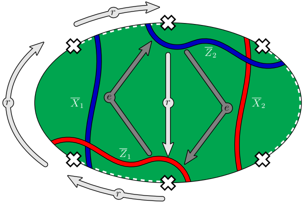
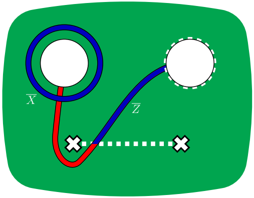
Picking holes and cutting corners to achieve Clifford gates with the surface code

Benjamin J. Brown

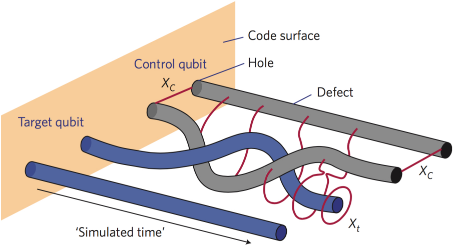




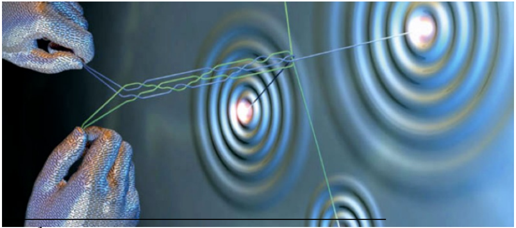
together with K. Laubscher, M. Kesselring and J. Wootton

Topological quantum computation

Code Deformations by braiding punctures[1]



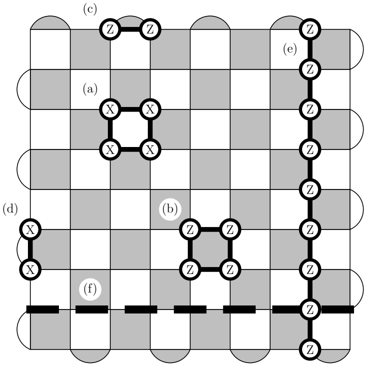
Braiding anyons[2]



[1]Figure from Nat. Phys. **5**, 19 (2009)
[2]Figure from http://www.csee.umbc.edu,

The planar code

We first introduce the familiar planar code



- The planar code is a stabilizer code, s.t.

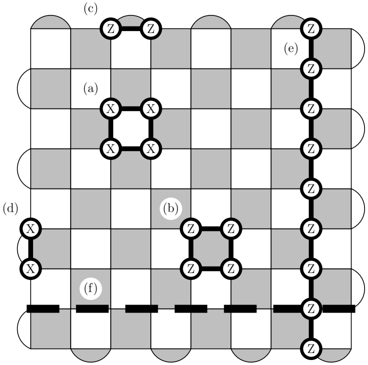
$$S|\psi\rangle = (+1)|\psi\rangle$$

  for elements $S \in \mathcal{S}$ of the stabilizer group $\mathcal{S}$ where $|\psi\rangle$ are codewords
- Codewords are manipulated by logical operators $\overline{X}$ and $\overline{Z}$
- (It follows that) logical operators have an unchanged action on the codespace under multiplication by stabilzers

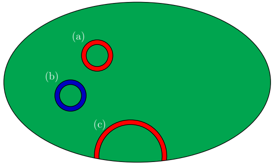
Alexei Kitaev Ann. Phys. (2003), Dennis *et al.* (2002)

The planar code

Multiplying (stringlike) logical operators by stabilizers continuously deforms strings

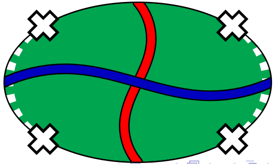


- Stabilizers are represented as closed loops

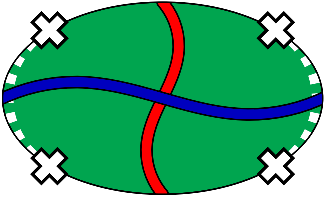


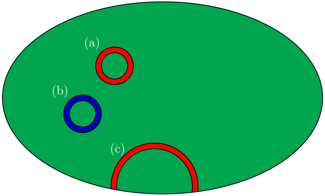
where red(blue) strings indicate strings of Pauli-Zs(Pauli-Xs)

- We also require different (rough and smooth) boundaries to terminate different types of strings



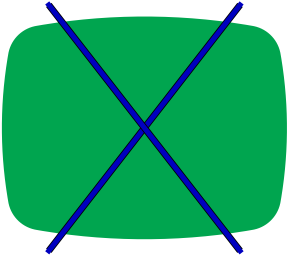
Strings can be interpreted as world lines of particles



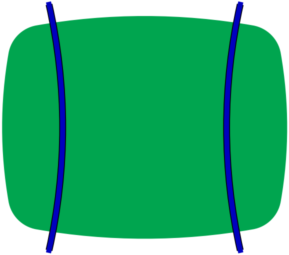


Strings can be interpreted as world lines of particles

Particles of the same type have bosonic exchange statistics

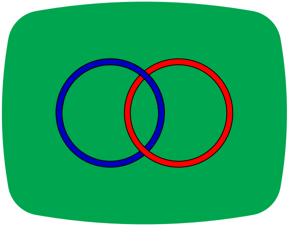


$$= (+1)$$

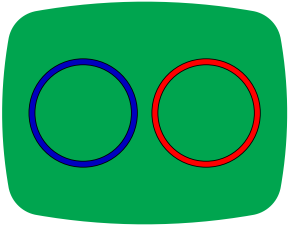


Strings can be interpreted as world lines of particles

Exchanging two particles of different types give non-trivial exchange statistics (e-charges and m-charges)

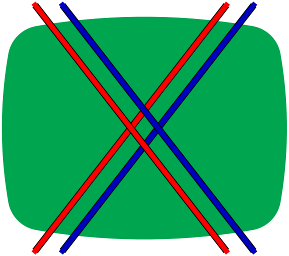


$$= (-1)$$

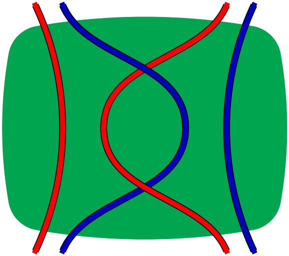


Strings can be interpreted as world lines of particles

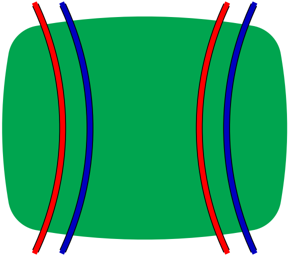
Composite excitations behave like fermions



$$= (+1)$$



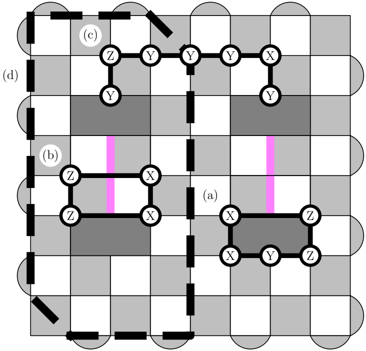
$$= (-1)$$



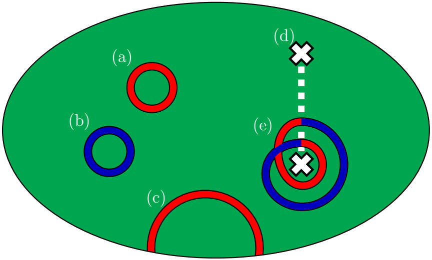
(This follows from facts given in the previous two slides)

We can also encode qubits using twist defects

Dislocations change the string type from X to Z, and their end points are Majorana modes



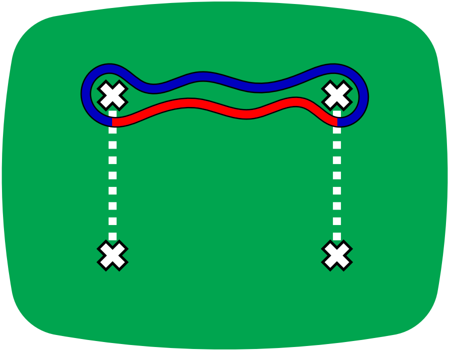
- We will mostly work with this diagrammatic langauge away from the lattice



- Dislocation lines change blue strings to red strings and vice versa.

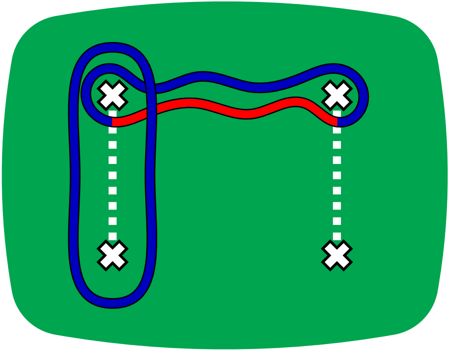
Interpreting twist defects as Majorana modes

Twist defects can absorb fermions



Interpreting twist defects as Majorana modes

We can only measure the charge parity of pairs of twists



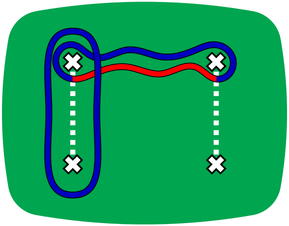
Interpreting twist defects as Majorana modes

With these observations we see that twist defects have the fusion rules of Ising anyons (Majorana modes)

$$\sigma \times \sigma = 1 + \psi$$

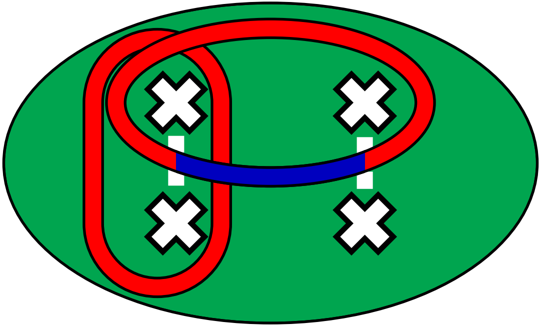
$$\sigma \times \psi = \sigma$$

$$\psi \times \psi = 1$$



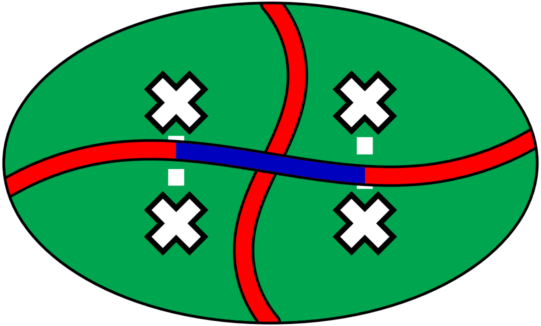
In the corners of the planar code

We consider four twist defects on the surface code



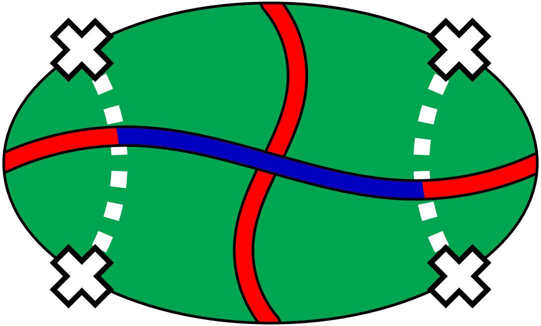
In the corners of the planar code

We deform the logical operators such that they terminate at the lattice boundary



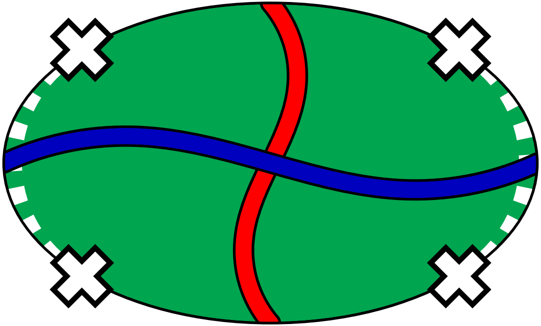
In the corners of the planar code

The physics of the previous model is unchanged if we move the defects to the boundary



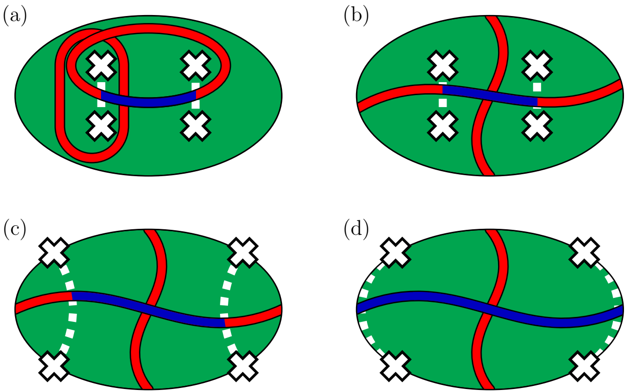
In the corners of the planar code

Moreover, we can move the dislocation lines to the boundary to recover the planar code



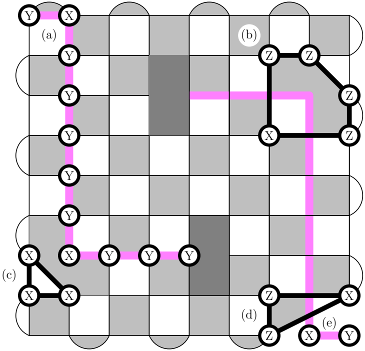
In the corners of the planar code

planar code corners $\Leftrightarrow$ Majorana modes



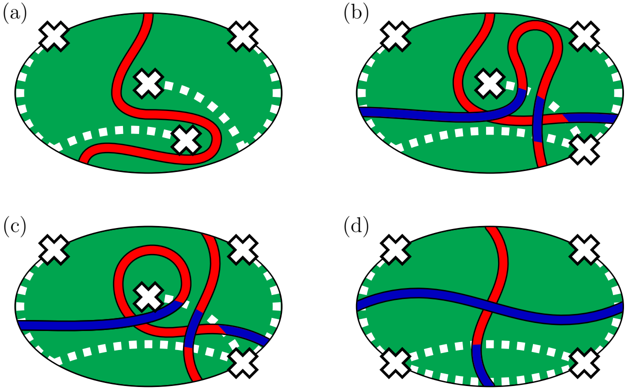
Braiding corners

We can move holes into the bulk by code deformation



Braiding corners

Exchanging corners allows us to perform single-qubit Clifford gates

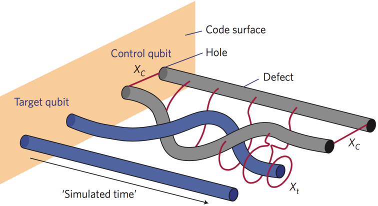


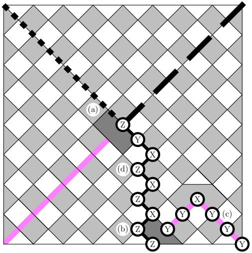
Other schemes for two-dimensional quantum computation

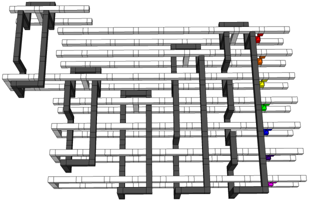
Braiding holes, color codes or corner braiding?

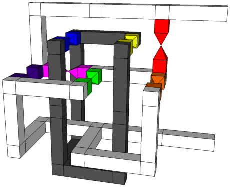
braiding corners

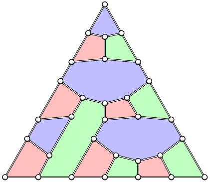
Braiding holes







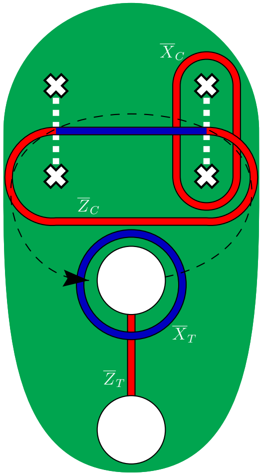




Color codes

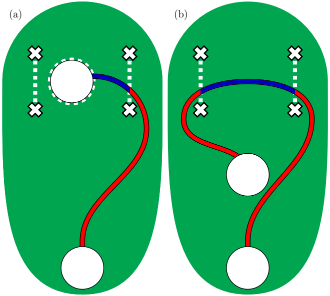
Combining surface code computational schemes

We can entangle a qubit encoded with four twists with a qubit entangled over two holes by braiding

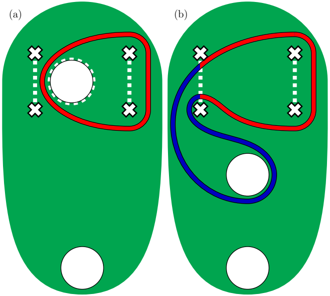


Combining surface code computational schemes

Diagrams showing that logical operators map accordingly
(other logical operators map trivially)



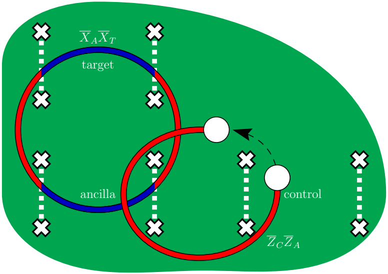
$$\overline{Z}_T \to \overline{Z}_C \overline{Z}_T$$



$$\overline{X}_C \to \overline{X}_C \overline{X}_T$$

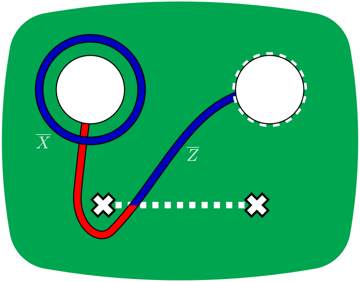
Combining surface code computational schemes

Entangling operations by parity measurements as in dislocation code schemes [Hastings and Geller] are achieved by braiding holes around static twist defects



Combining surface code computational schemes

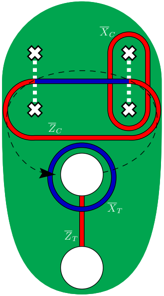
We can also design other encodings over holes and twists

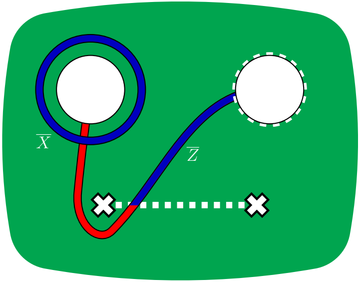


We call this a hybrid qubit

Combining surface code computational schemes

Hole qubits, twist qubits and hybrid qubits have complimentary gate capabilities

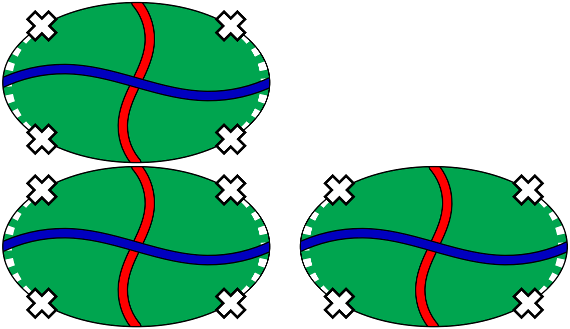




- **Twist qubits** perform all single qubit Clifford gates, but require ancilla for two qubit gates
- **Hole qubits** do not require ancilla qubits for entangling gates
- **Hybrid qubits** have one single-qubit gate, and achieve two-qubit gates easily

We can fault-tolerantly map between different encodings

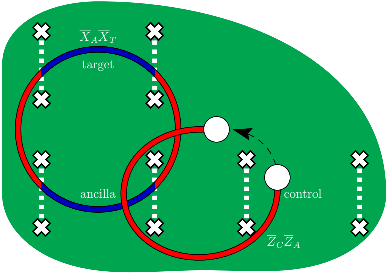
Two-qubit gates - lattice surgery

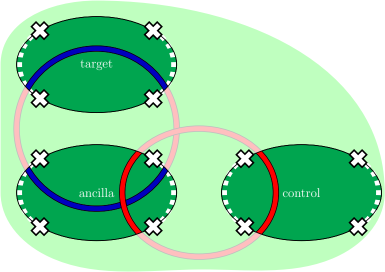


$\mathcal{O}(L)$ physical ancilla qubits are placed between pairs of planar codes to perform logical parity measurments

Two-qubit gates - lattice surgery

By regarding corners as Majorana modes lattice surgery appears much more familiar as measurement only topological quantum computation

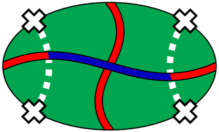




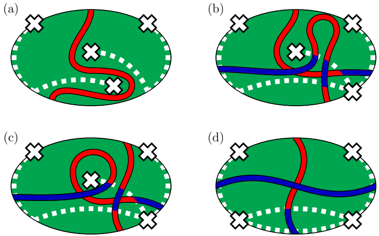
Summary

We have developed and unified several methods of surface code quantum computation

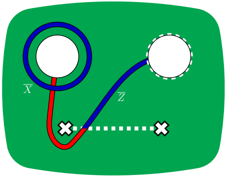
- Twists ⇔ corners



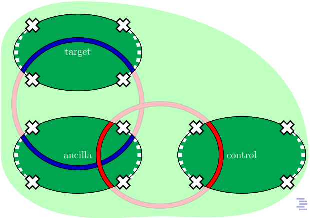
- We achieve all the Clifford gates with the planar code



- New hybrid encodings with different gates to known encodings

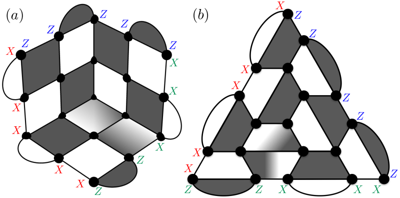


- Lattice surgery ⇔ measurement only TQC



Relationship to other recent work

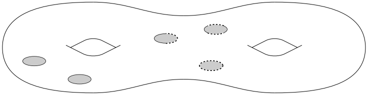
Yoder and Kim arXiv:1612.04795

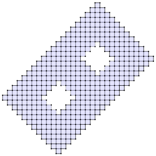


surface code with $n \sim 3d^2/4$ qubits

code with three corners and one twist

Delfosse, Iyer and Poulin arXiv:1606.07116





'packing' logical qubits into surface codes.

Can more qubits be packed with bulk twists?

Minimising space-time resource costs

Perhaps we can find more resource efficient quantum circuits by combining different computational schemes?