Surprising facts about quantum error correction

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Université de Sherbrooke

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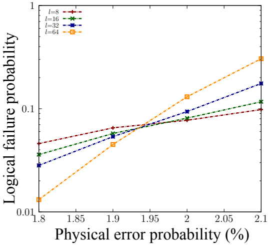
The accuracy threshold theorem



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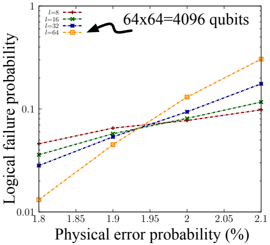

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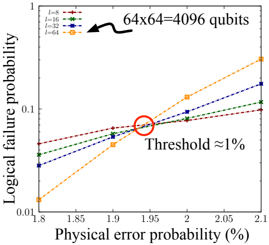

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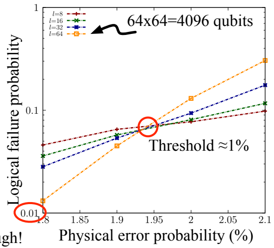

Not good enough!

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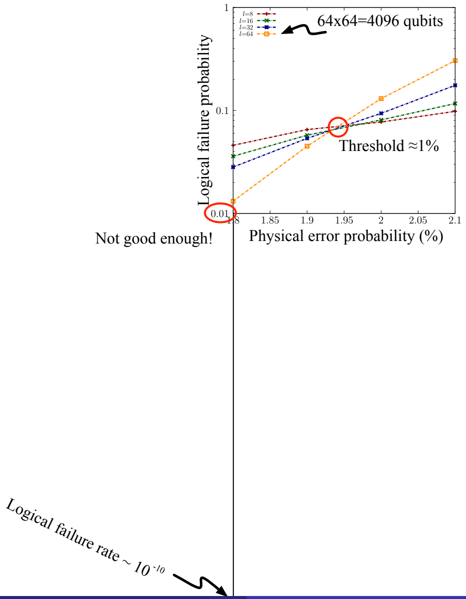

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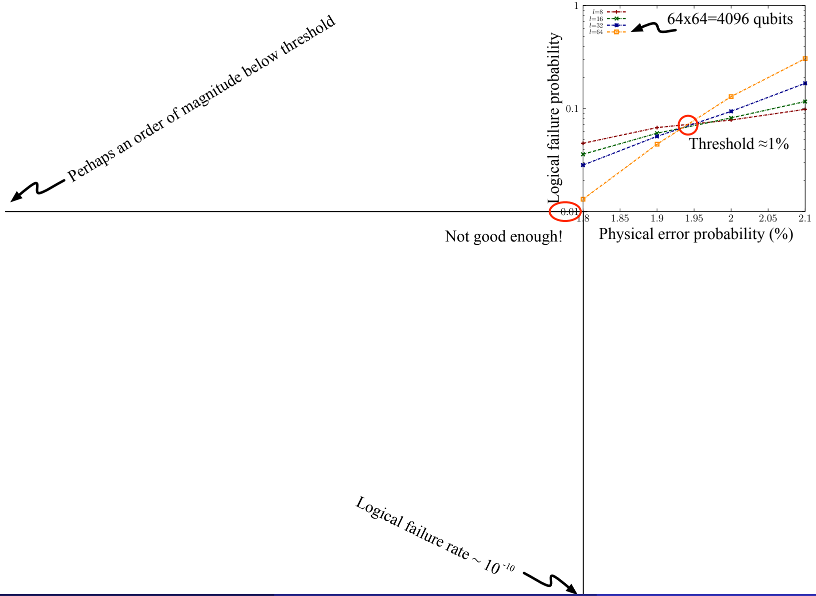

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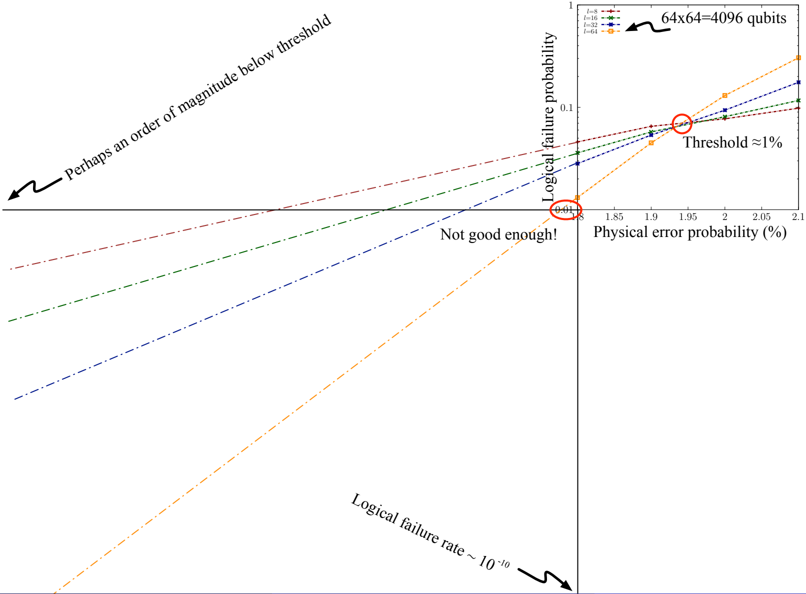

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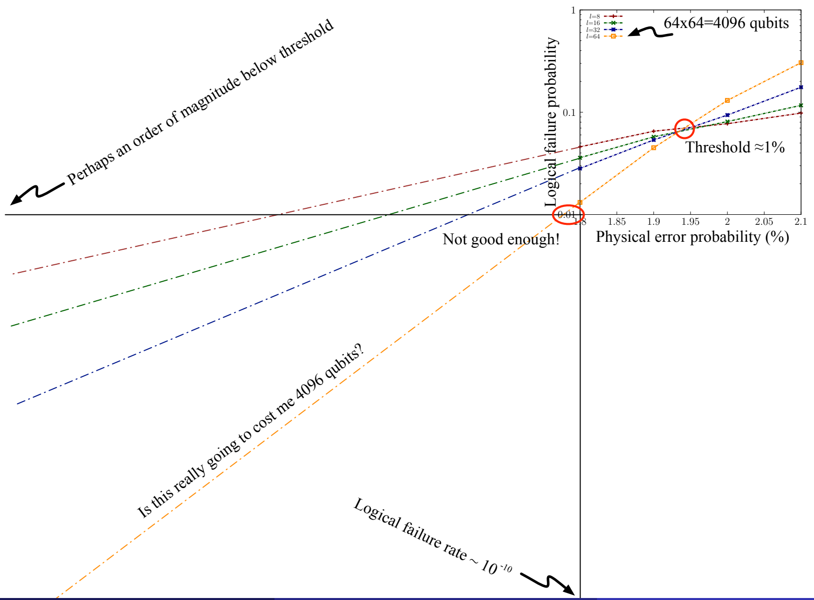

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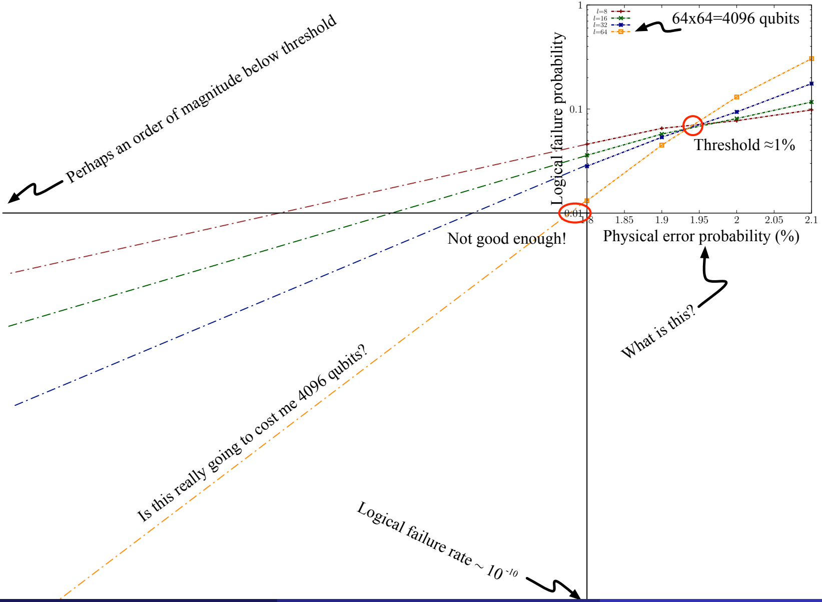

Outline

Fault-tolerant overhead

- We want to execute a quantum algorithm with N logical gates.
  - $N \sim 10^{12}$-10^{15} to simulate a small molecule like Fe_2S_2.
- Each gate is error-corrected to accuracy δ, so errors build up to
  - $N\delta$ if they add coherently (worst case, systematic bias).
  - $\sqrt{N}\delta$ if they add stochastically.
- δ needs to be $\sim 1/\sqrt{N}$ to $1/N$ to prevent harmful error build up.
  - 10^{-6} to 10^{-15} for quantum chemistry (pretty vague).
- If the physical noise rate ϵ is sub threshold, then fault-tolerant error correction can produce logical gates of accuracy δ with overhead polylog($\frac{1}{\delta}$).

Given a physical noise rate ϵ, how much error correction do I need to achieve a logical noise rate δ?

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Quantum many-body methods

- Realistic noise models cannot be efficiently simulated.
  - Interacting quantum many-body problem.

Our contribution

Study fault-tolerance with realistic noise models using numerical many-body techniques

- Tensor network methods
  - Density matrix renormalization group (DMRG).
  - Projected entangled pairs state (PEPS).
  - Multi-scale entanglement renormalization ansatz (MERA).
  - etc.

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What goes into a simulation?

- Prepare some known code state $|\bar{\psi}\rangle$
- Applying some noise $\mathcal{E}$ to $\rho = |\bar{\psi}\rangle\langle\bar{\psi}|$.
  - When $\mathcal{E}$ is some stochastic noise, we can sample the noise instead of applying $\mathcal{E}$.
- Sample the syndrome bits $\mathrm{pr}_j(\pm) = \frac{1}{2}(1 \pm \mathrm{Tr}[\mathcal{E}(\rho)S_j])$.
- Decode, i.e., find a correction operation C based on the observed syndrome.
- Apply the correction to the post-measurement state ρ'.
- Evaluate the logical transformation that has been applied to the logical state.
- Repeat for different input states $\bar{\psi}$ to perform logical process tomography.
  - We actually use Jamilkowski isomorphism instead.

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If we can do all of this...

Simulation

INPUT

- Noise $\mathcal{E}$.

OUTPUT

- A syndrome s.
- The probability of that syndrome pr(s).
- The logical channel conditioned on that syndrome $\mathcal{E}_s^L$.

Given this we can estimate...

- Average channel $\overline{\mathcal{E}}_L = \sum_s \mathrm{pr(s)}\mathcal{E}_\mathrm{s}^\mathrm{L}$
- Average logical error $\sum_s \mathrm{pr(s)}\|\mathcal{E}_\mathrm{s}^\mathrm{L} - \mathrm{id}\|$
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Surface code

- The surface code logical basis states $|\bar{\psi}\rangle = |\bar{0}\rangle, |\bar{1}\rangle, |\bar{+}\rangle$, or $|\bar{-}\rangle$ are PEPS and are complete for logical process tomography.
- $\rho = \mathcal{E}(|\bar{\psi}\rangle\langle\bar{\psi}|)$ is a PEPO for any PEPSO CPTP map $\mathcal{E}$.
- Computing the probability of a syndrome bit $\frac{1}{2}(1 \pm \mathrm{Tr}[\rho S_j])$ boils down to contracting a 2D TN.
- The post-measurement state $\rho' \propto (1 \pm S_j)\rho(1 \pm S_j)$ is also a PEPO.
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- Compatible with
  - Uncorrelated noise.
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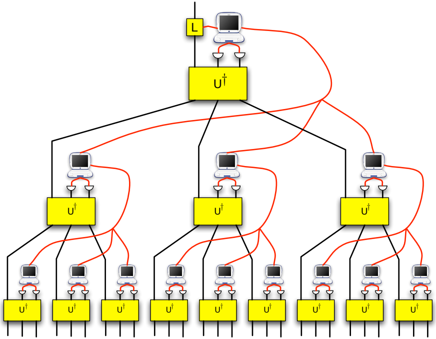

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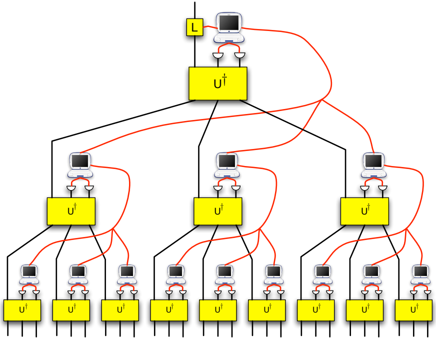

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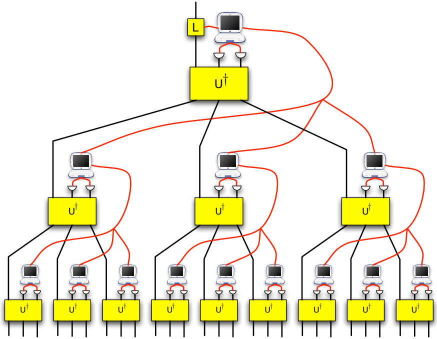

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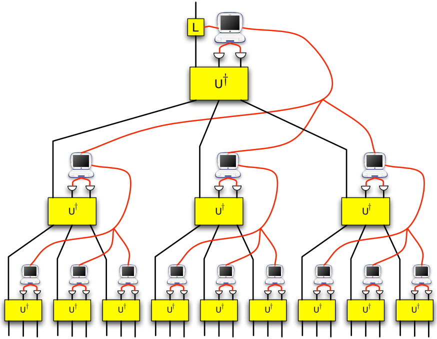

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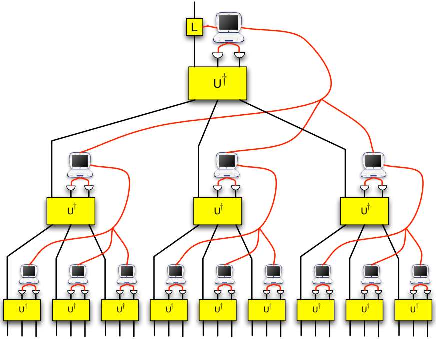

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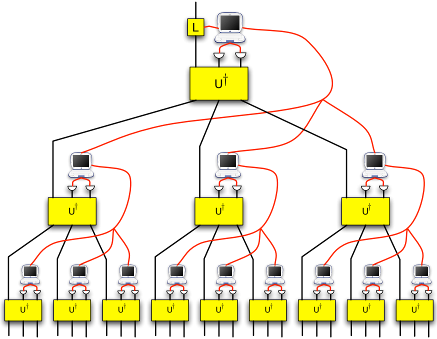

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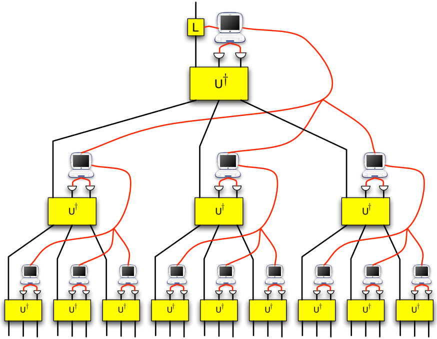

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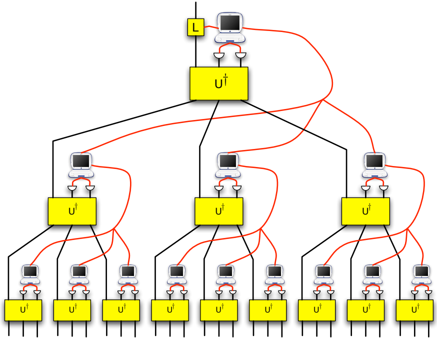

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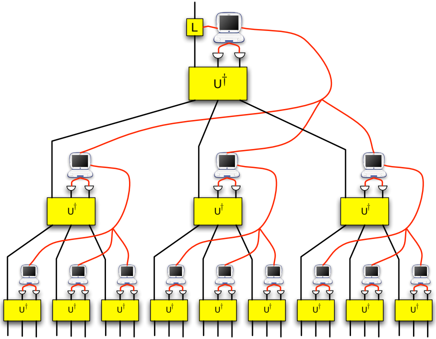

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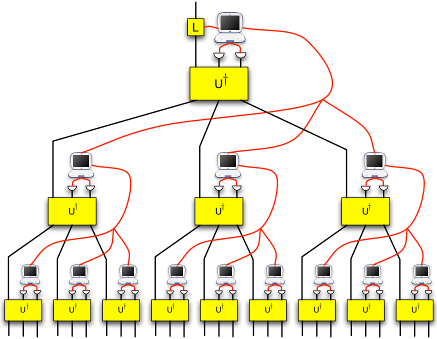

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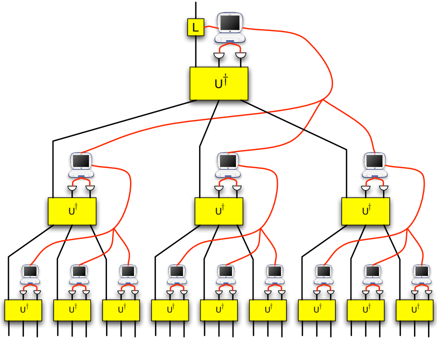

Outline

Does this question even make sense?

Given a physical noise rate ϵ, how much error correction do I need to achieve a logical noise rate δ?

- How do I quantify the physical noise rate?
  - Infidelity $\epsilon = 1 - \int d\psi F[\psi, \mathcal{E}(\psi)]$ has a nice statistical interpretation, measured by randomized benchmarking.
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Better approximation to the diamond norm

$\|\cdot\|_\diamond$ used for FT analysis and $1 - F$ accessed by randomized benchmarking differ significantly.

- The diamond norm could always be extracted from full process tomography of the channel.
- Can we combine randomized benchmarking results with other easily accessible quantities to estimate diamond norm?
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Kueng, Long, Doherty, & Flammia arXiv:1510.05653

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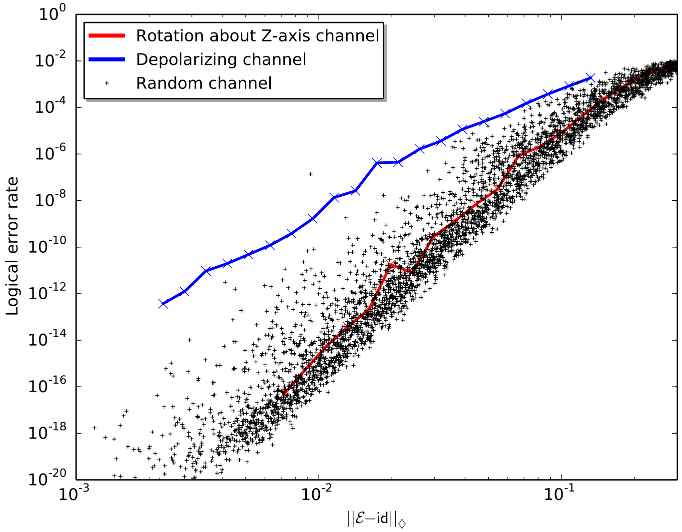

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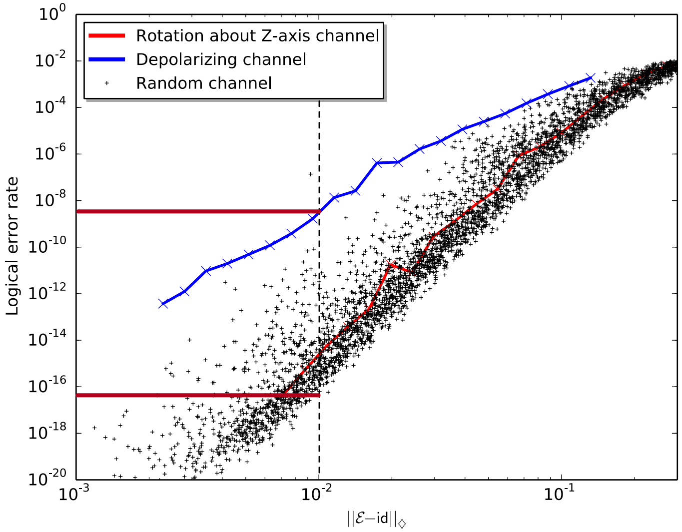

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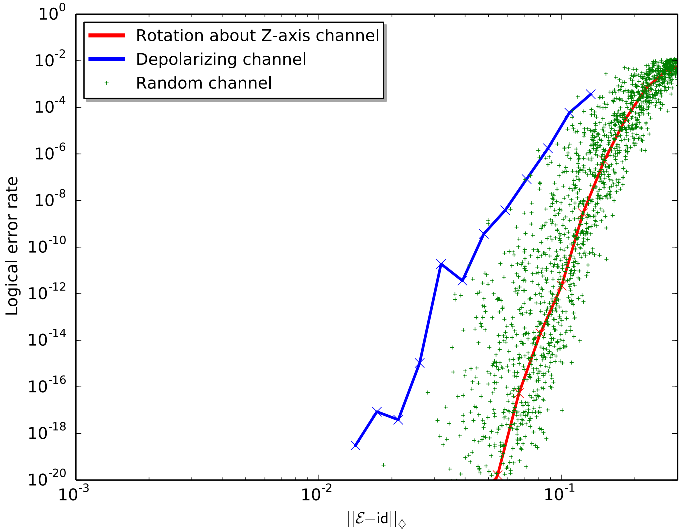

Predictability of noise metrics

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It is not possible to even very crudely predict the logical failure rate of a FT scheme given only the noise rate of the physical channel, as measured by any of the standard error metrics (Infidelity, Diamond norm, Channel entropy, Error probability, Euclidian norm, Trace norm).

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- Analytical approach: stochastic adversarial noise model with rate ϵ should provide an upper bound.
  - Pros: Simple, widely applicable.
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  - For N qubits, need to manipulate $2N \times 2N$ matrices.
- If the noise consists of Clifford operations too, the whole process can be efficiently simulated.
  - In particular Pauli noise models (e.g. depolarizing).
  - These are the 'free-fermion' models of fault-tolerance.

Monte Carlo simulations

FOR t=1:M

1. Assume the system is initially in a valid code state.
2. Execute a fault tolerant circuit on this encoded state
   - Sprinkle noise operations at a rate ϵ over the circuit.
3. Verify if the logical information has been altered by the noise.

END

Report logical failure rate $\delta =$ Number of reported failures/M.

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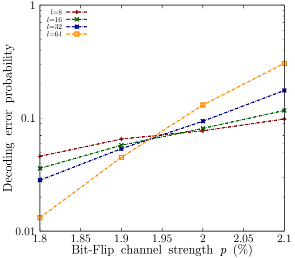

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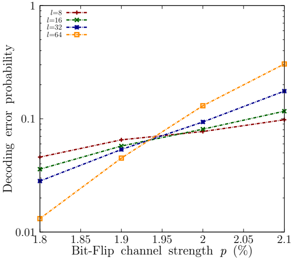

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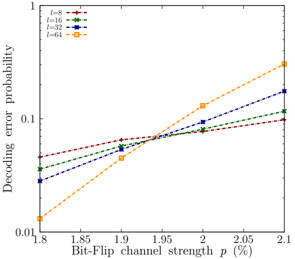

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Low noise rate & non-Pauli noise

Our simulations methods are based on the idea of simulating the full dynamics $\mathcal{E}$ and sampling the syndromes, not the errors, just like in a real experiment.

- Thus, they are not limited to Pauli noise.
- When sampling Pauli errors, the correction is either right or wrong.
- When sampling syndromes, and simulating $\mathcal{E}$ entirely, there is always some residual logical error.
  - This is why we are able to report 10^{-18} logical error rates.
- We also used importance sampling methods to decrease statistical fluctuations:

$$\sum_s \mathrm{pr}(s) \|\mathcal{E}_s^L - \mathrm{id}\| = \sum_s q_s \left(\frac{\mathrm{pr}(s)}{q_s} \|\mathcal{E}_s^L - \mathrm{id}\| \right)$$

  - The intrinsic variance of the logical error is very large: outliers are dominant.

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Outline

Twirled approximation

MC simulations are limited to unphysical Pauli noise models.

- We need to approximate the physical channel $\mathcal{E}$ by a Pauli channel $\mathcal{P}$.
  - Ignore the non-Pauli contributions to the channel.
- E.g. Rotation $R_z(\theta) = e^{i\theta Z} = \cos\theta I + i\sin\theta Z$ error

$$\rho \to (\cos\theta I + i\sin\theta Z)\rho(\cos\theta I - i\sin\theta Z)$$

is approximated by a stochastic Z error (dephasing)

$$\rho \to \cos^2\theta\ \rho + \sin^2\theta\ Z\rho Z.$$

- This twirled, or 'tight-binding' approximation might be much nicer than the original model.

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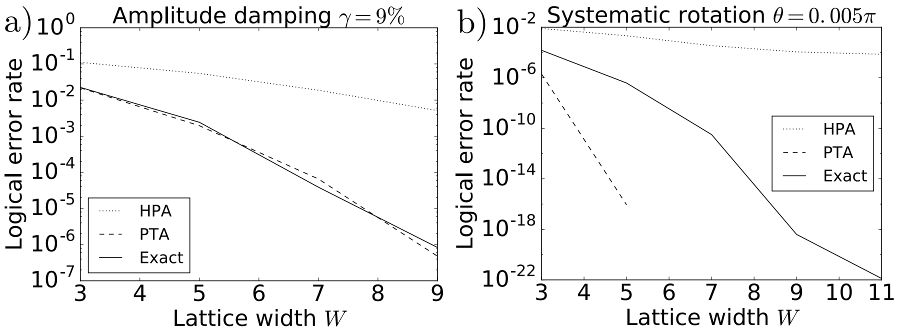
Pauli approximations, surface code threshold

	Exact	Pauli twirl	Honest Pauli
Depolarising (ϵ)	$18.5 \pm 1.5\%$	$18.5 \pm 1.5\%$	$18.5 \pm 1.5\%$
Damping (γ)	$39 \pm 2\%$	$39 \pm 2\%$	$6 \pm 1\%$
Z-rotation (θ)	$> 0.40\pi$	0.34π	0.11π

- Depolarizing, lattice up to size $9 \times 9 = 81$ qubits.
- Amplitude damping, lattice up to size $9 \times 17 = 153$ qubits.
- Depolarizing, lattice up to size $11 \times 11 = 121$ qubits.

Pauli approximations, surface code overhead



Usefulness of Pauli approximations

Conclusions

- It is not possible to even very crudely predict the logical failure rate of a FT scheme from known Pauli approximations.
- The twirl approximation gets a good threshold estimate in the examples we looked at.

- For the amplitude damping channel:
  - the twirled gave a good prediction,
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Outline

Pauli approximations for the purpose of decoding

There are two levels of difficulty: decoding and simulating.

- Even for Pauli noise, decoding is in general a hard problem, but there are efficient algorithms for some classes of codes.
- For non-Pauli noise, the problem becomes even harder.
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  - Suboptimal but efficient.
- Same comments apply to correlated noise models (either Pauli or not).

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- Same comments apply to correlated noise models (either Pauli or not).

Our simulations methods, combined to efficient (approximate) contraction schemes of TN provide efficient decoders for a wide variety of non-Pauli and/or correlated noise models.

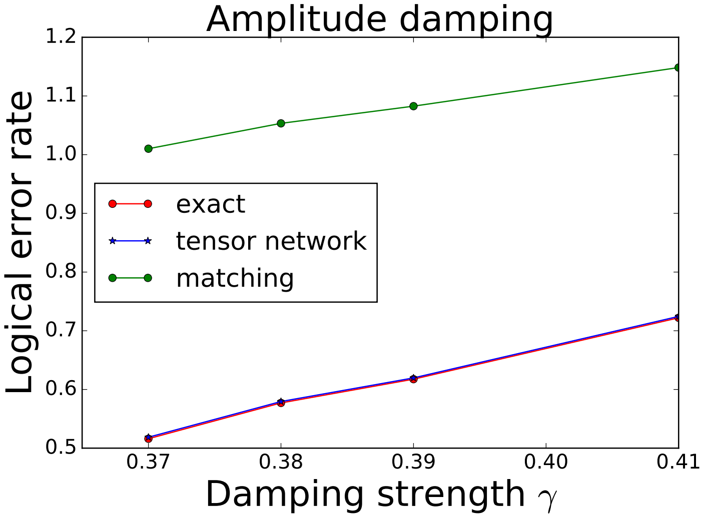
Pauli approximations for the purpose of decoding

There are two levels of difficulty: decoding and simulating.

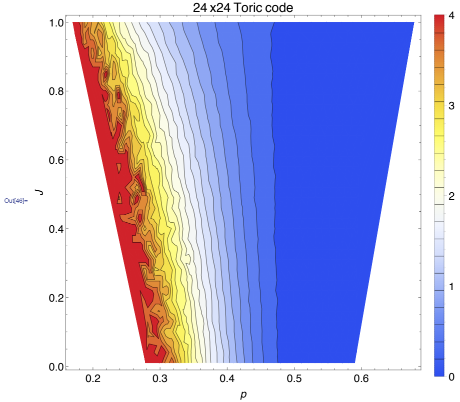
- Even for Pauli noise, decoding is in general a hard problem, but there are efficient algorithms for some classes of codes.
- For non-Pauli noise, the problem becomes even harder.
- For decoding purposes, one could assume one of the Pauli approximations.
  - Suboptimal but efficient.
- Same comments apply to correlated noise models (either Pauli or not).

Our simulations methods, combined to efficient (approximate) contraction schemes of TN provide efficient decoders for a wide variety of non-Pauli and/or correlated noise models.

Pauli approximations for surface code decoding



Correlated erasures on surface code



The erasure pattern is given by spin down configuration of a classical ferromagnetic Ising model in a magnetic field favoring spin ups.

Color shows
$-\log_{10}$(logical error rate)

Given a physical noise rate ϵ, how much error correction do I need to achieve a logical noise rate δ?

- We use methods from quantum many-body physics to address this question.
  - Surface code.
  - Concatenated code.
- We have found that:
  - for a fixed physical noise rate, as measured by any standard metric, the logical failure rate can fluctuate by several orders of magnitude.
  - the diamond norm is no better than other norms in that regard.
  - the logical failure rate of Pauli approximations can differ by several orders of magnitude.
  - for a given diamond norm physical error rate, incoherent noise is the worst.
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- Given an experimentally motivated noise model have the tools to predict its FT performances.
- These predictions can guide experimentalists:
  - If I changed this parameter, would it have a noticeable effect?

What features of a noise model are critical to FT QEC?

- We are using machine learning techniques, combined to our simulations, to find these parameters.
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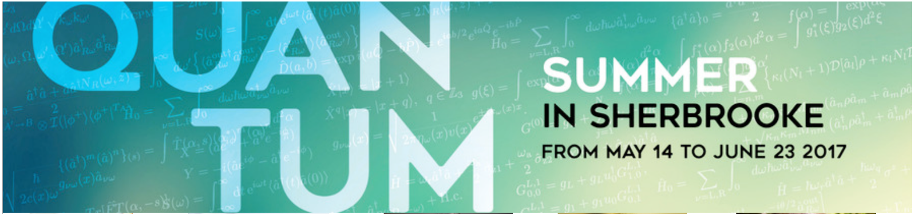


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