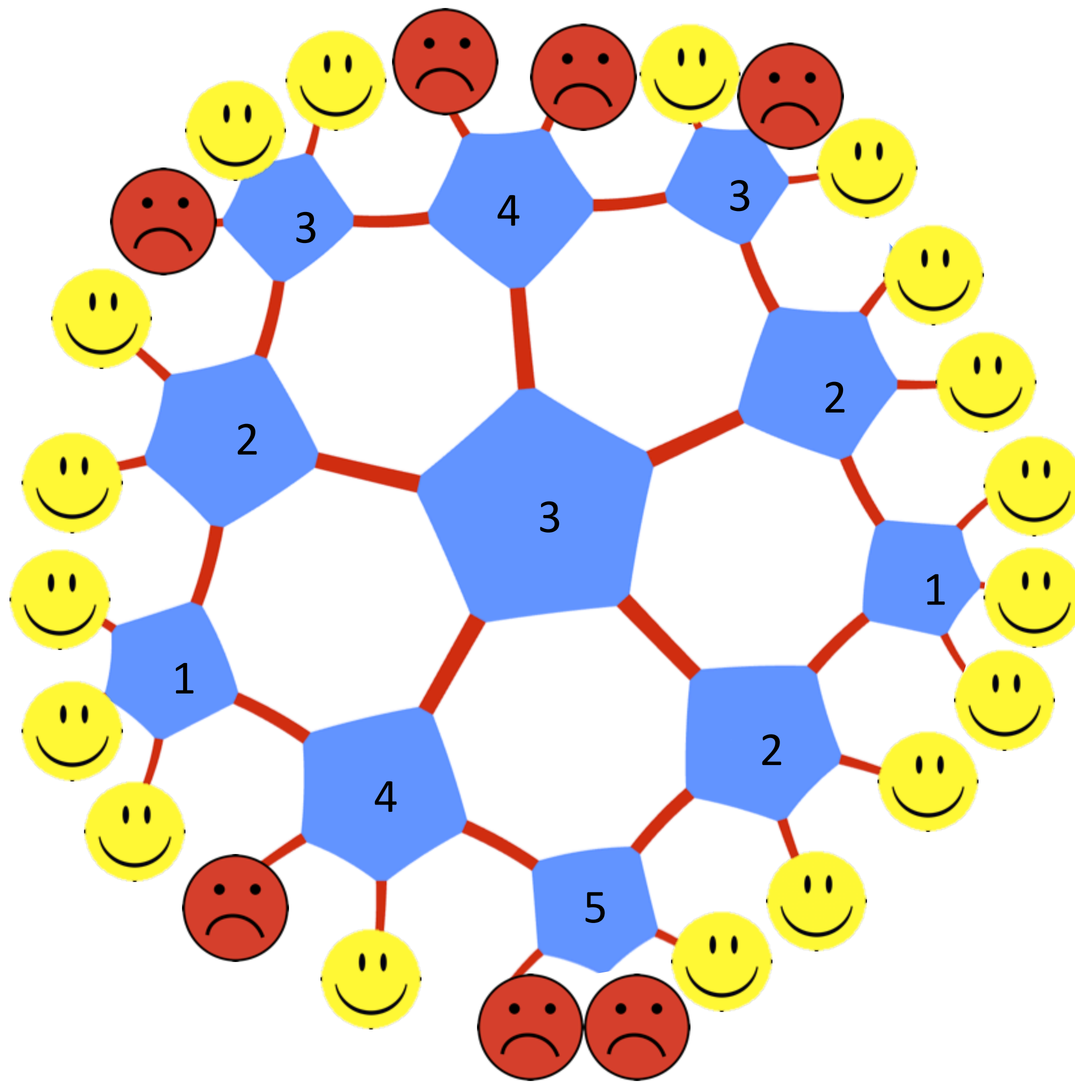


Greedy geodesic algorithm

Data: $A \triangleq$ Boundary region
Result: $g_A \triangleq$ Greedy geodesic of A
Result: $R_A \triangleq$ Region between A and g_A

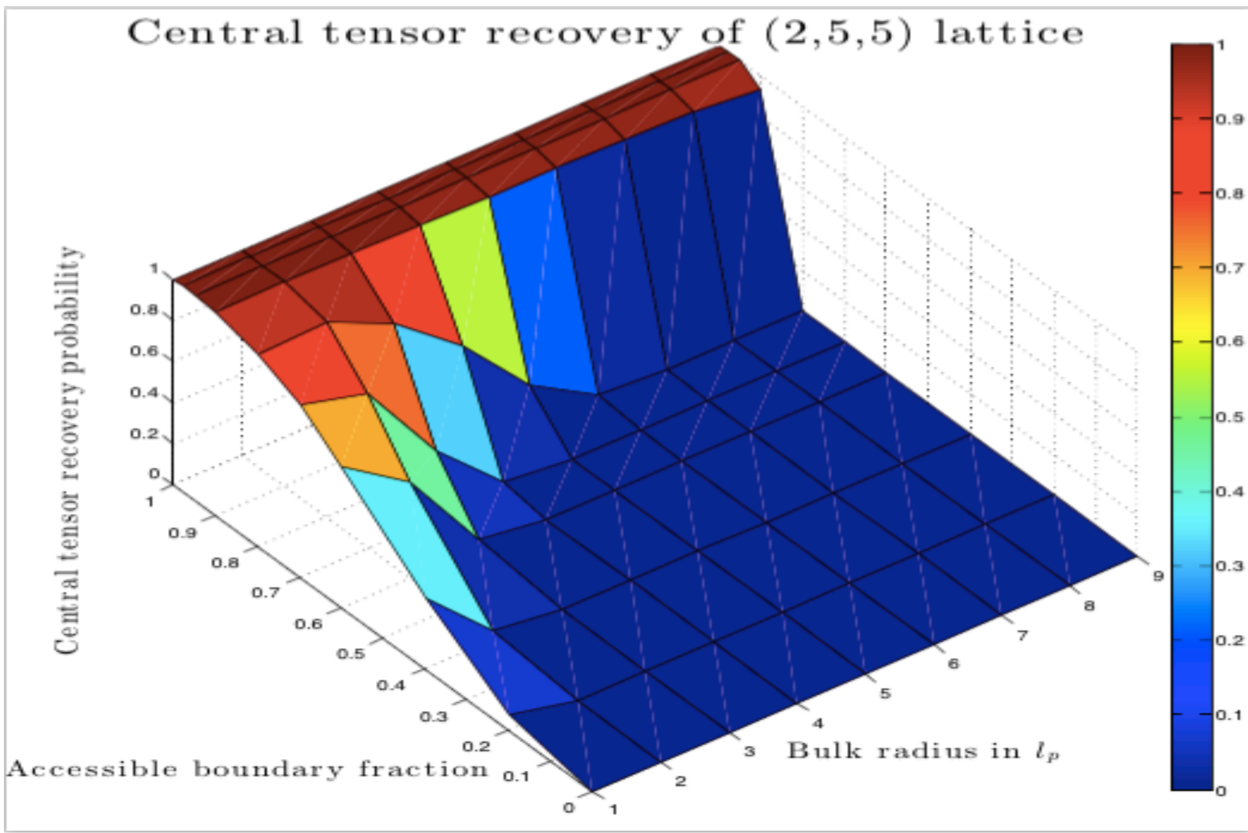
$g_A := A$;
$R_A := \emptyset$;
while $\exists T : T \notin R_A$ *and* $|\partial T \cap g_A| \geq |\partial T|/2$ **do**
  $g_A := g_A \oplus \partial T$;
  $R_A := R_A \cup T$;
end
return g_A

Erasure recovery

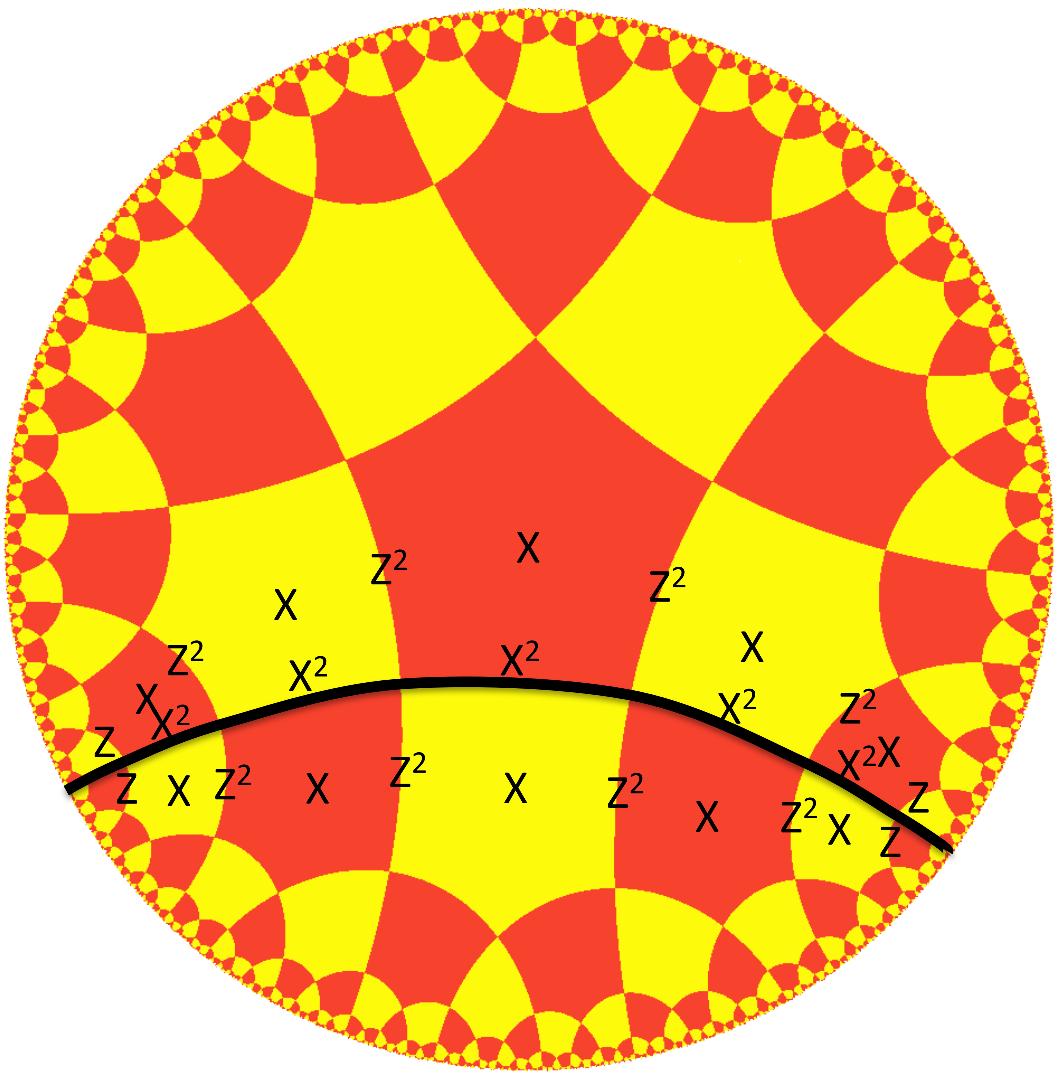


Code property checklist

- Does the central qubit have a threshold?
- Code distance = (3 for suburban logicals)



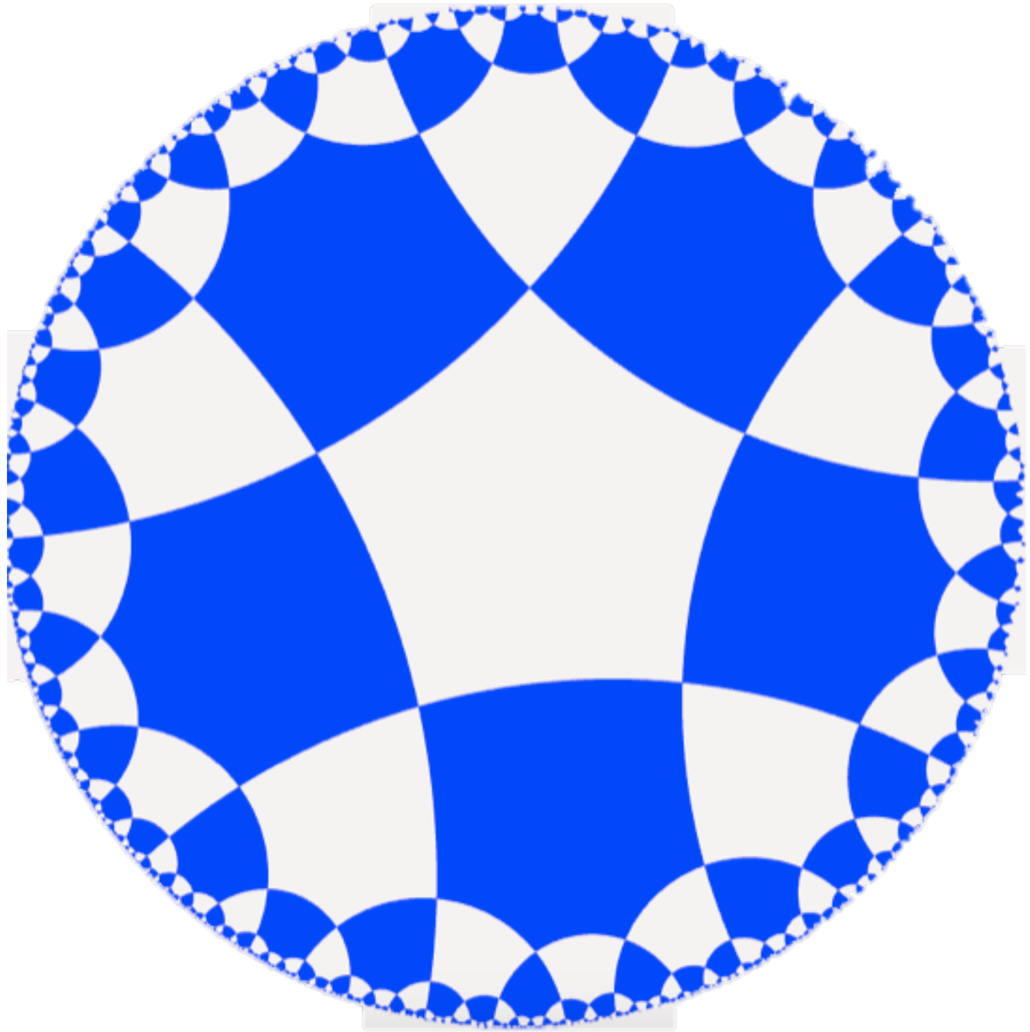
Weight 4 logical ops. affecting downtown logicals



Digression to Kaleidotile

(on drawing more than 6 polygons on a hyperbolic lattice)

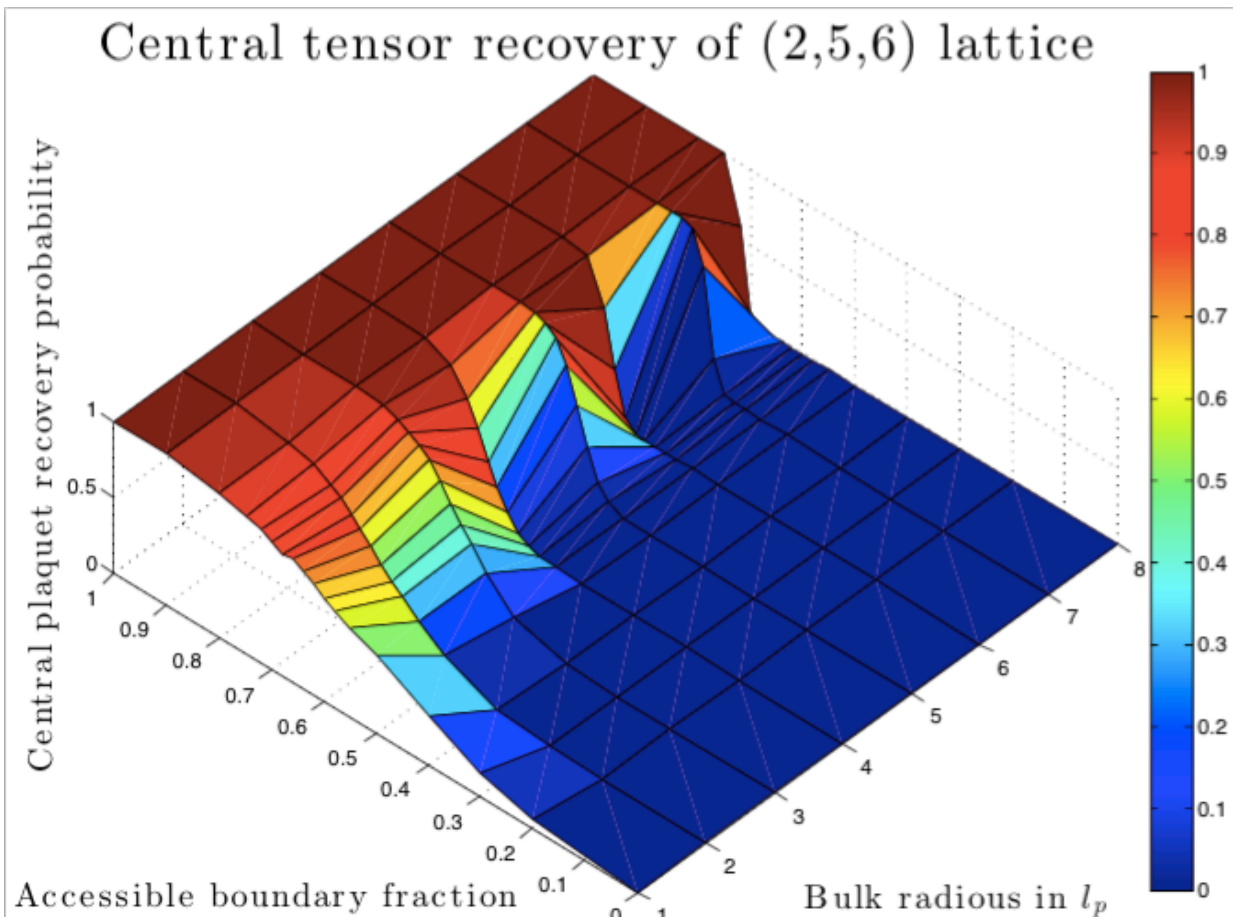
Make the code less dense



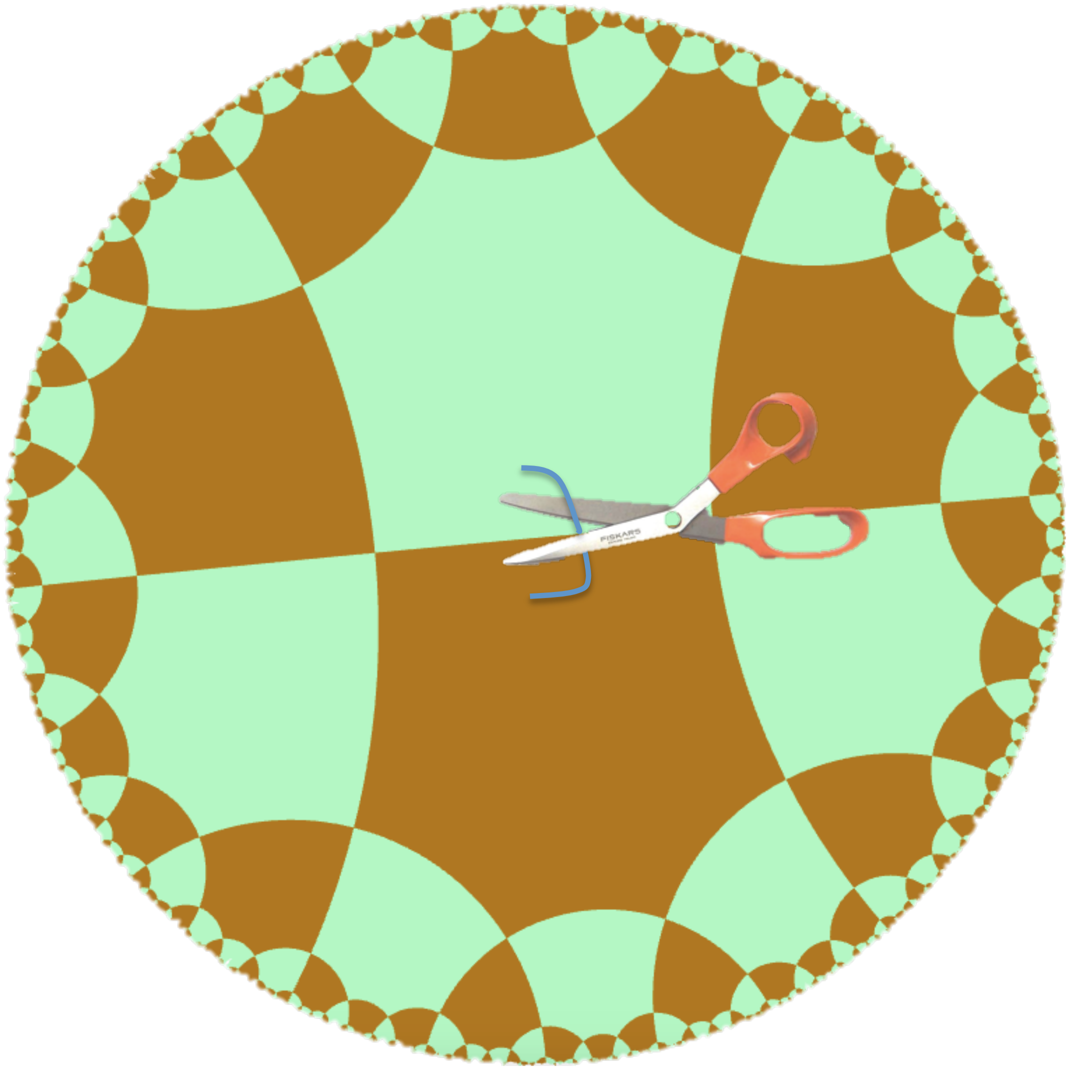
Pentagons and hexagons (4 polygons per vertex)

Code property checklist

- Erasure threshold with a n.n. correlated noise.
- Numerical erasure threshold for greedy recovery.

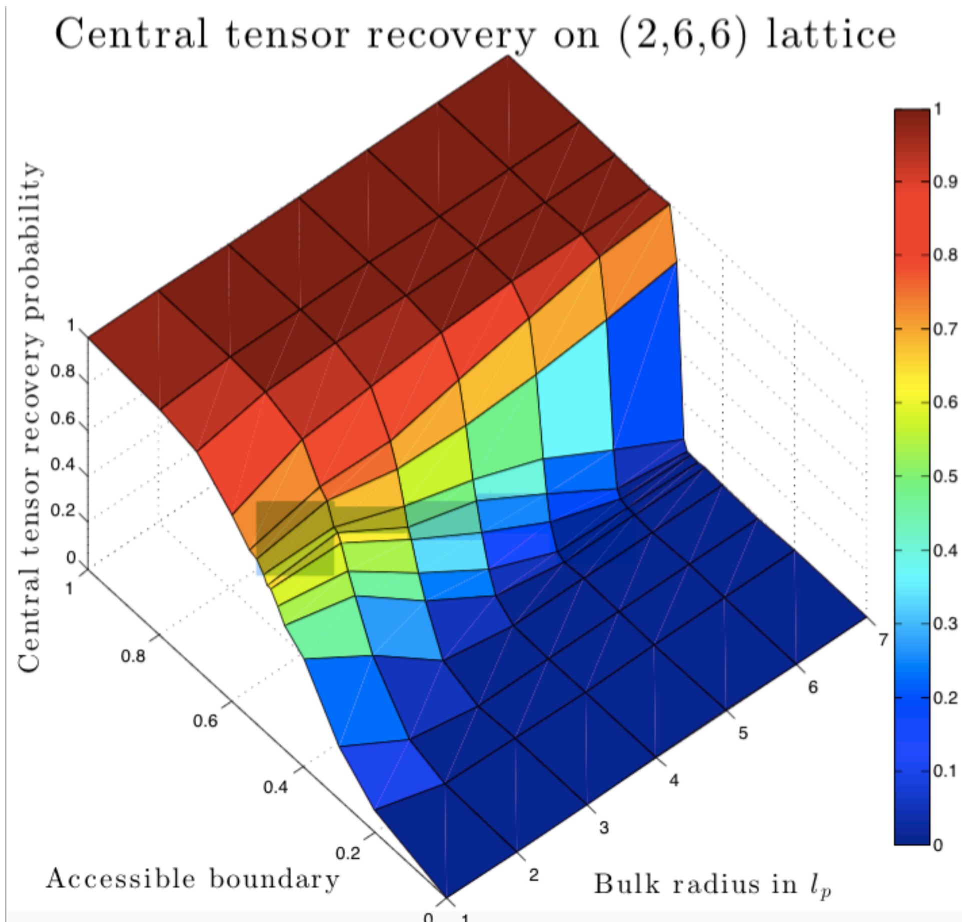


Arbitrarily high erasure threshold?

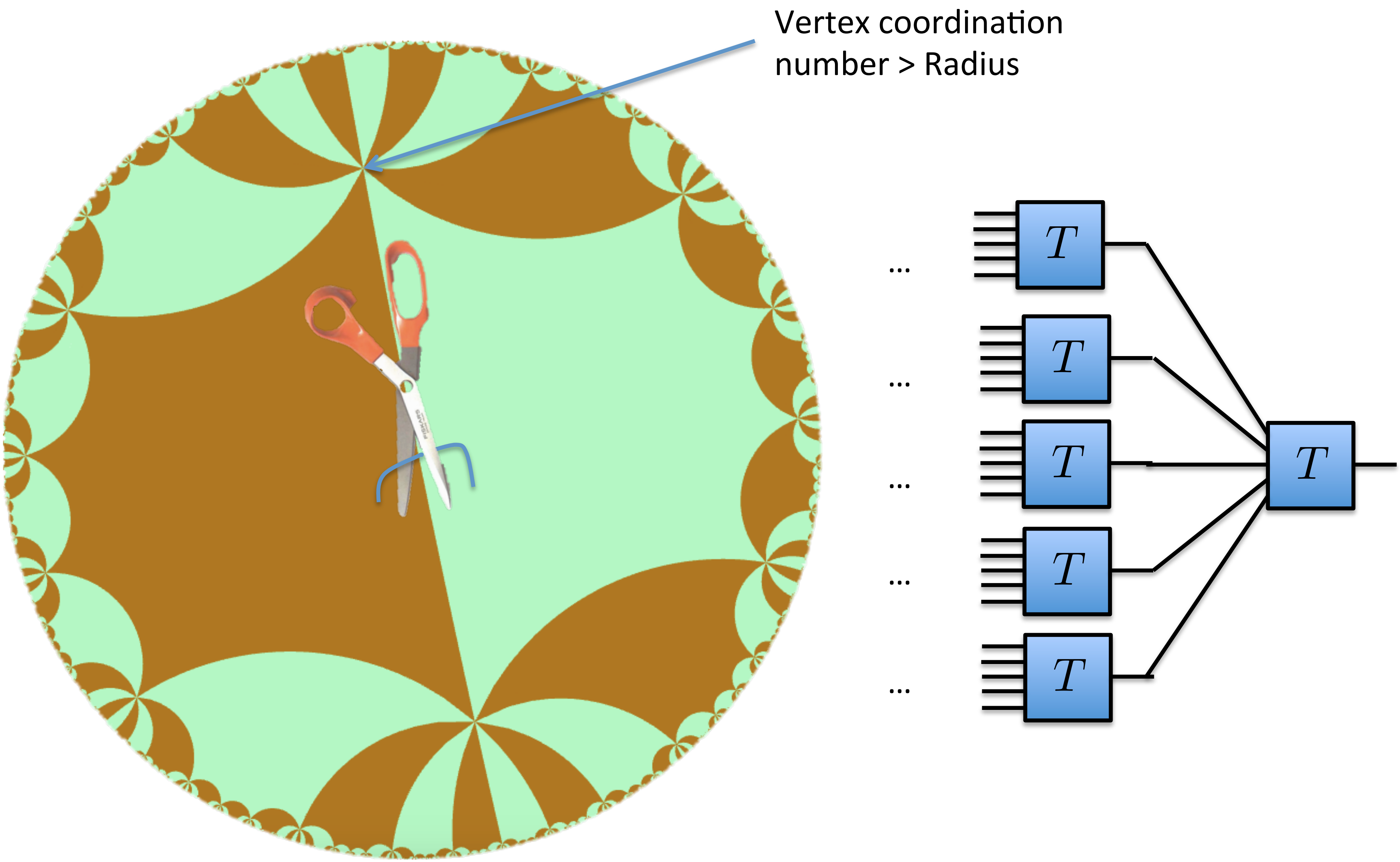


Almost optimal threshold

- Numerical greedy recovery threshold ~0.52.
- Actual threshold of 0.5?



The concatenated code limit



How general can we be?

- [[6, 0, 4]]$_2$ -> [[2k, 0, k+1]]χ
  Exists for any k if $\chi > O(k^{1/2})$
- Arbitrary lattice.
- Any negative curvature?
- Limit the ~~number~~ density of bulk legs.

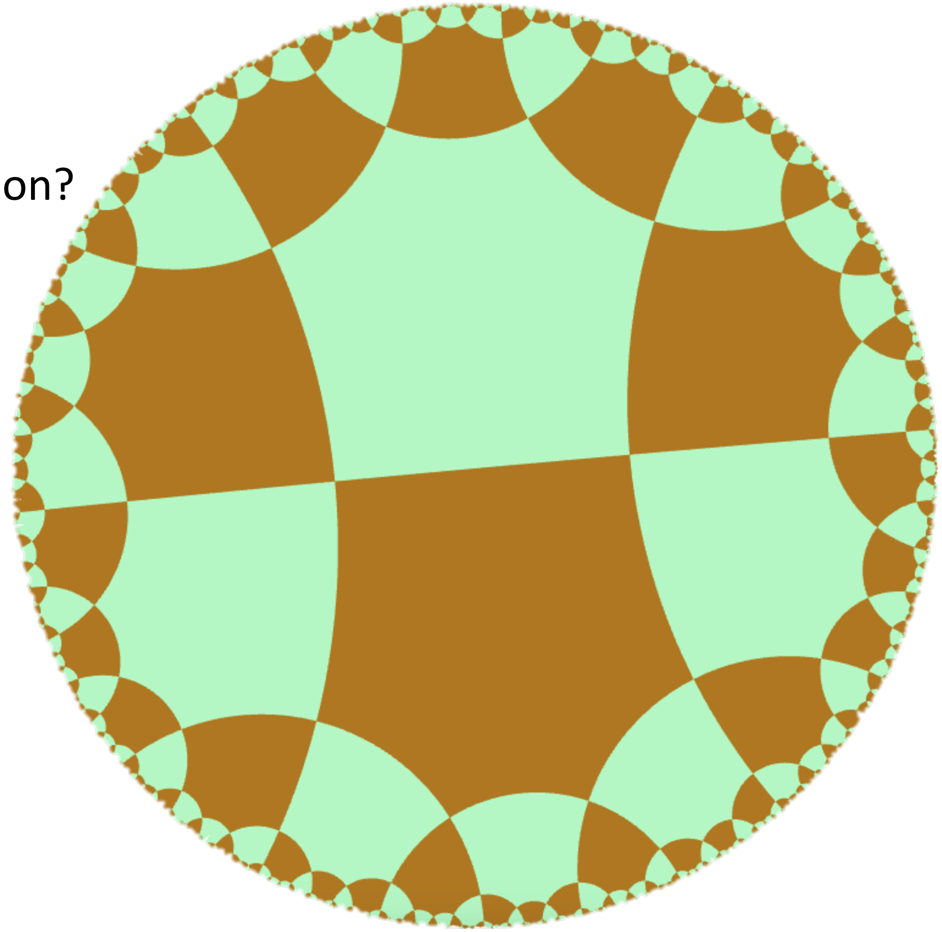
What is the right vacuum?

Holographic state

No bulk/logical legs.

What is the normalization?

Does it satisfy Ryu-Takayanagi?



Ryu-Takayanagi -> Entanglement entropy = length of bulk geodesics.

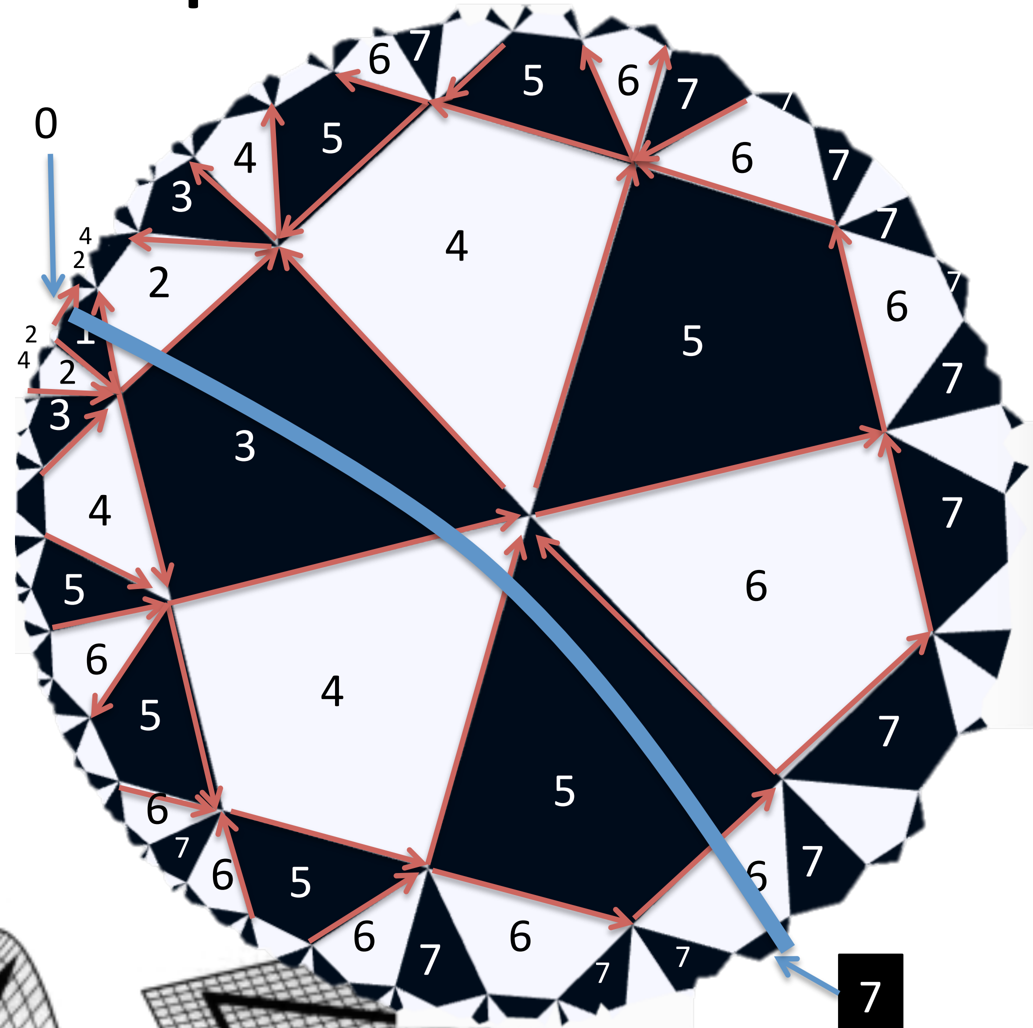
Circuit interpretation

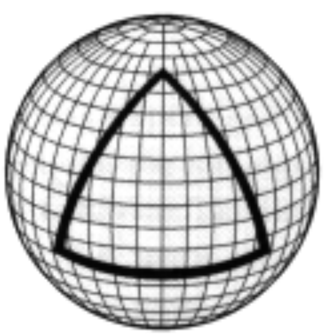
Flux: #Incoming = #Outgoing 

DAG: Directed acyclic graph 
No time-like curves

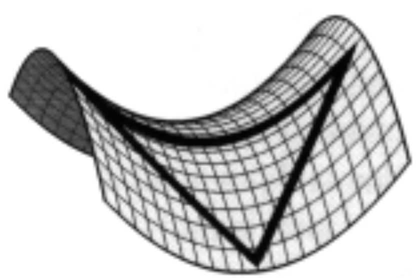
No local (bulk) maxima for distance

Non-positive curvature

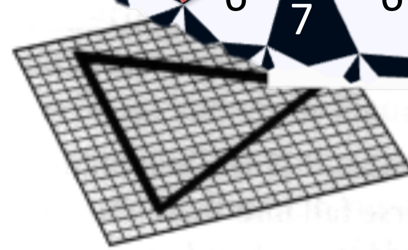




Positive Curvature



Negative Curvature



Flat Curvature

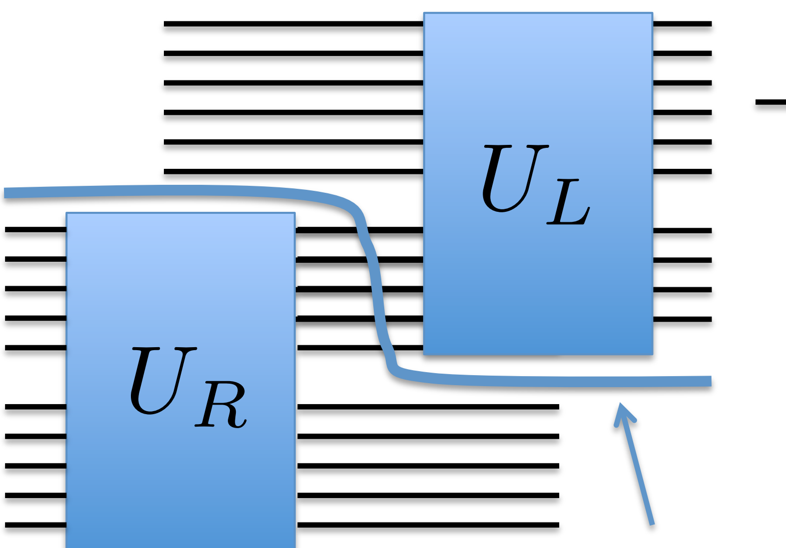
Holographic state

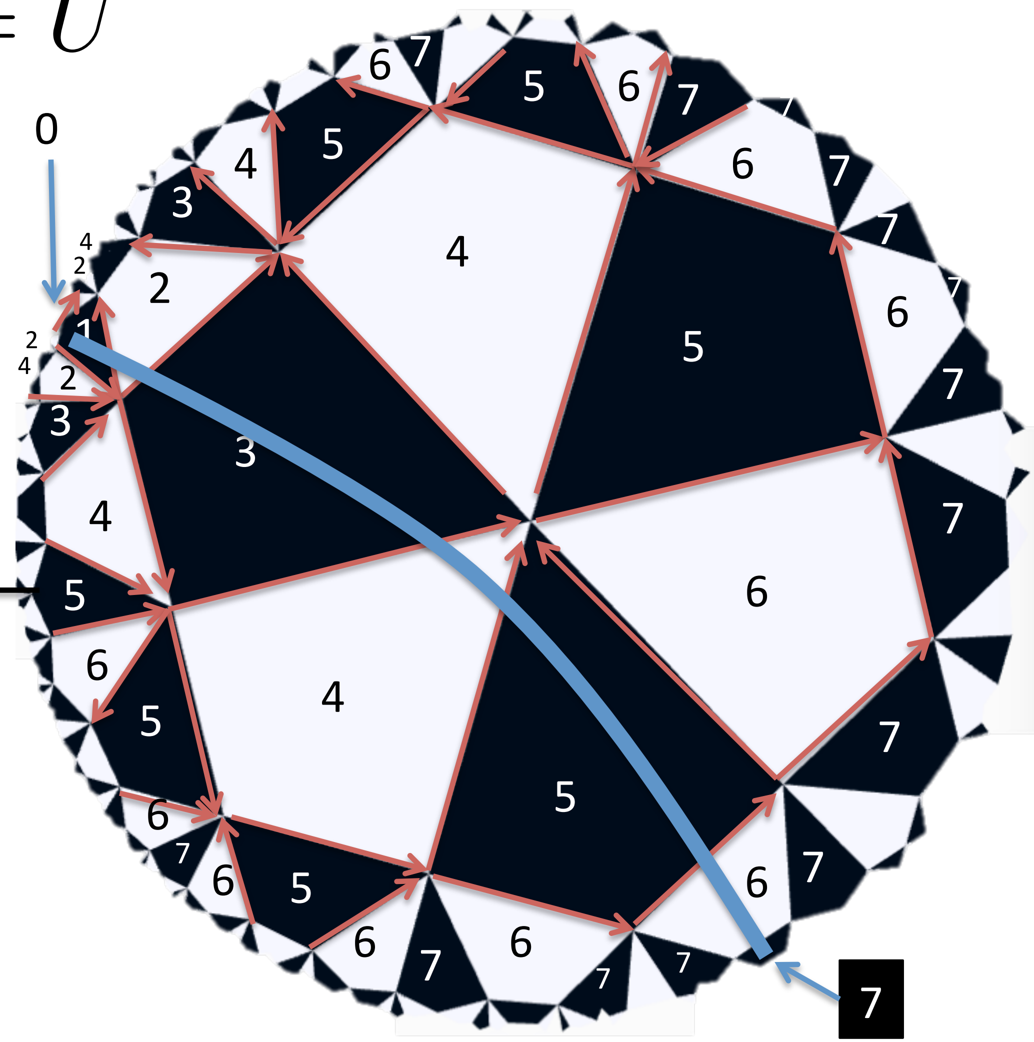
No bulk/logical legs.

Is it normalized state? $T = U$

Does it satisfy Ryu-Takayanagi? Exactly!

$$T = U = U_L U_R$$

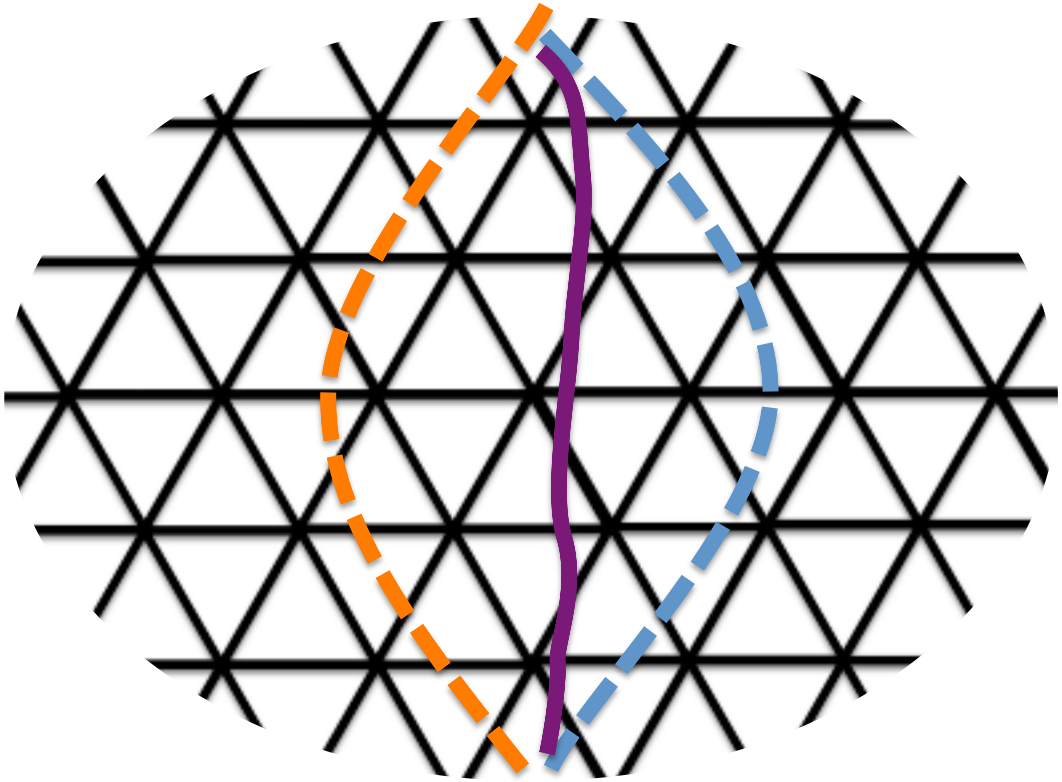




Geodesics and the greedy algorithm

Greedy geodesics region R_A for A contains all simple geodesics.

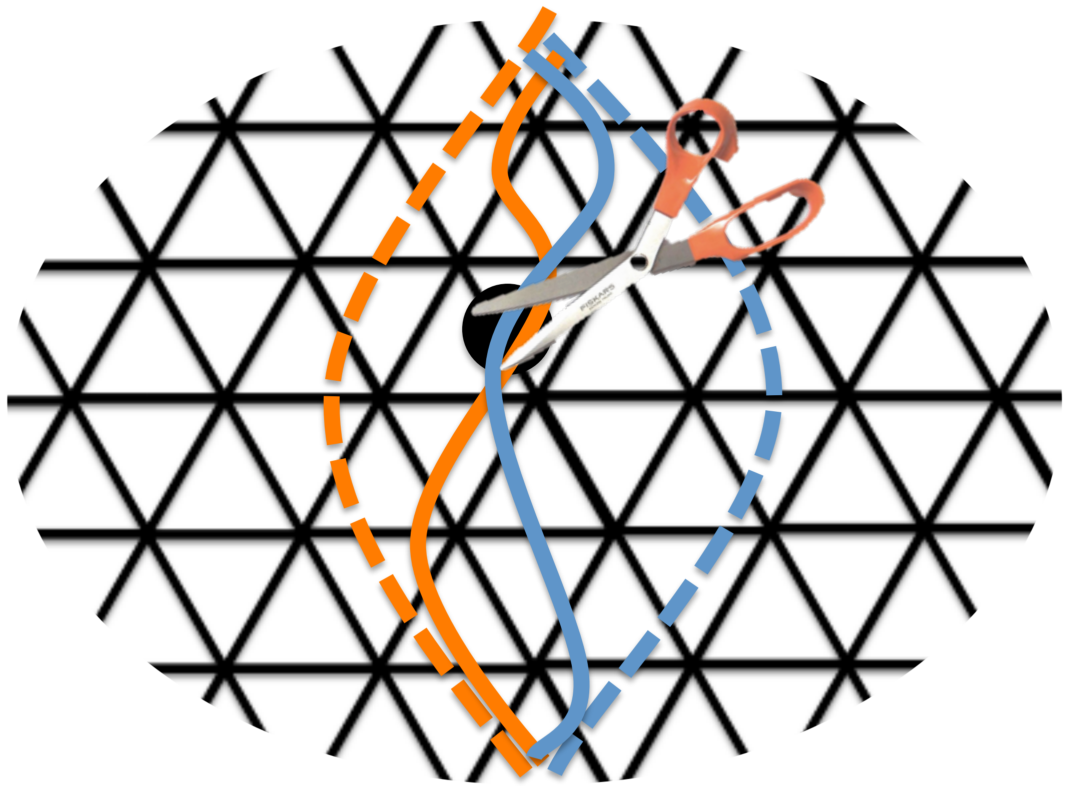
Proof: For holographic states, just follow the arrows.



Greedy geodesics recede with cuts

Greedy geodesics recede with cuts.

Proof: $R_A \cap R_{\bar{A}} \cap R_L = \emptyset$



Give me the stabilizers!

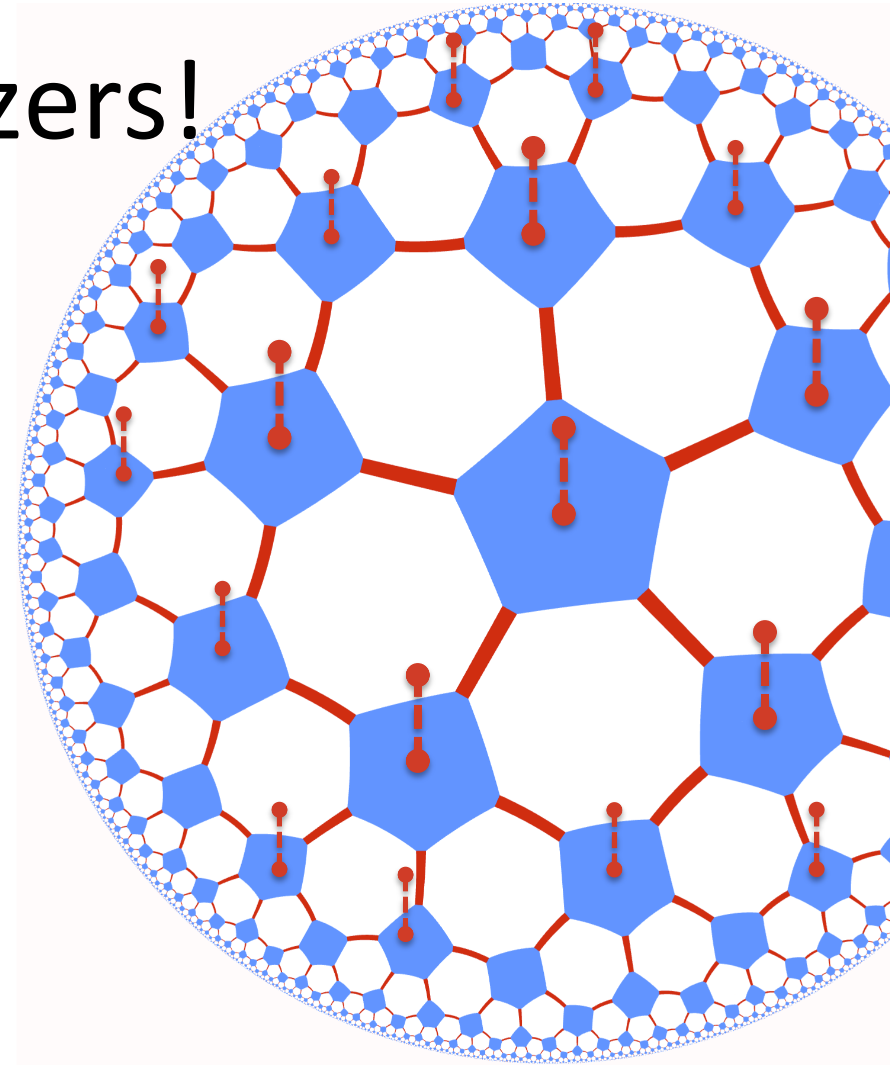
Holographic code: has bulk/logical legs.

Is the encoder normalized? 

If we can build a DAG-Flux with all bulk as input.

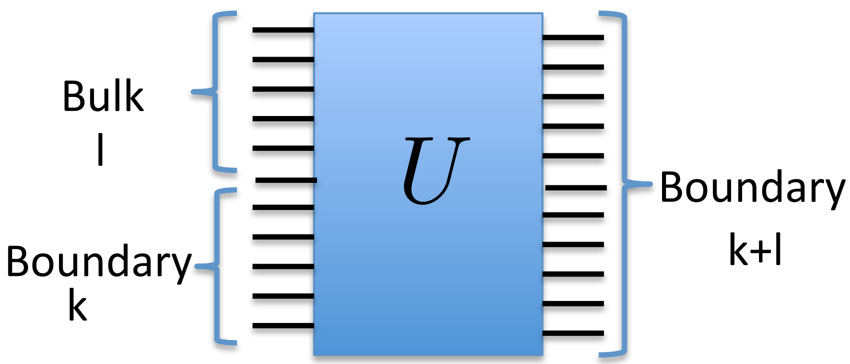
$$T = U$$

Iff greedy algorithm succeeds.



|Boundary|=2k+l

|Bulk|=l



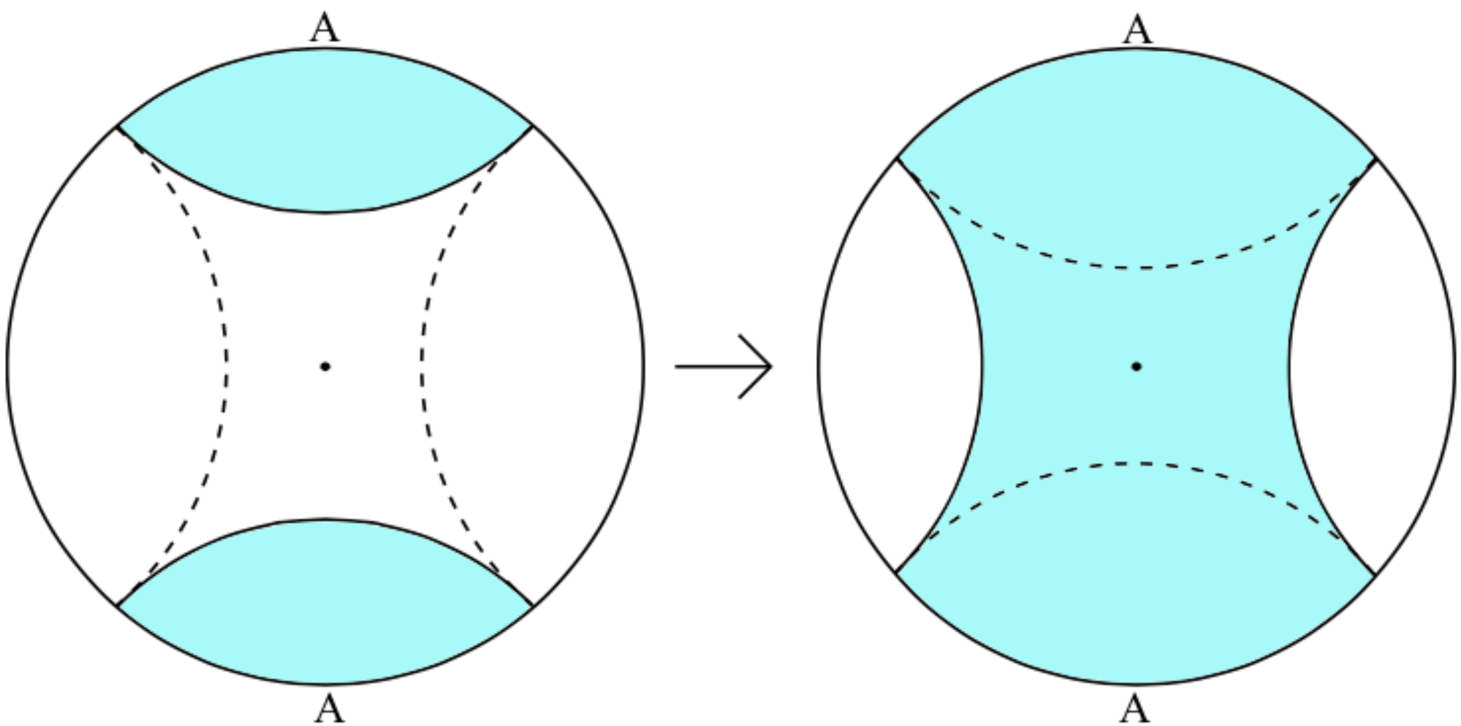
$$\mathcal{S} = \langle X_j \otimes U^\dagger X_j^\dagger U, Z_j \otimes U^\dagger Z_j^\dagger U \rangle$$

When does the greedy algorithm fail?

Finding multiregion minimal geodesic

Greedy algorithm
≈
Causal wedge

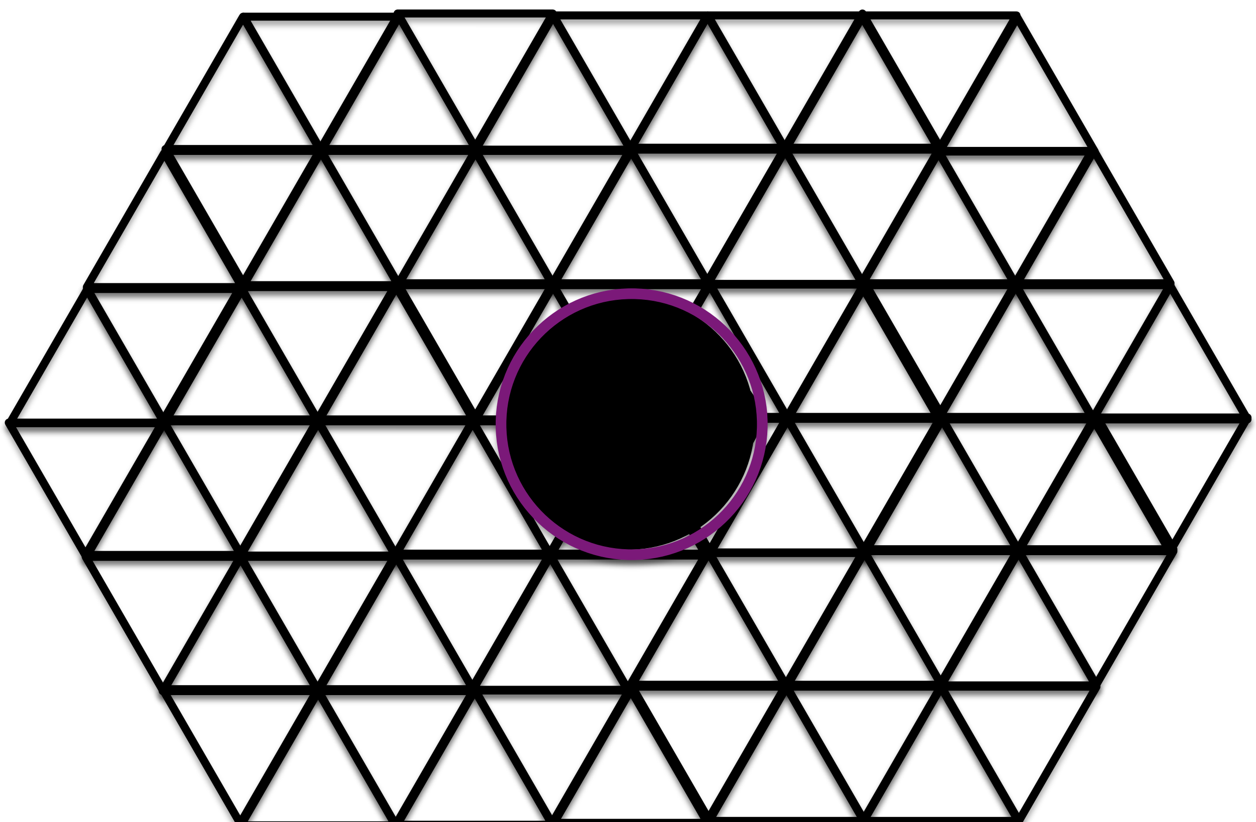
Optimal algorithm
≈
Entanglement wedge



Black holes and Bekenstein-Hawking

Consider the perimeter of an encoded region

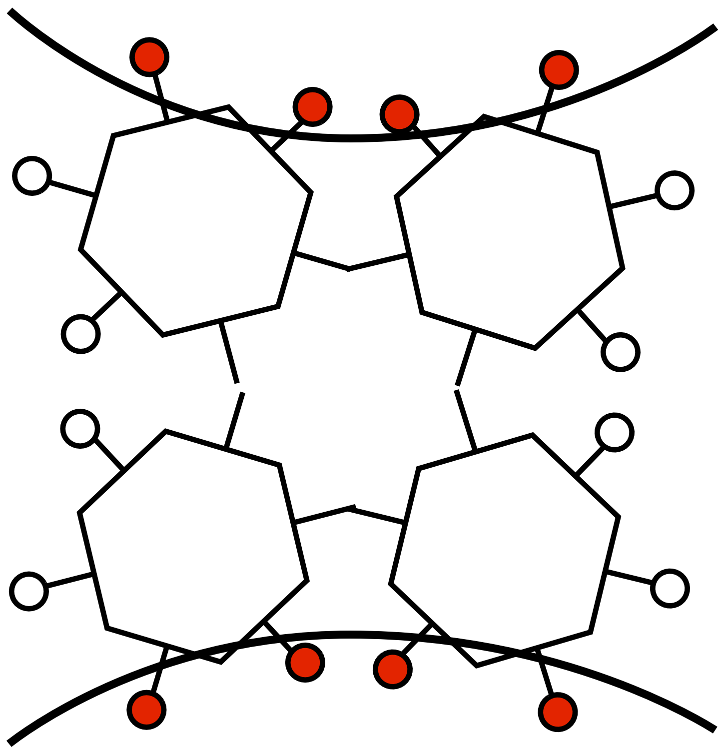
$$|R \cap R_L| \leq \partial R$$



Ryu-Takayanagi Corrections

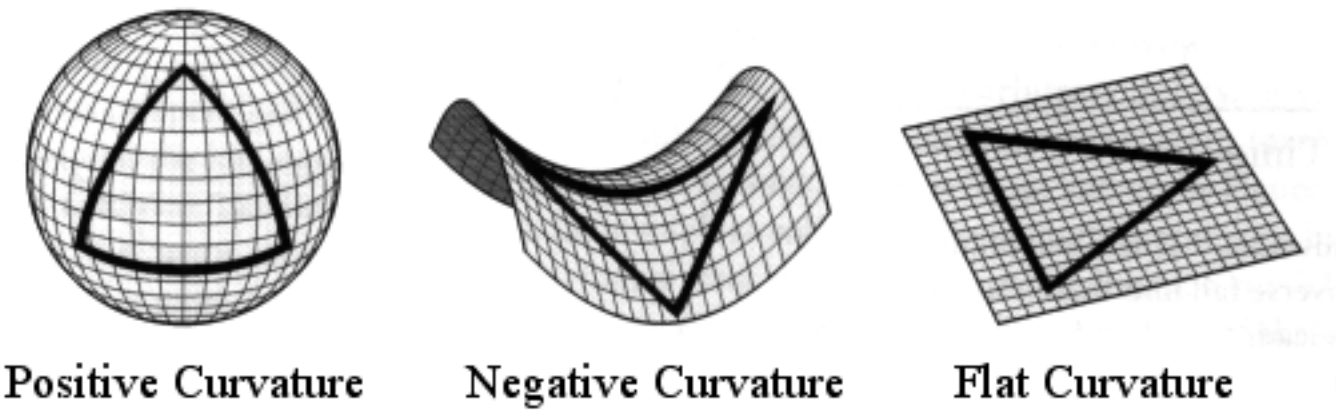
Minimal curve length = 8

Entanglement entropy = 6



Thoughts on discrete curvature

Local curvature in discrete lattices



For triangles

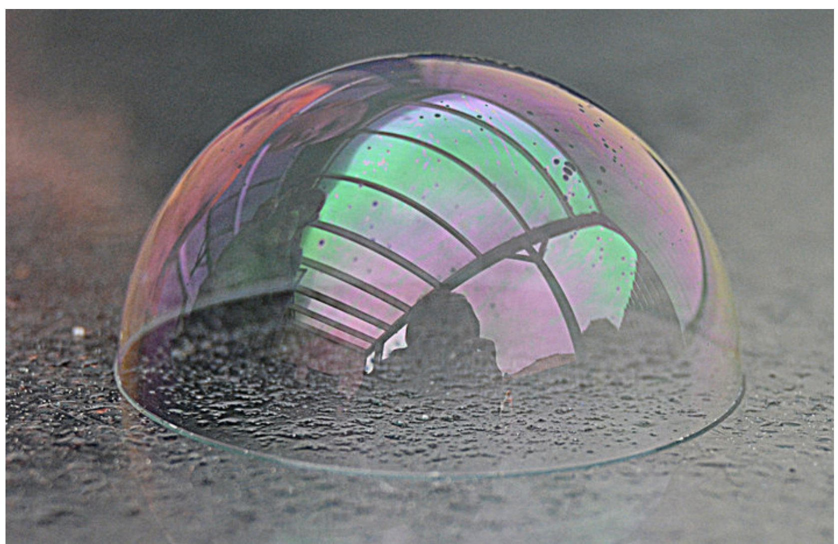
$$\int c \; dV = \phi_1 + \phi_2 + \phi_3 - \pi$$

For general polygons.

$$\int c \; dV = \sum_{j=1}^{N} \phi_j - (N-2)\pi$$

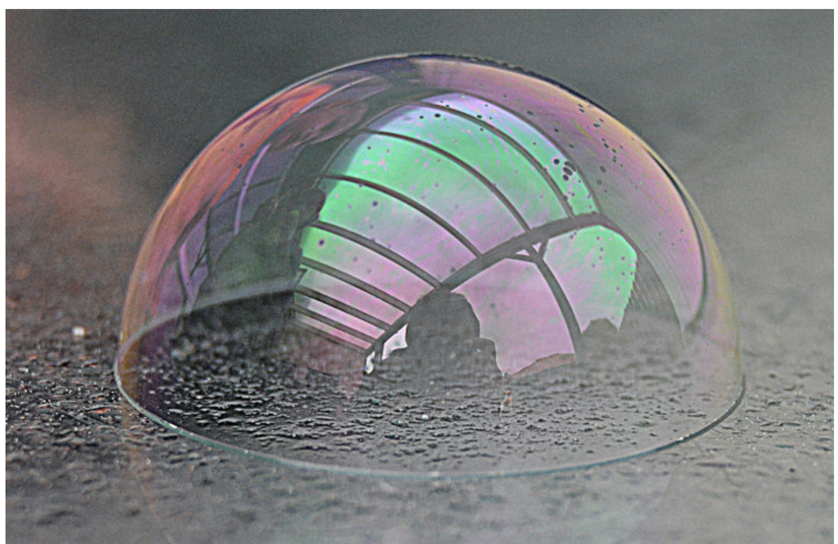
No interior maxima from No contractible bubble

- Conjecture 1: If there is no convex region in the manifold with curvature greater than Pi, there will be no interior maxima to the distance function.



No contractible bubble

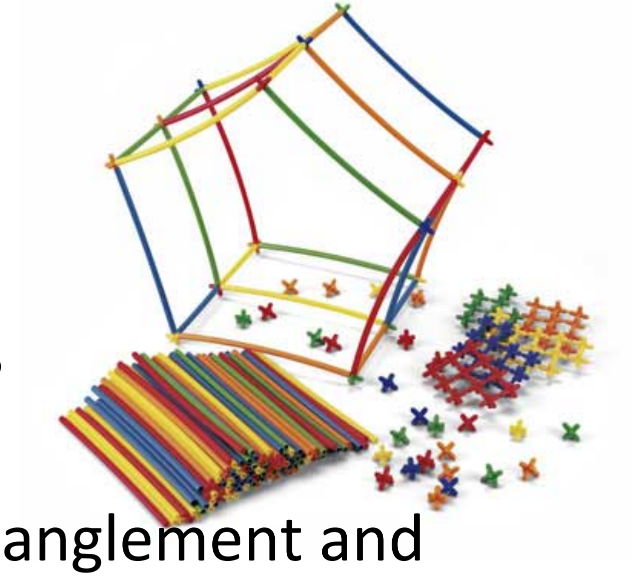
- Conjecture 2: If there there is a convex region in the manifold with discrete curvature >= Pi we may apply exact TNR.



Length scales

$$l_{\text{Planck}} \ll l_{\text{Coarse grain}} \ll R_{\text{AdS}}$$

Conclusions



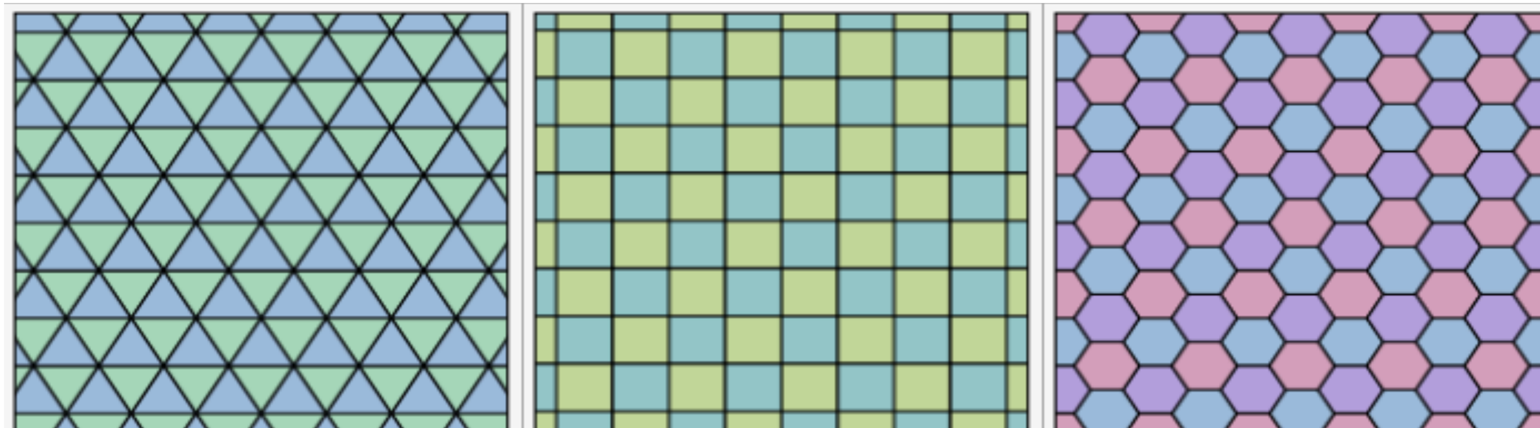
- Illustrated power of perfect tensors
  - For constructing QECC
  - For providing exact connection of entanglement and geometry
- Constructed a family of holographic QECC
  - With holographic properties
  - Showed the possibility of a threshold
- Constructed holographic "vacuum" states
  - Proved exact Ryu-Takayanagi entanglement entropy

Open problems

- Analyze and optimize code parameters.
- Error decoding algorithms!!
- Non-positive curvature from TNR
- Identifying bulk/boundary dynamics
- Emergence of frame independence from perfect tensors.
- Continuum limit in bond dimension CSS
- Lattice continuum limit
- Bounding of RT corrections
- RT proof generalization to higher D.

- GUT = Grand unifying tensor ☺

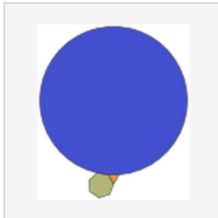
Regular flat lattices

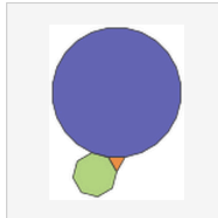


3^6 Triangular tiling | 4^4 Square tiling | 6^3 Hexagonal tiling

Flat angles

Trivalent


3.7.42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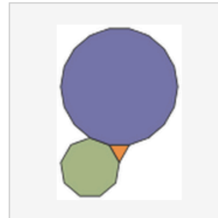
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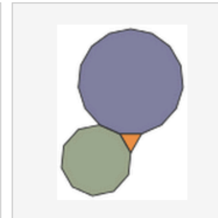
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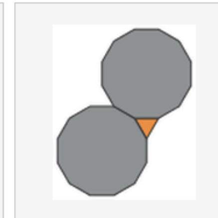
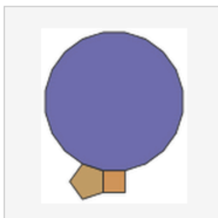
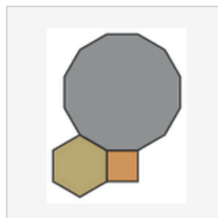

3.12.12


4.5.20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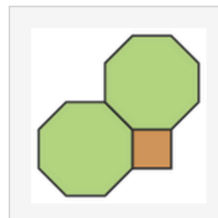
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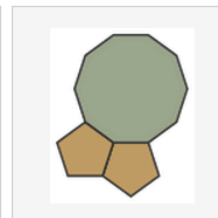
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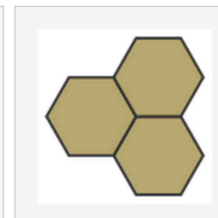
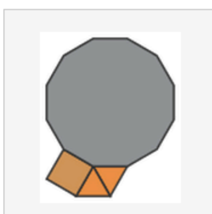
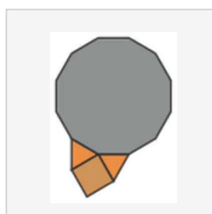
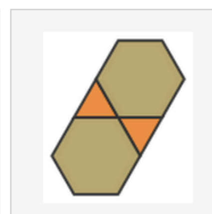

6.6.6

Fourvalent


3.3.4.12


3.4.3.12


3.3.6.6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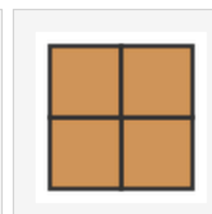
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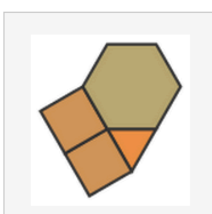
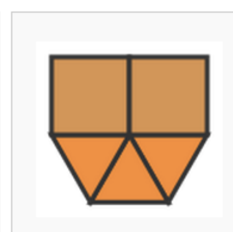

3.4.4.6


3.4.6.4

Five valent


3.3.3.3.6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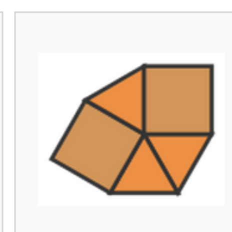
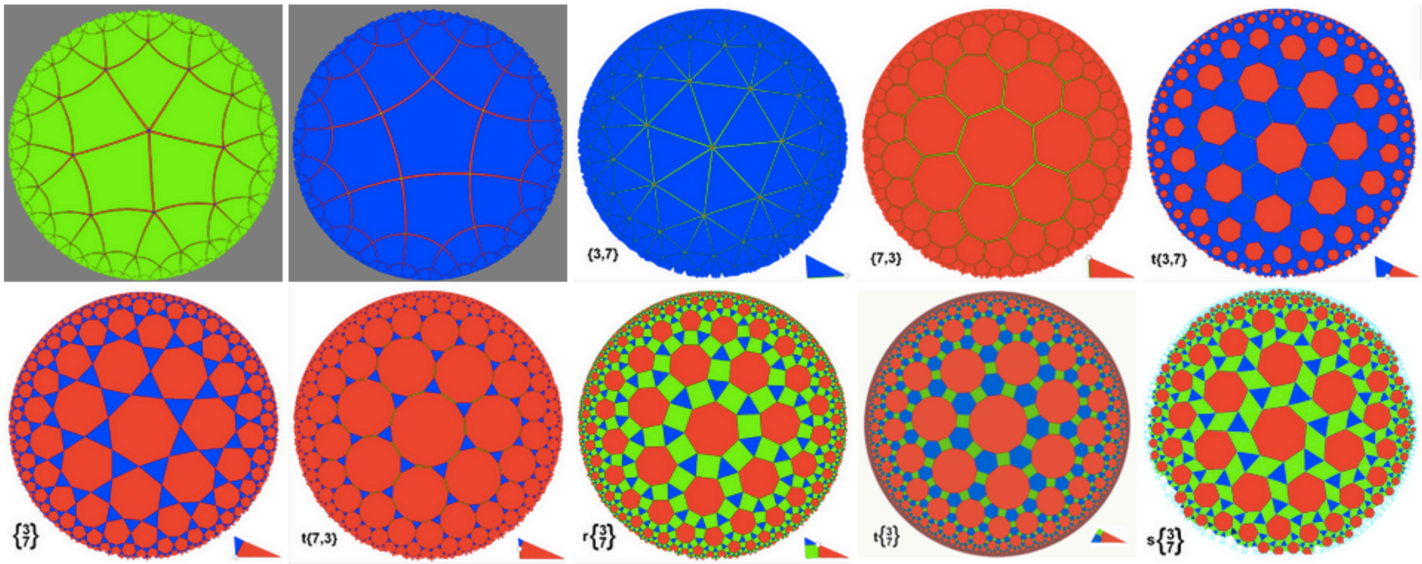

3.3.4.3.4

Small curvature hyperbolic tessellations



Small curvature regular lattices

f	v	c_f	c_v
3	7	$\pi/7$	$\pi/3$
4	5	$2\pi/5$	$\pi/2$
4	6	$2\pi/3$	π
5	4	$\pi/2$	$2\pi/5$
6	4	π	$2\pi/3$
7	3	$\pi/3$	$\pi/7$

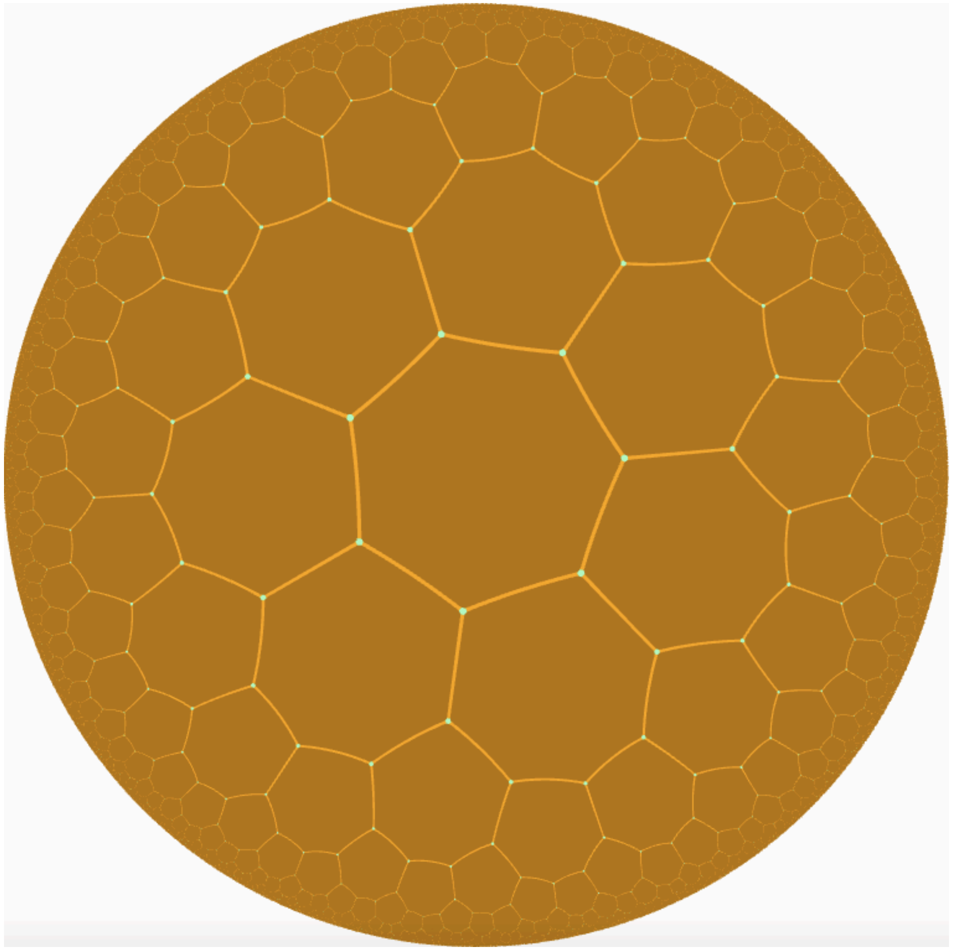
Smallest per vertex curvature (non-extensible)

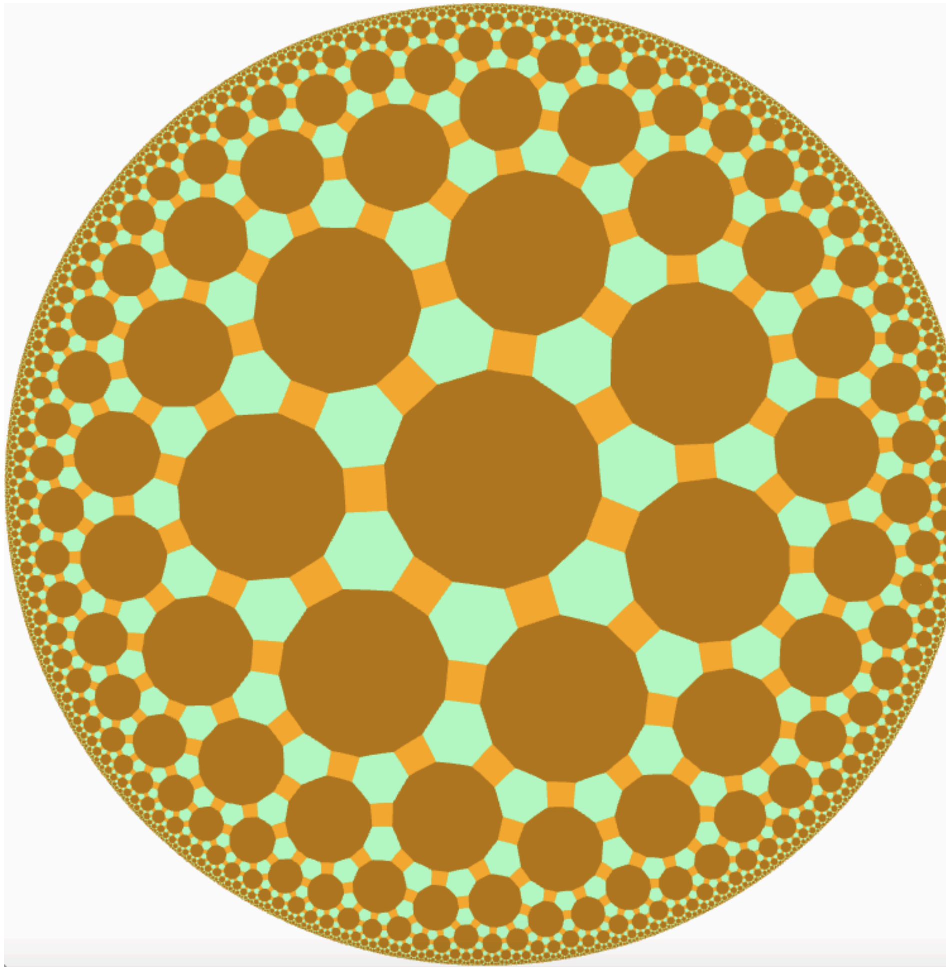
$$c_v = \pi/903, \quad n_1 = 3,\ n_2 = 3,\ n_3 = 7,\ n_4 = 43$$

Smallest per vertex curvature (even) extensible

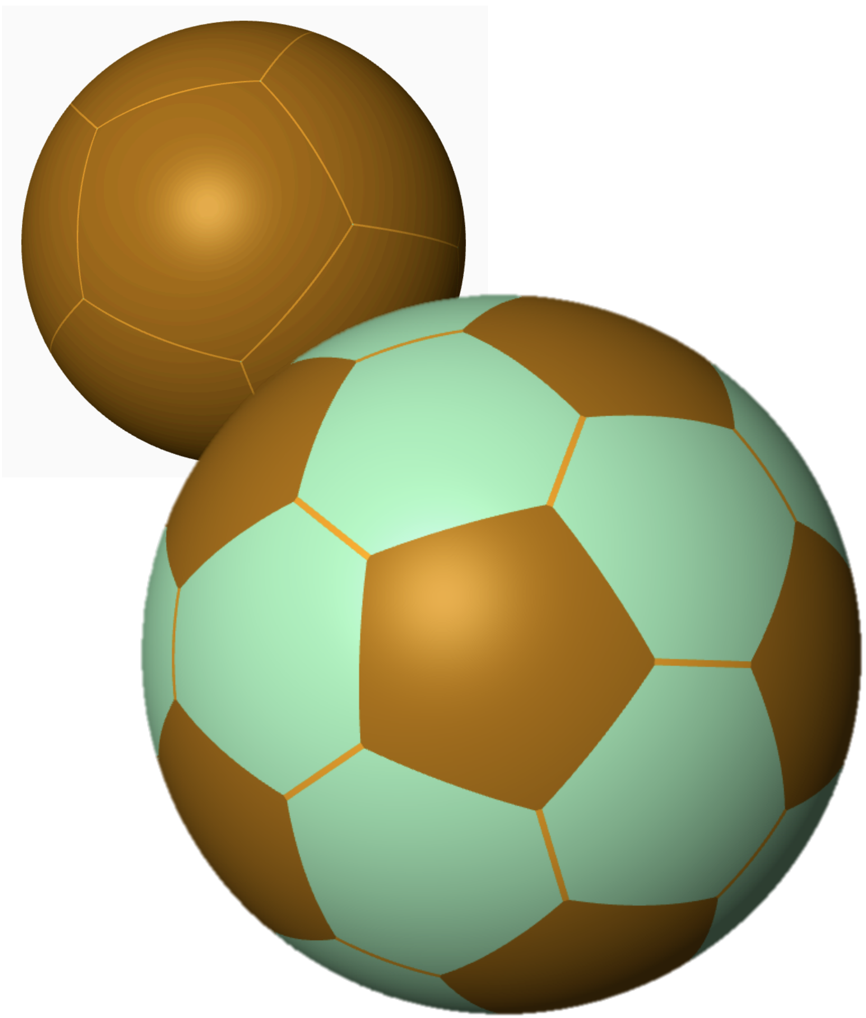
$$c_v = \pi/42, \quad n_1 = 4,\ n_2 = 6,\ n_3 = 14$$

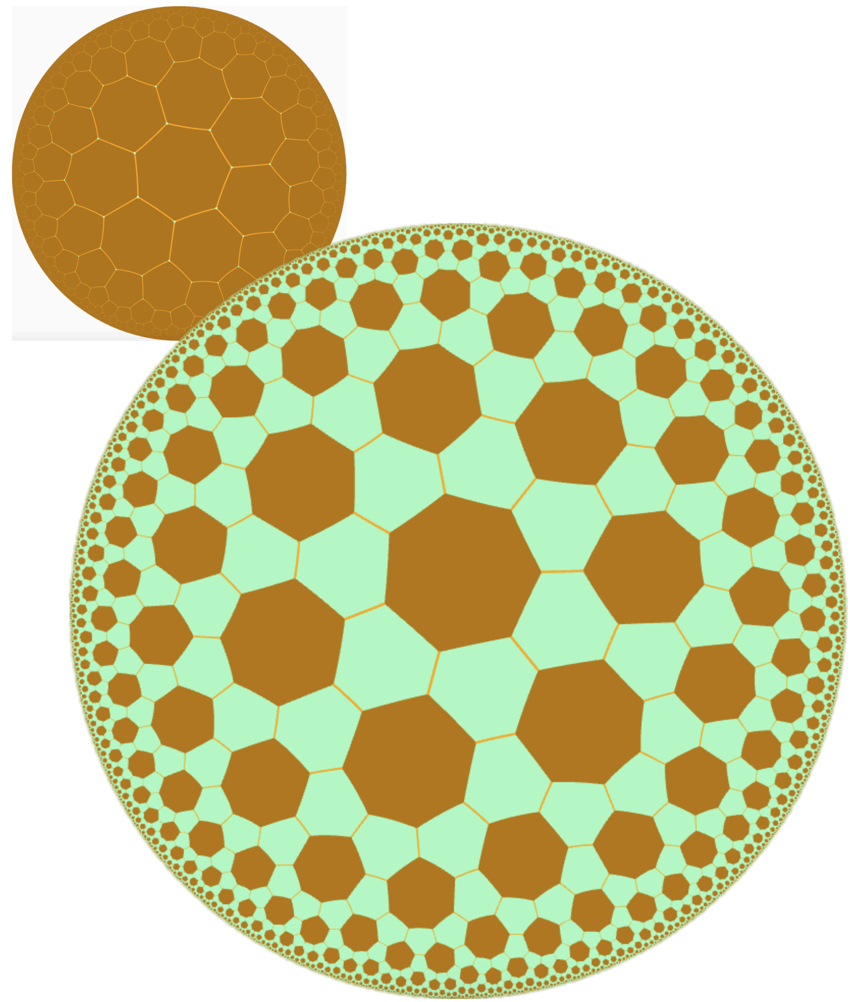
Minimum curvature regular & vertex regular





Leapfrog fullerene





Perfect tensors and relativity

- GRelativity is reference frame independent
- Evolution should be unitary along different timelike directions.
- Perfect tensors may provide the right way to discretize evolution in spacetime.

Thank You!