2D SET order in tensor networks

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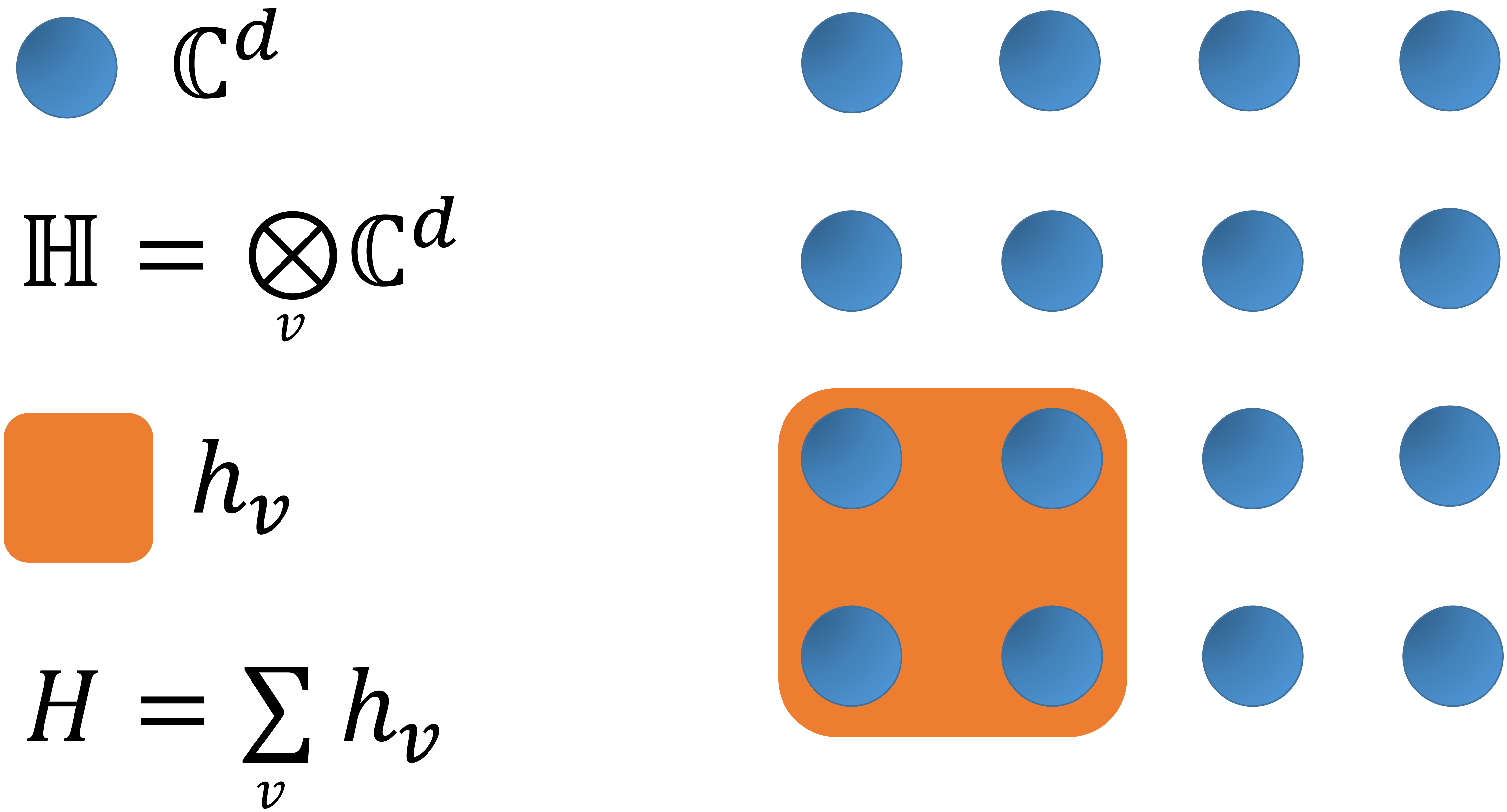
Verstraete Group

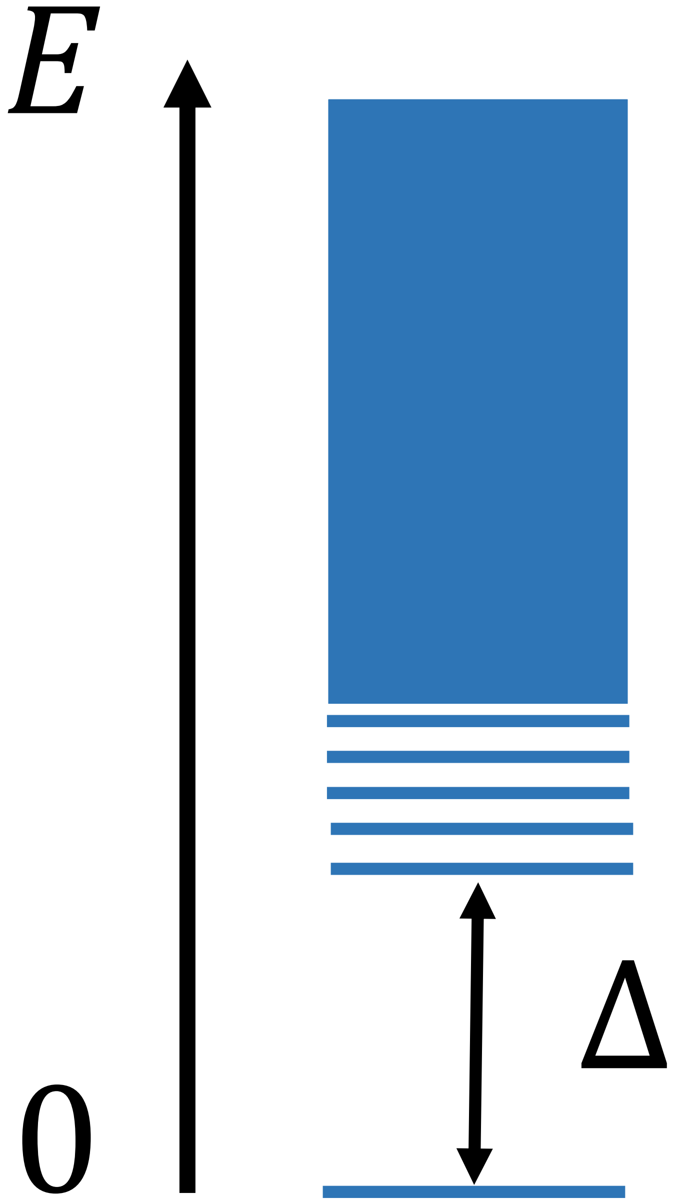
University of Vienna

arXiv:1412.5604

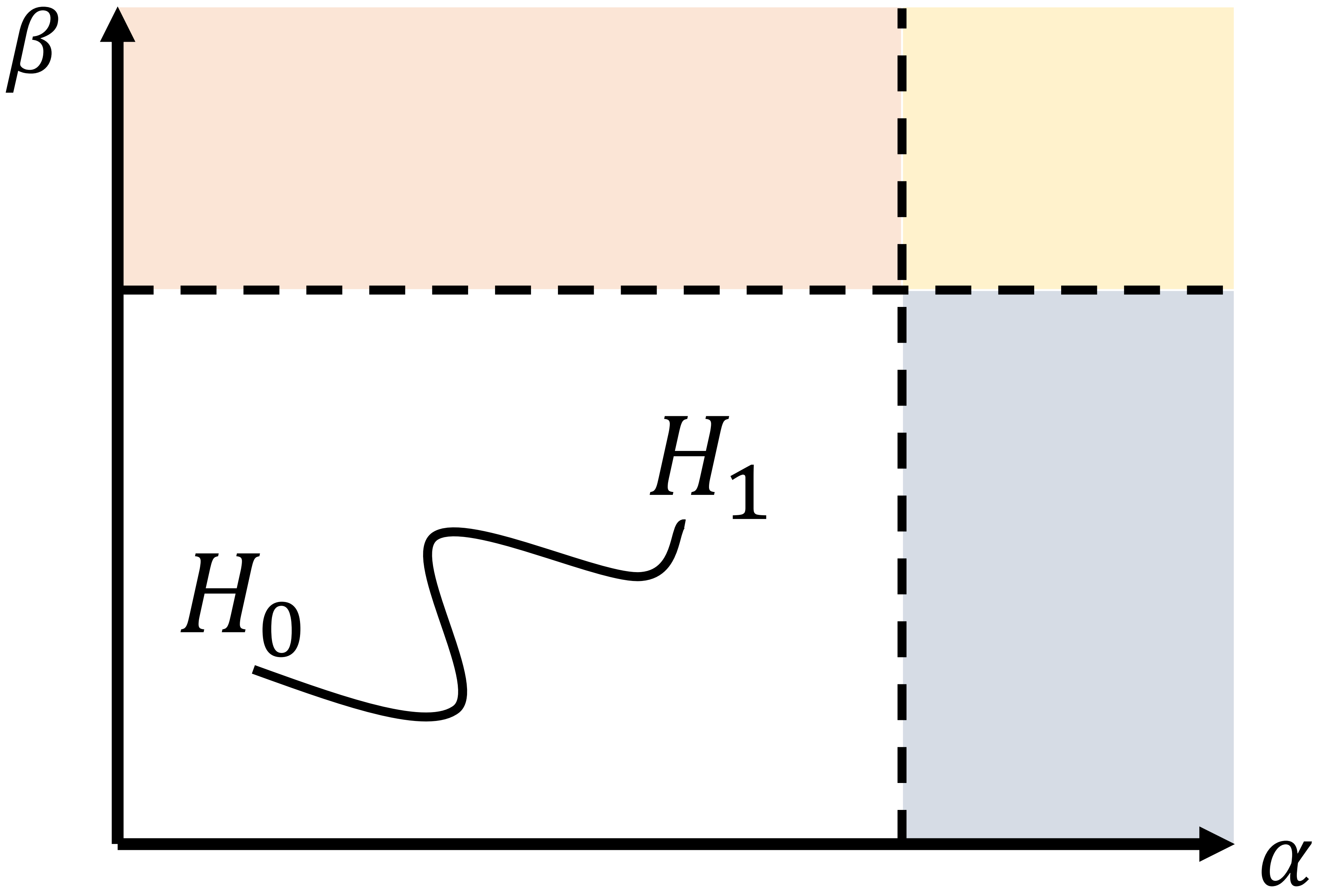
arXiv:1511.08090



$\mathbb{C}^d$
$\mathbb{H} = \bigotimes_v \mathbb{C}^d$
h_v
$H = \sum_v h_v$



E
0
Δ



β
α
H_0
H_1

$$\Pi O \Pi = c(O)\Pi$$

Π-ground space projector

O-local operator

Topological order

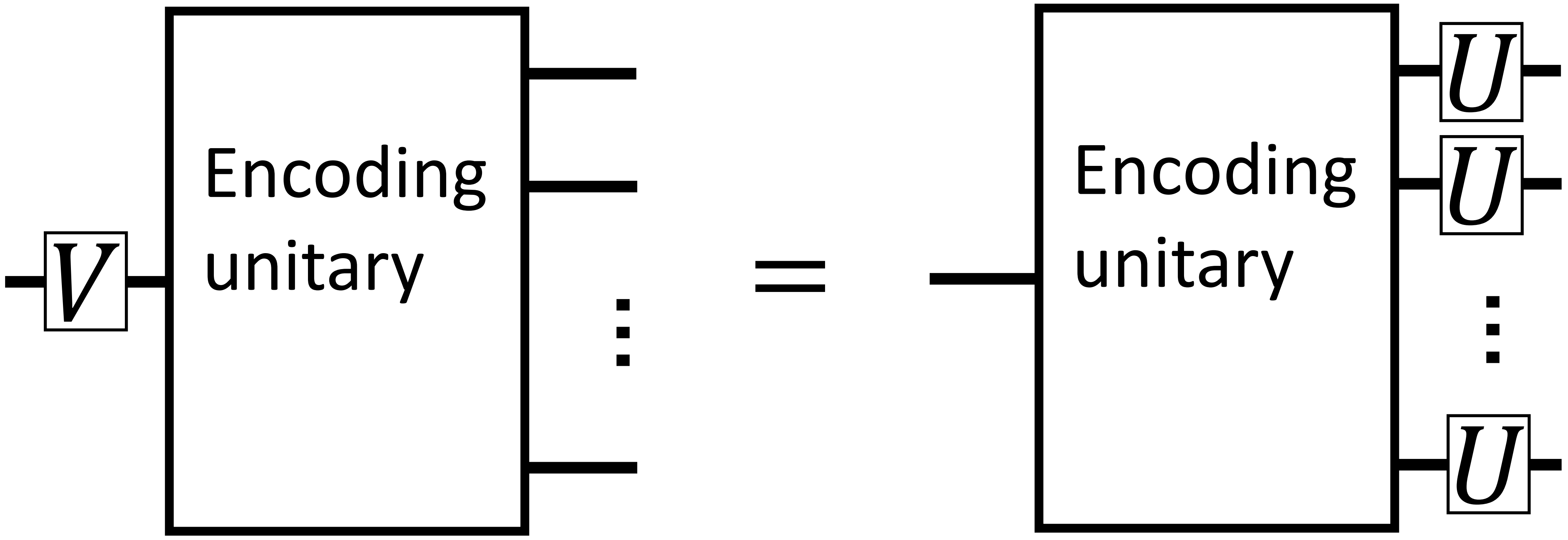
Quantum code

$$\Pi \mathrm{E}_\mathrm{b}^\dagger \mathrm{E}_\mathrm{a} \Pi = c_{ab}\Pi$$

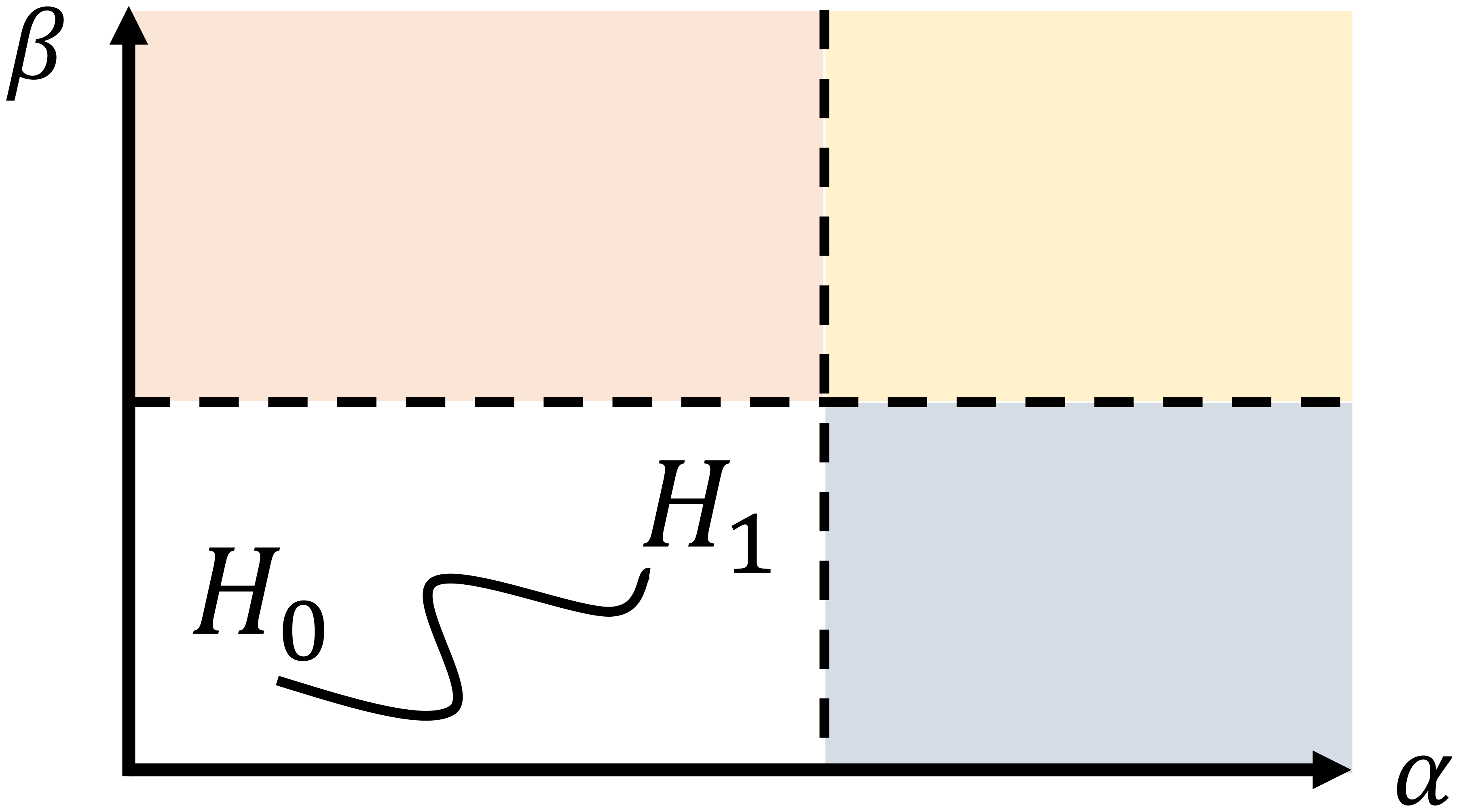
Π-code space projector

E_a-error operator

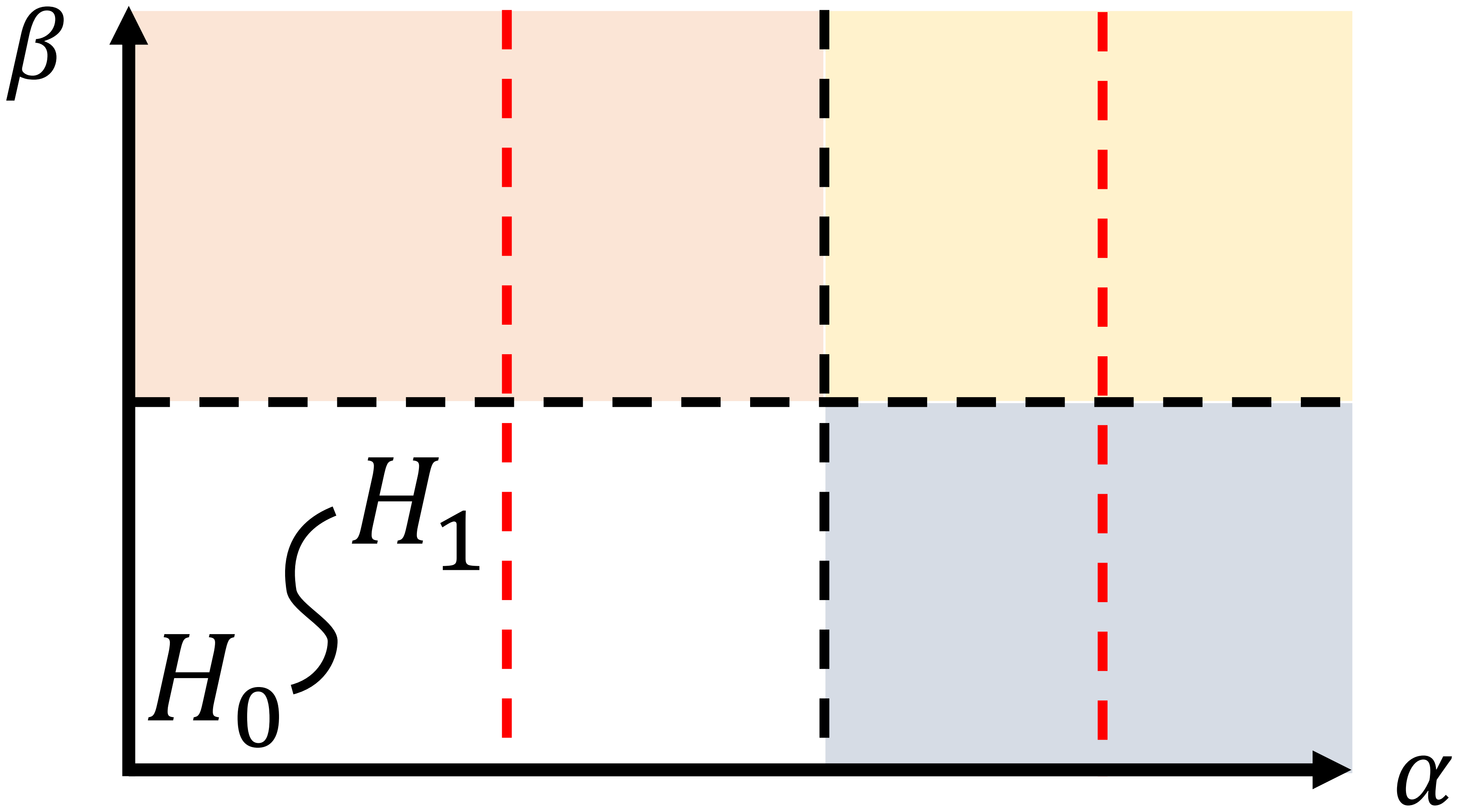
Transversal gates



Transversal $\subset$ Locality preserving



$$H_s = \sum_v h_v^s,$$



$$H_s = \sum_v h_v^s, \qquad U_g^{\otimes N}, g \in G, \qquad [h_v^s, U_g^{\otimes N}] = 0$$

Transversal Gates on Quantum Codes

Eastin Knill etc.
⇒ Restrictions?

→

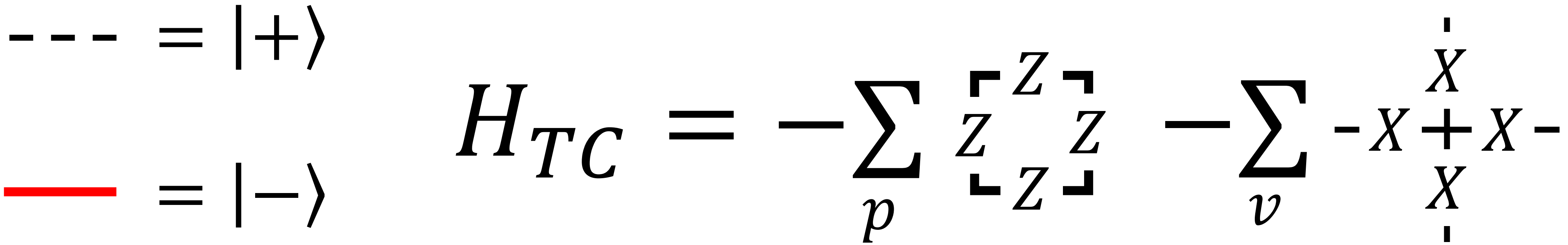
←

All gates based on anyon symmetries can be made transversal

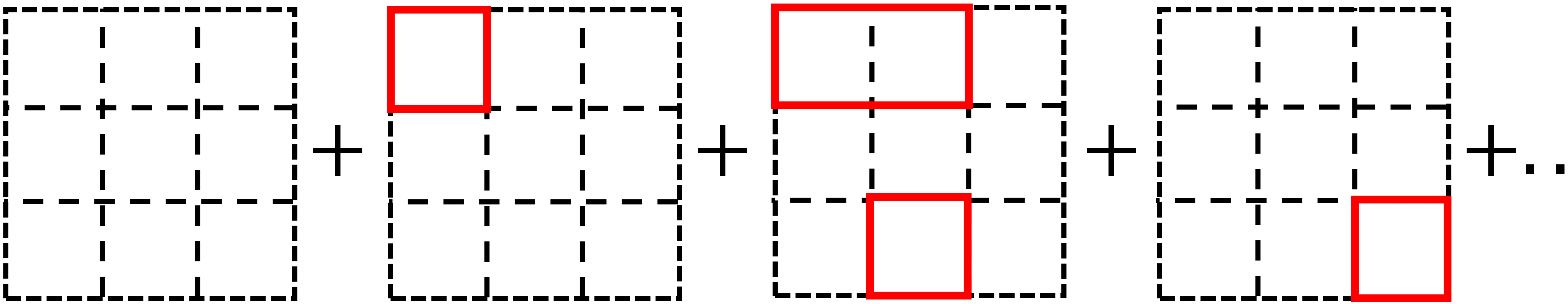
Symmetry Enriched Topological order

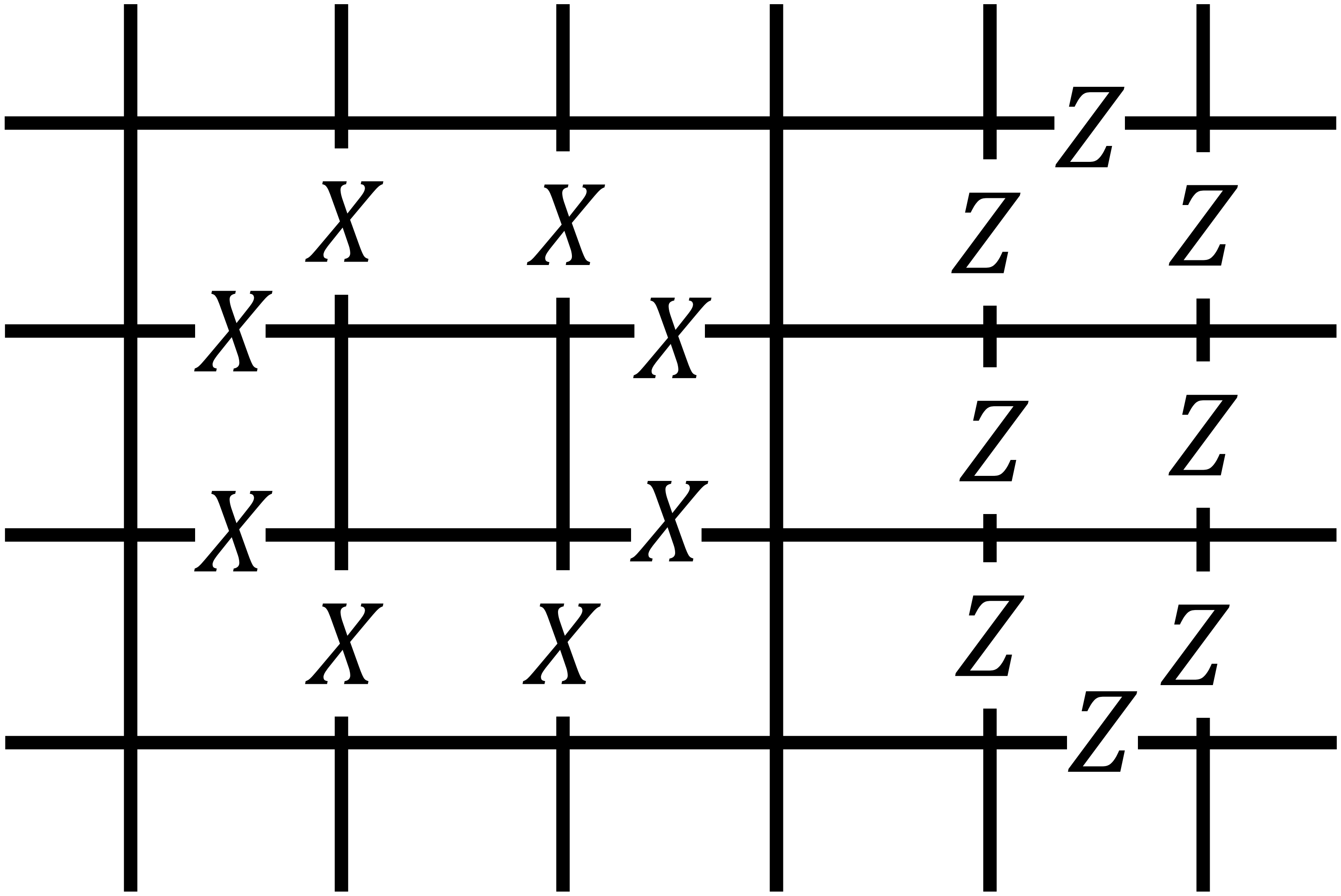
A Motivating Example

Toric code



$|\psi_0\rangle =$




X
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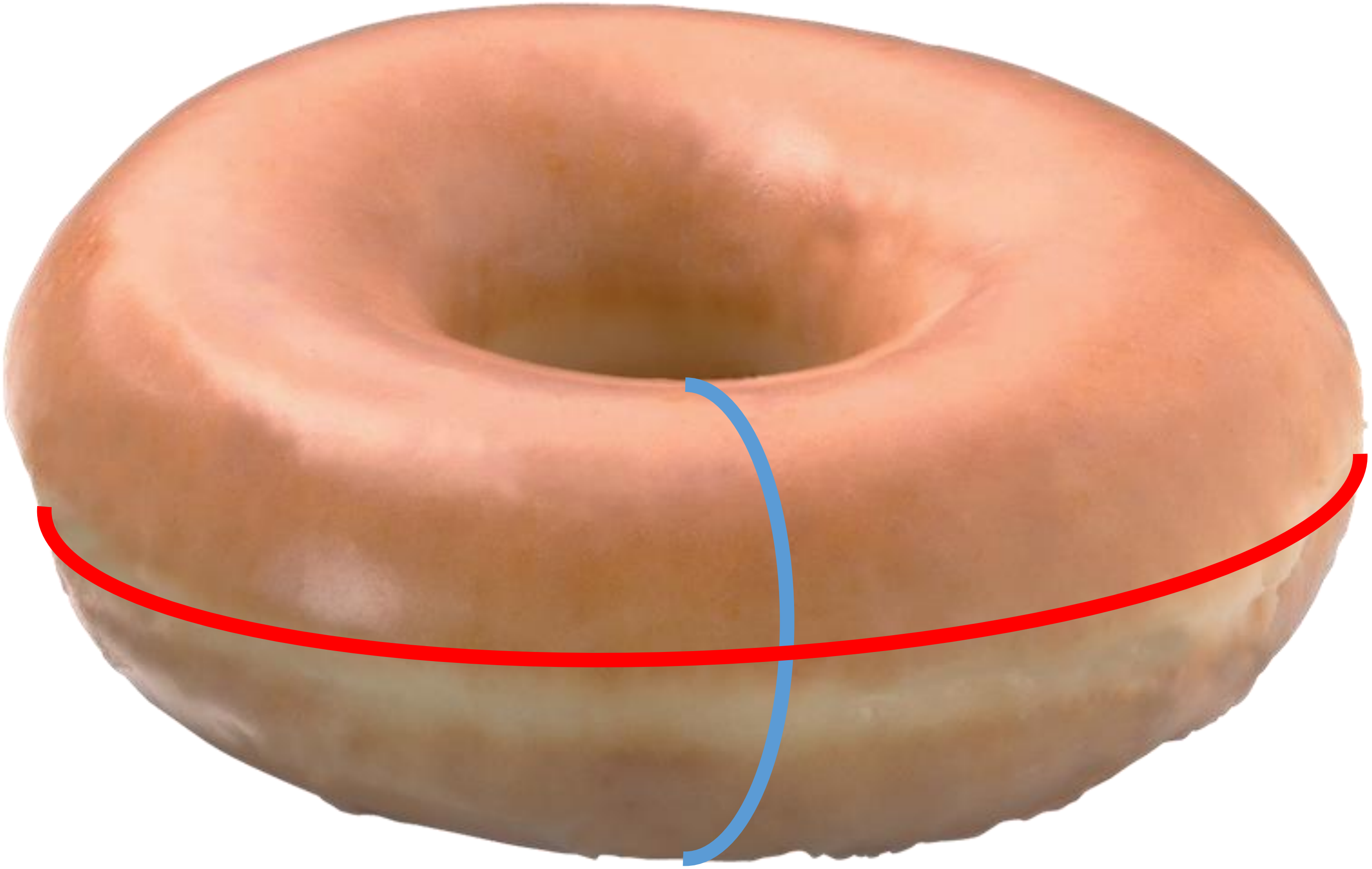
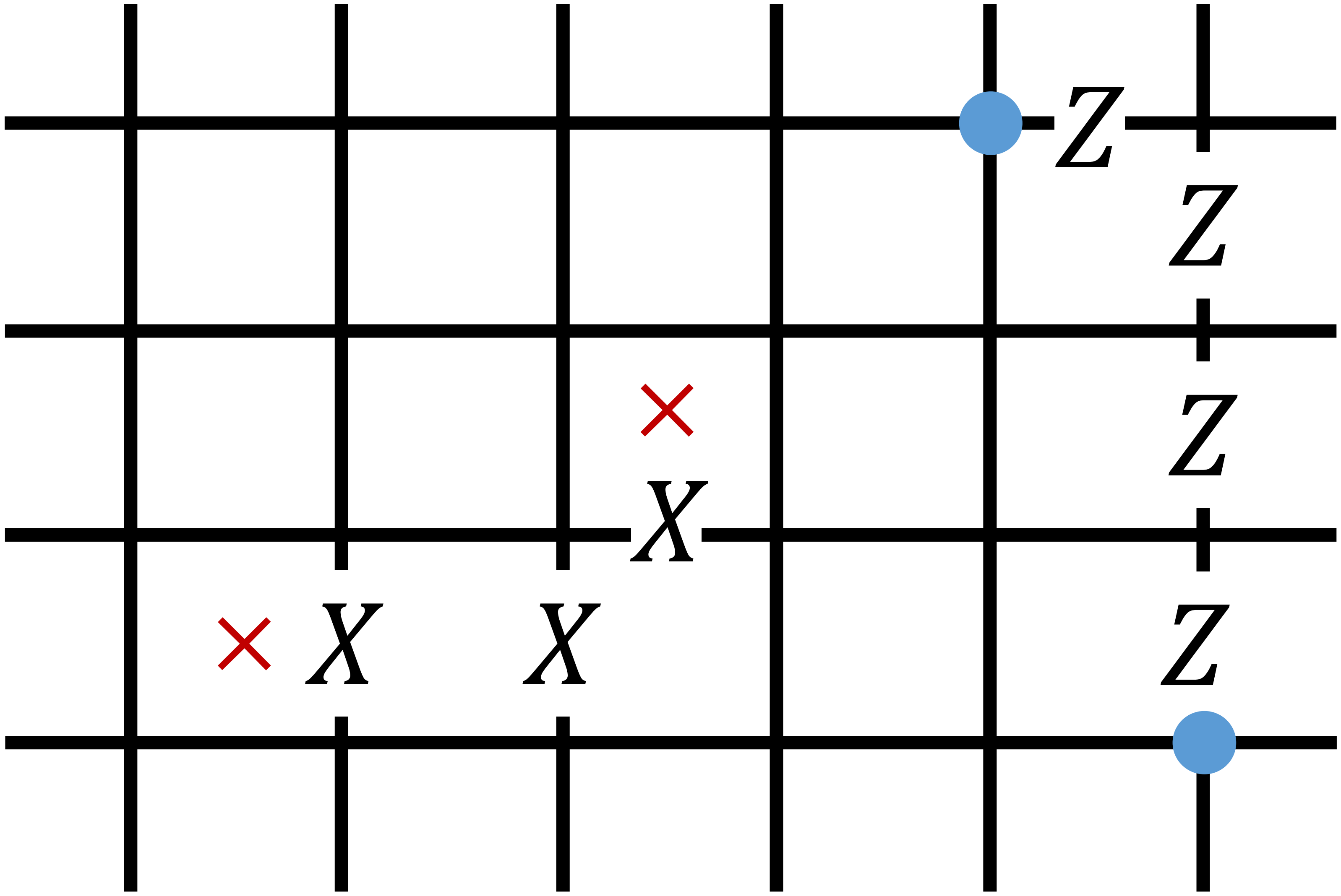




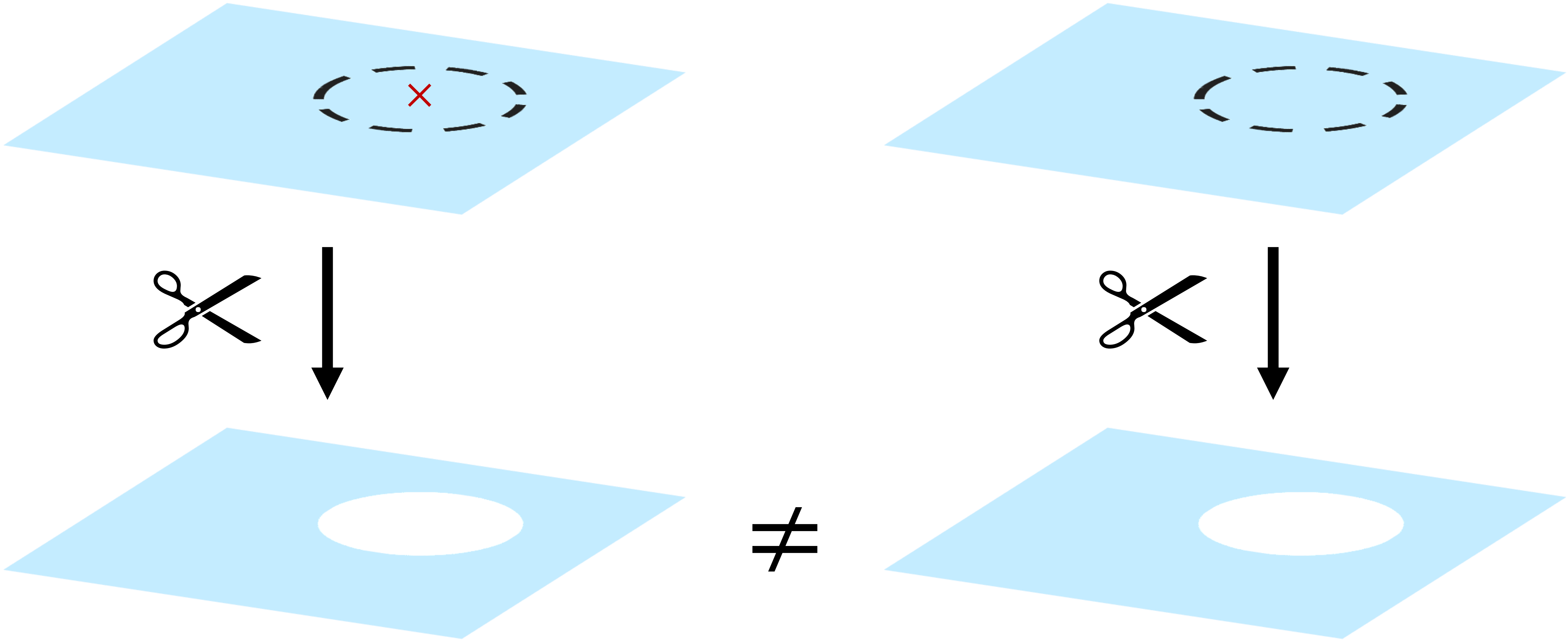
Z
Z
Z
Z
X
X
X

Anyons / Superselection sectors

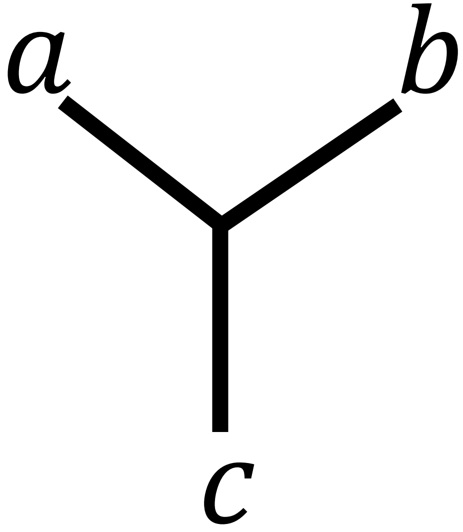
(Modular Tensor Categories)

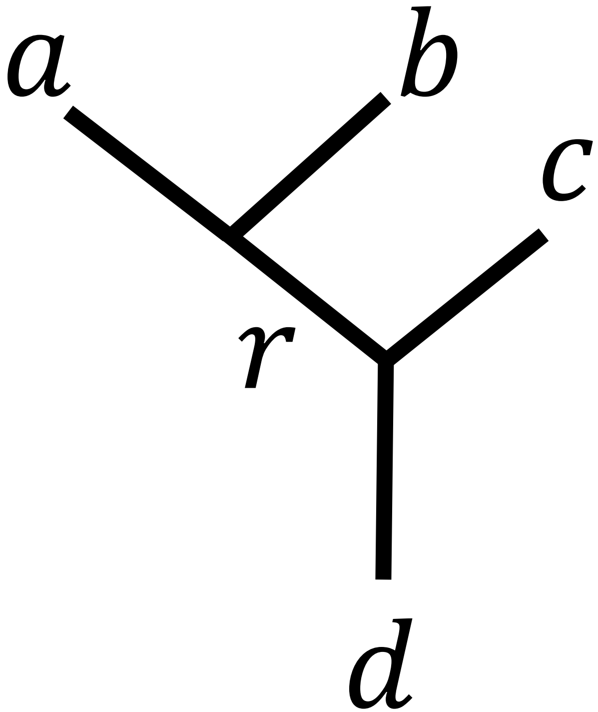
arXiv:cond-mat/0506438 Kitaev

$$[\rho] := \{\rho' \mid \rho' = U\rho U^{-1}, \exists U\}$$

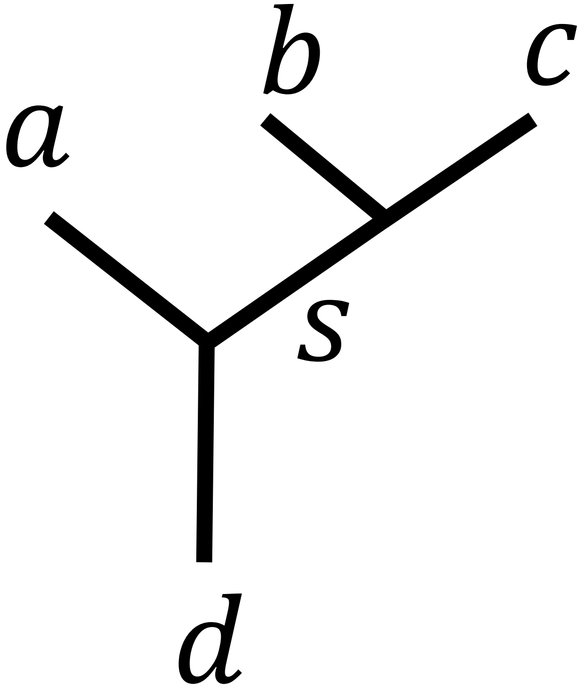


$$a \times b = c + d + \cdots$$
$$= \sum N_{ab}^{c}\, c$$

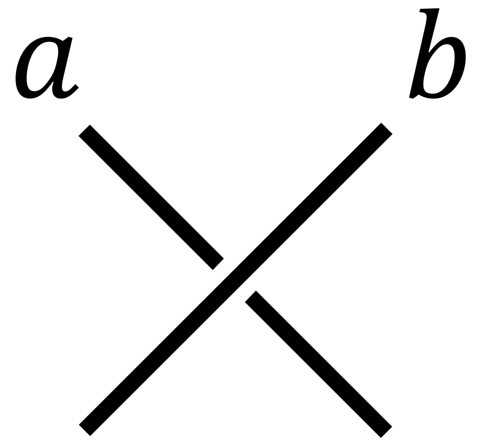


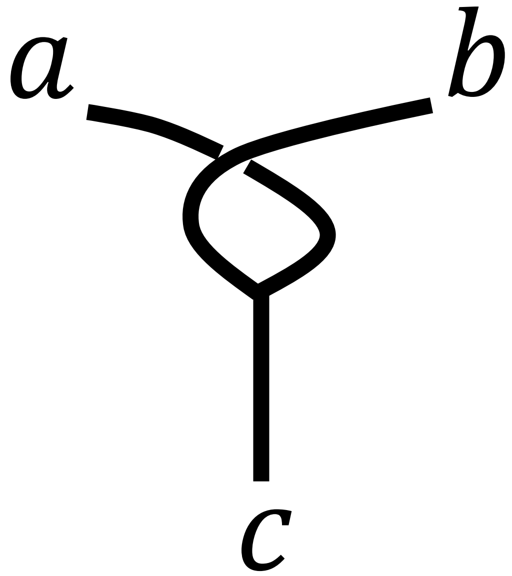


$$= F_{d;rs}^{abc}$$

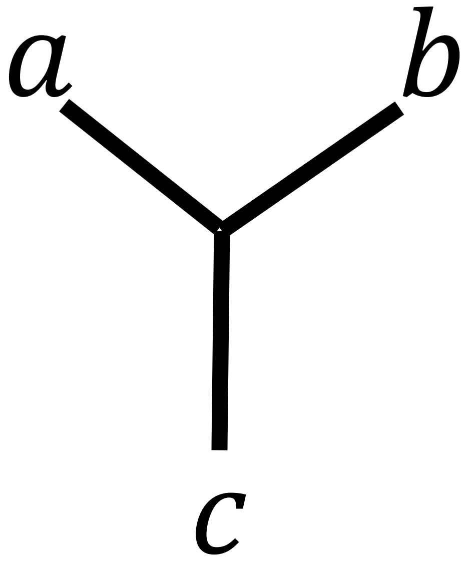


$$R^{ab} =$$

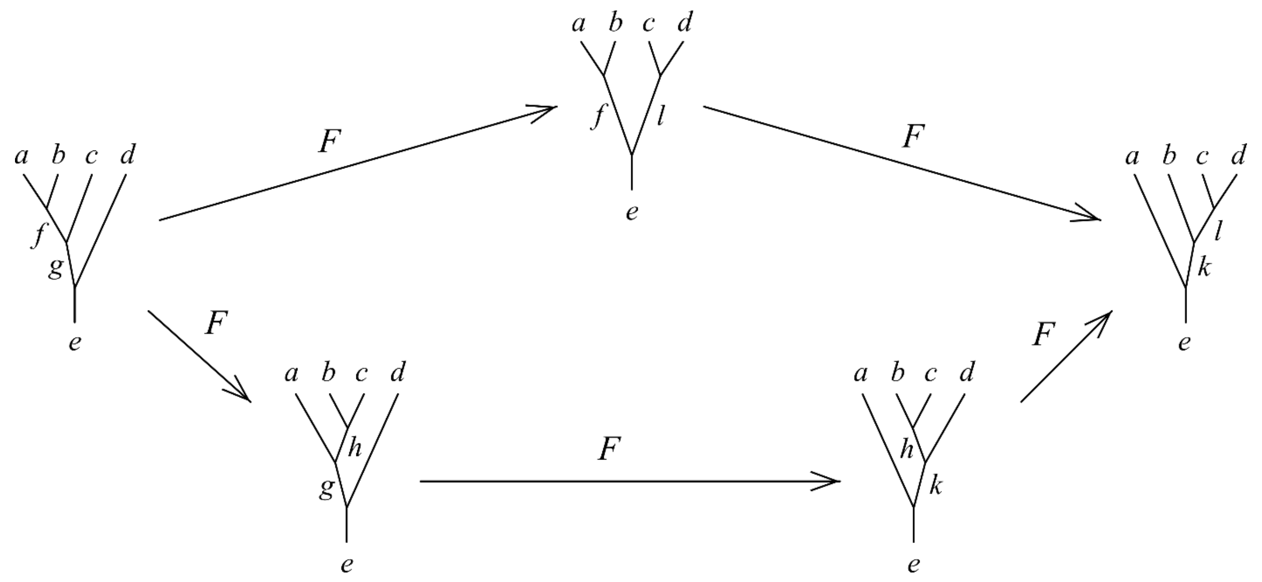




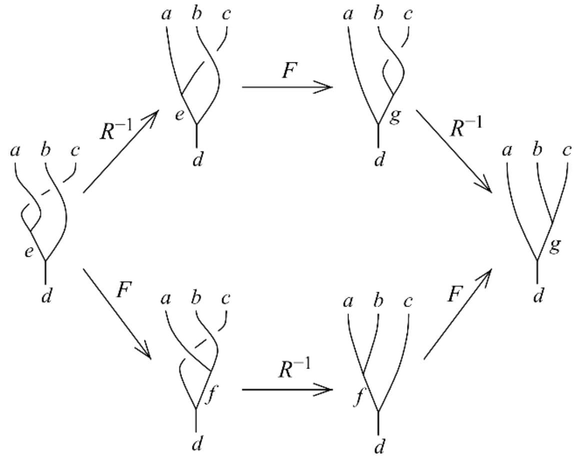
$$= R_c^{ab}$$

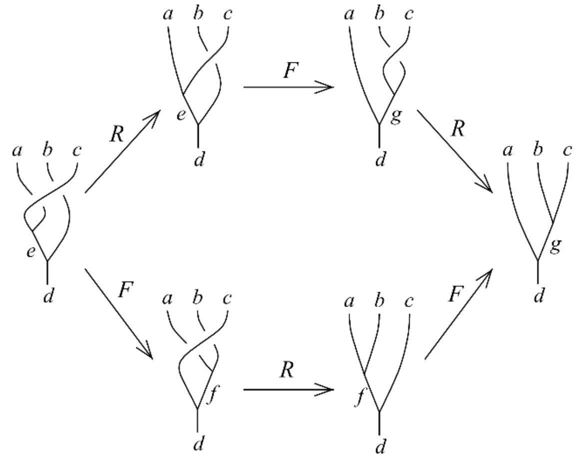


Pentagon



Hexagons





Toric code anyons: $\mathrm{D}(\mathbb{Z}_2)$

$$\mathcal{C} = \{1, e, m, em\}$$

$$e \times m = em$$

$$e \times e = m \times m = 1$$

F symbols trivial

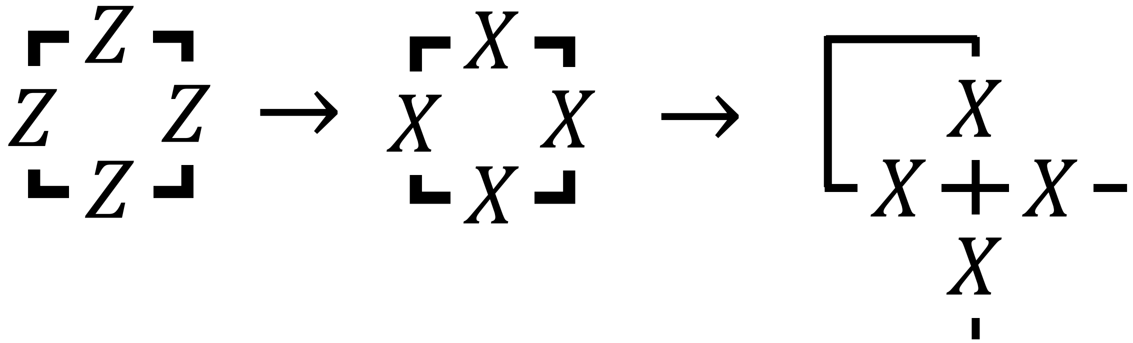
$$R_{em}^{e,m} = -1 = R_{m}^{e,em} = R_{e}^{em,m}$$

$$U: \quad \begin{array}{c} e \leftrightarrow m \\ em \rightarrow em \end{array}$$

$$U: \quad \begin{array}{c} e \leftrightarrow m \\ em \rightarrow em \end{array}$$

$$U H_{TC} U^{\dagger} = H_{TC},$$

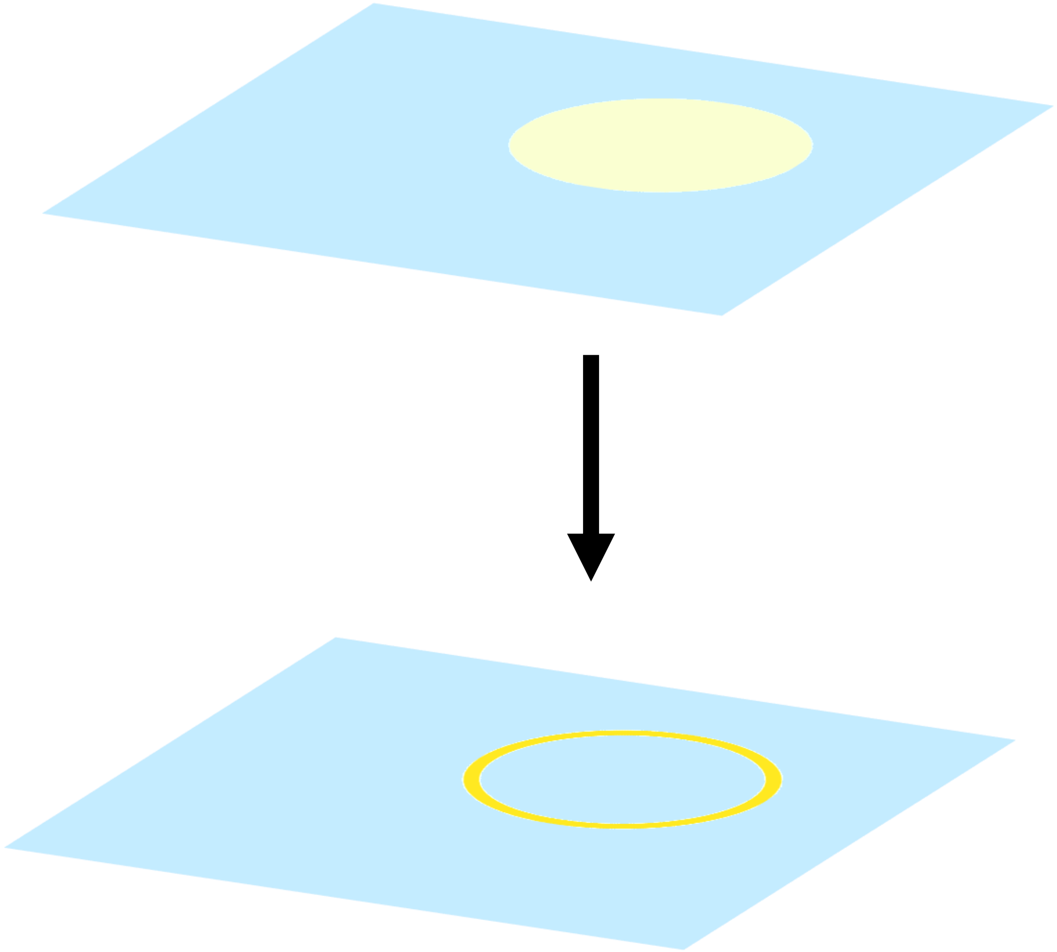
U:

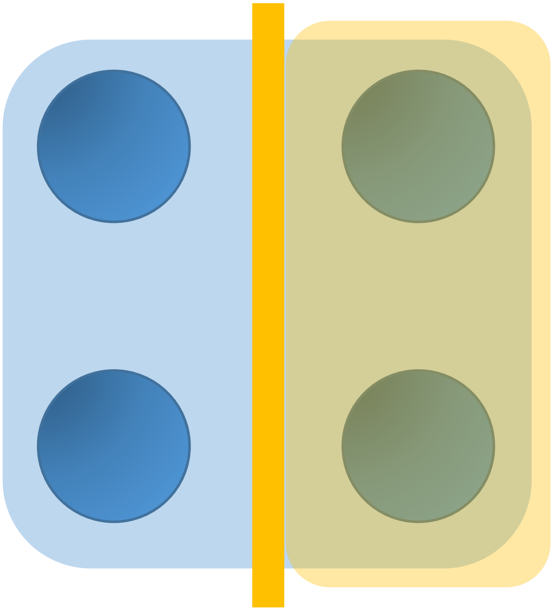


$$U_{\text{Torus}} = \mathrm{H} \otimes \mathrm{H} \; SWAP$$

Symmetry Domain walls

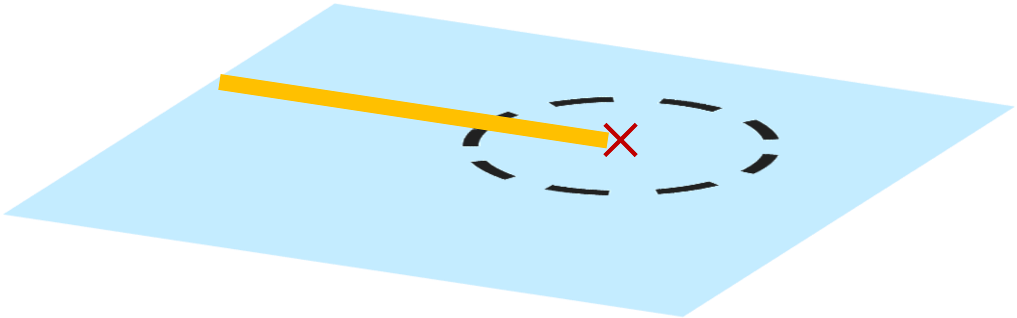
$$h_{|} \rightarrow \pi_g h_{|} \pi_g^{\dagger}$$

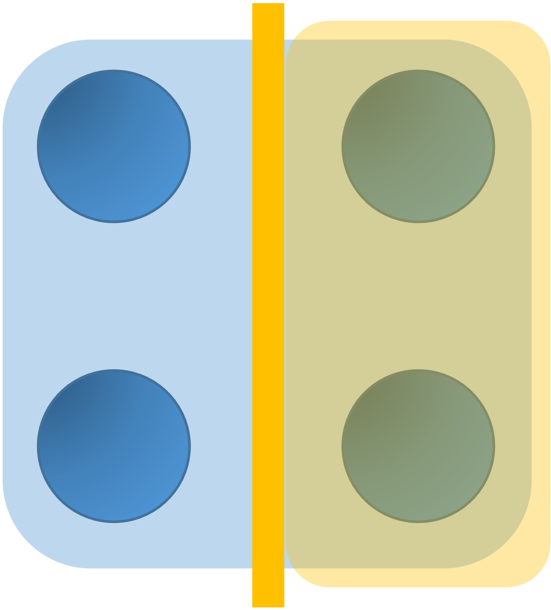




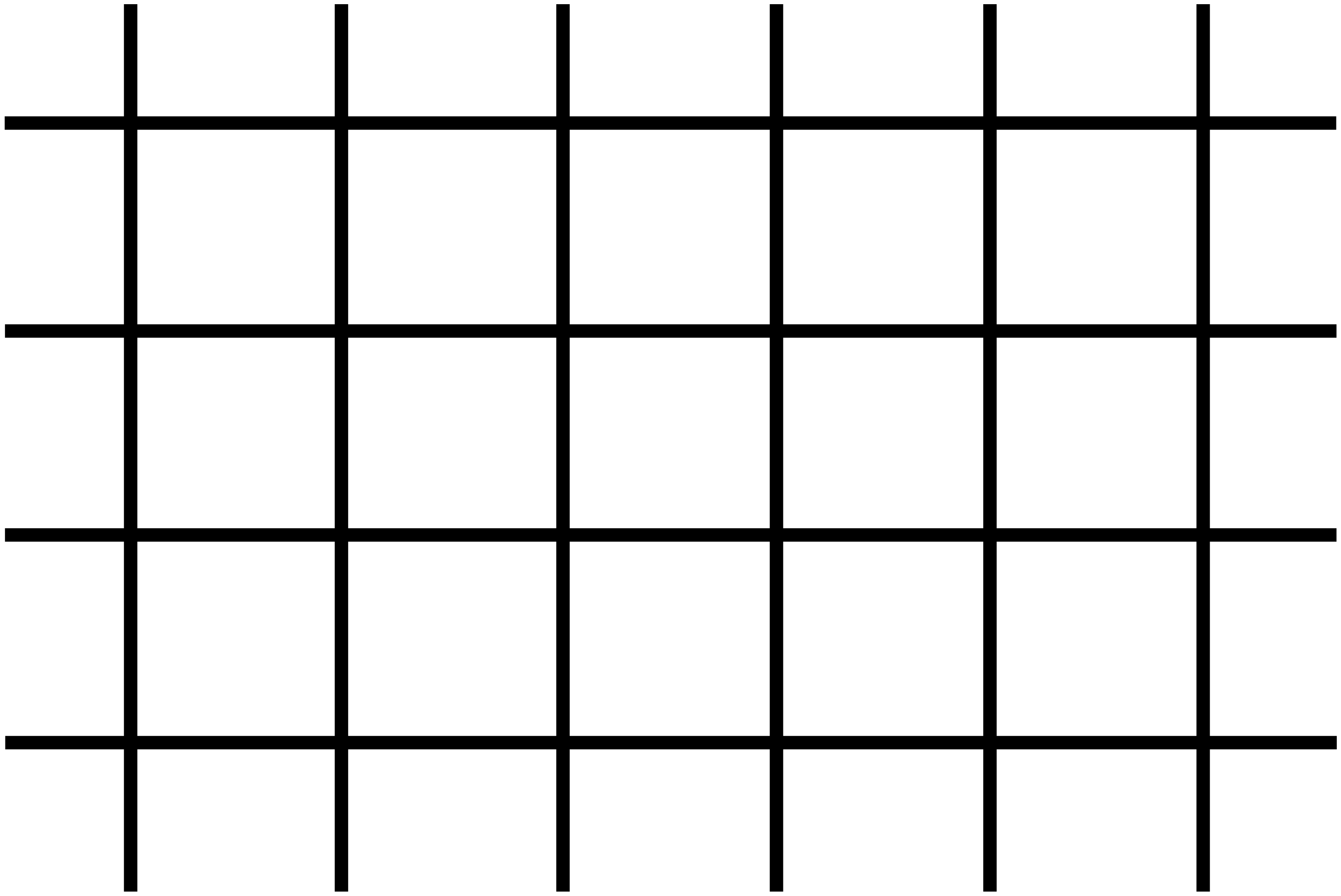
Symmetry Defects

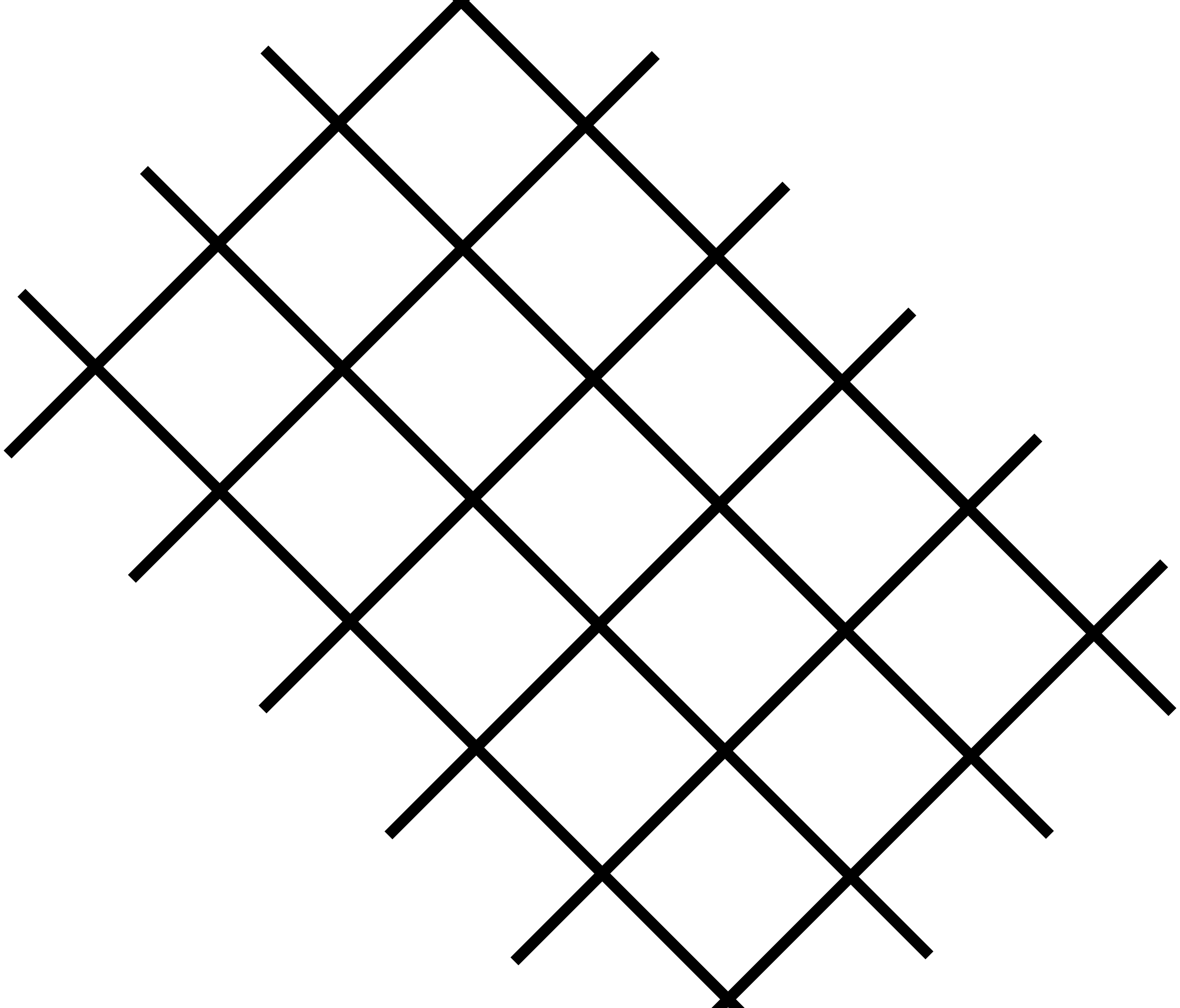
$$h_{|} \rightarrow \pi_g h_{|} \pi_g^{\dagger}$$

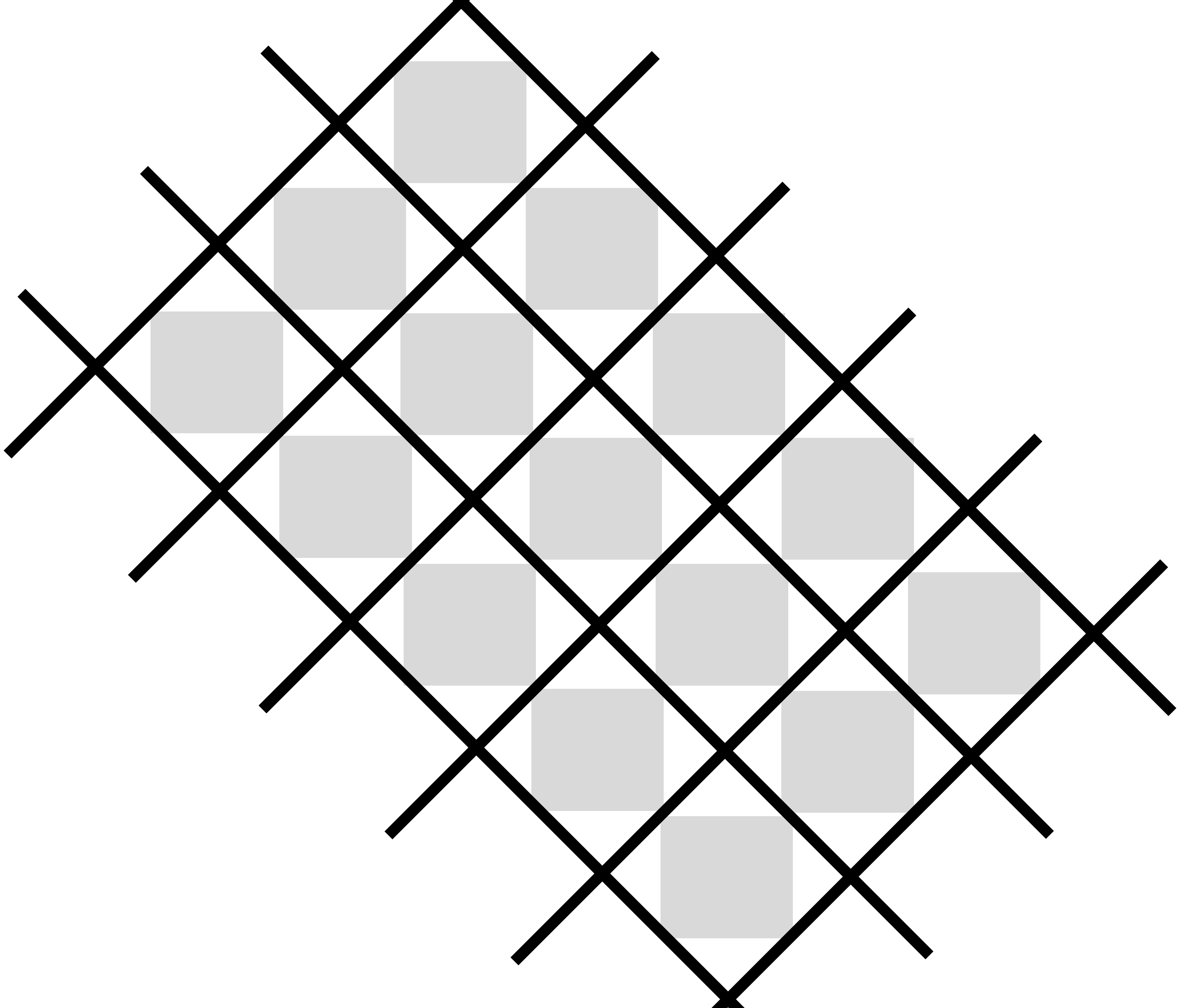


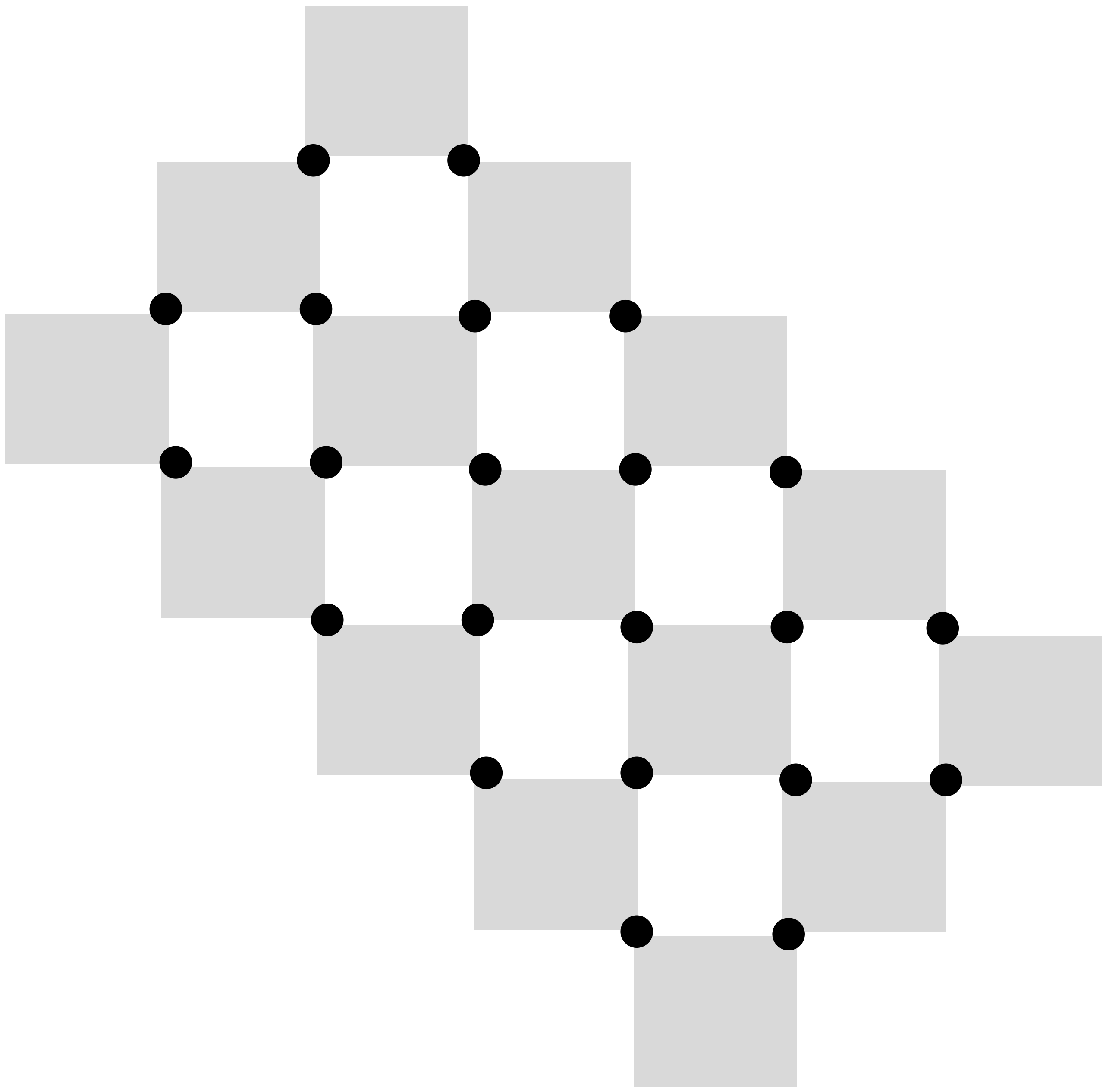


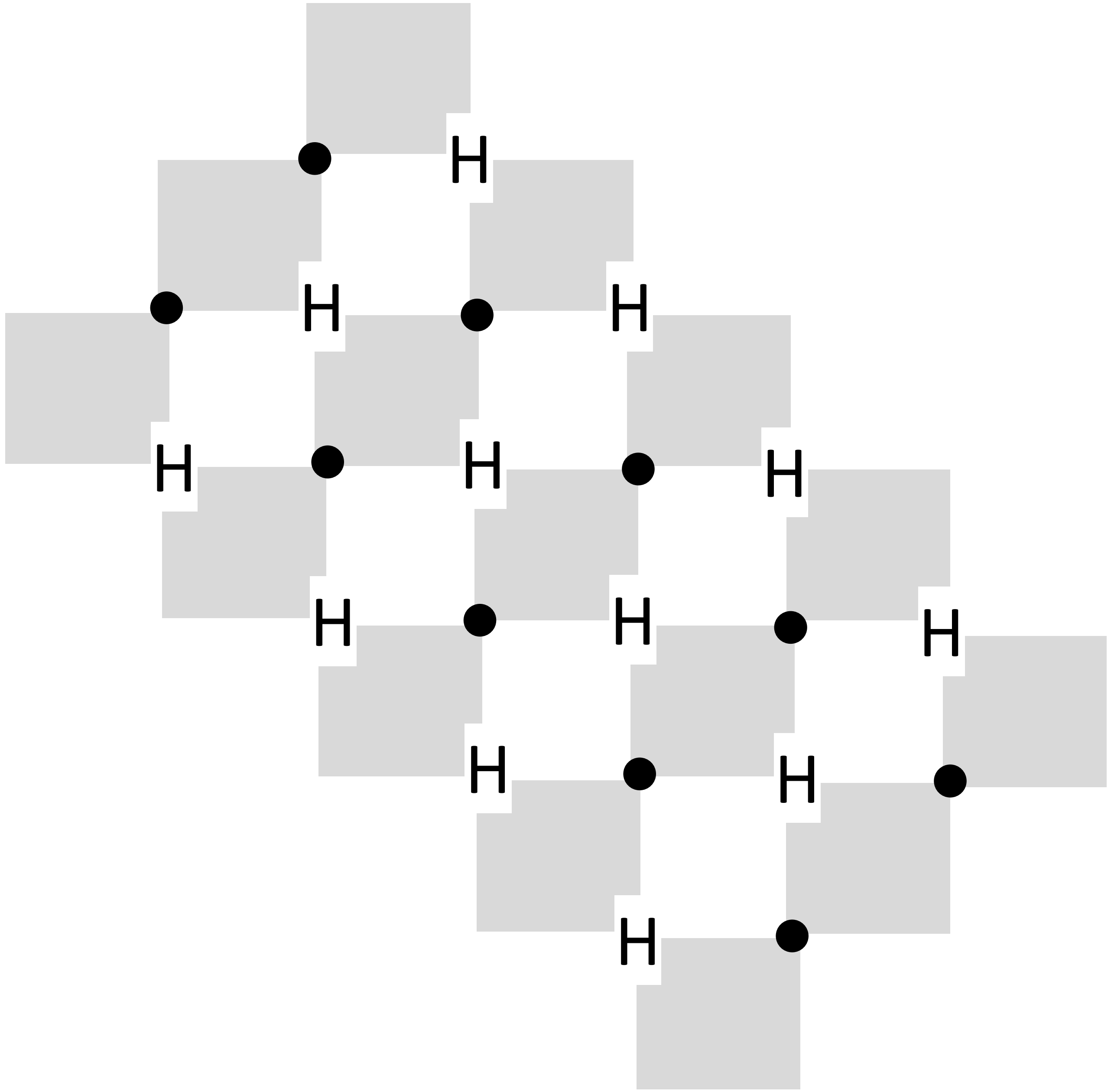
arXiv:cond-mat/0506438 Kitaev
arXiv:1004.1838 Bombin










H
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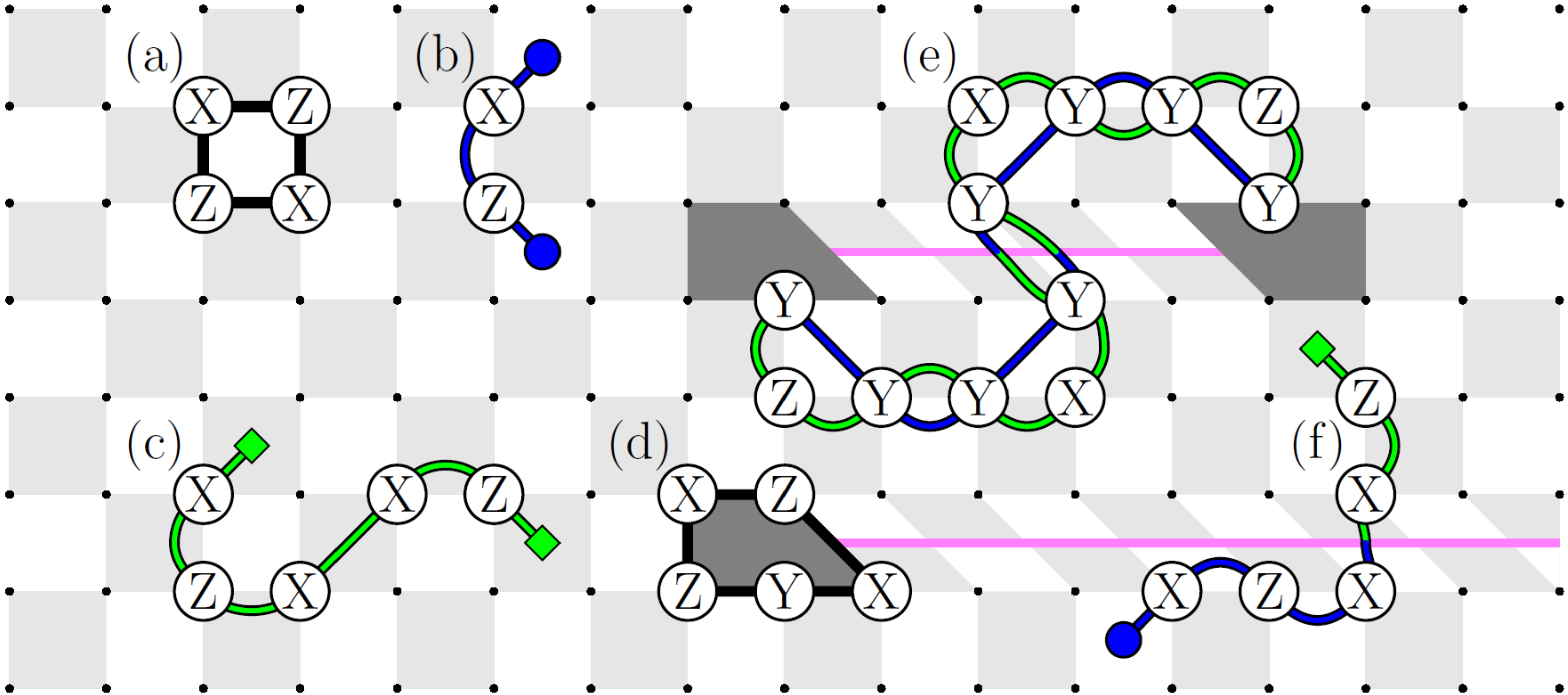


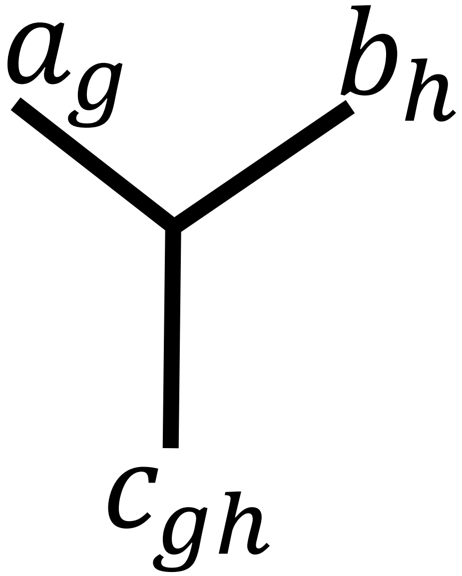
arXiv:1303.4455 Brown, Bartlett, Doherty, Barrett

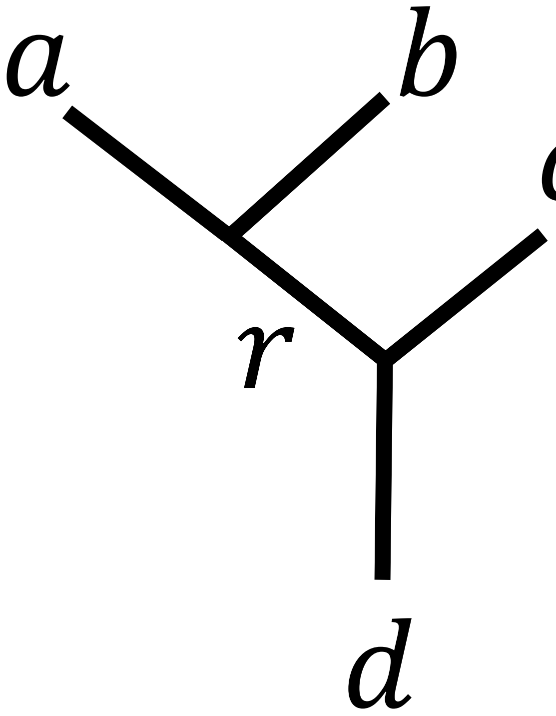
Defects in a topological phase

(Unitary G-Crossed Braided
Fusion Categories)

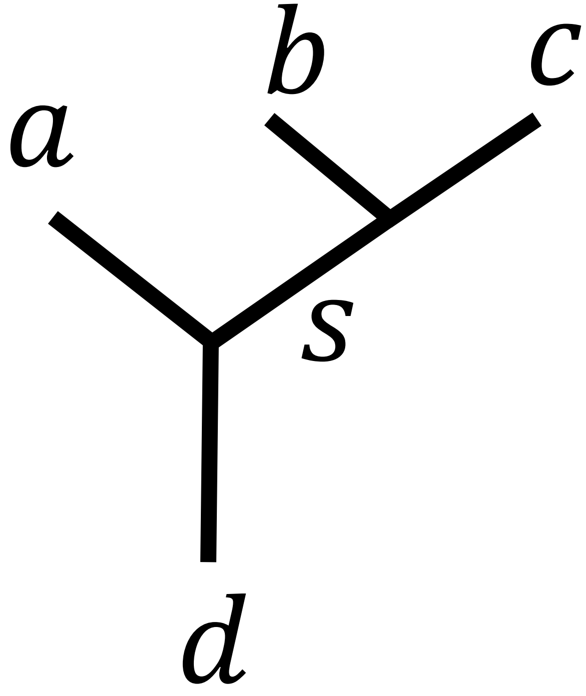
arXiv:1410.4540 Barkeshli, Bonderson, Cheng, Wang

$$a_g \times b_h = c_{gh} + d_{gh} + \cdots$$
$$= \sum N_{ab}^{c}\, c_{gh}$$

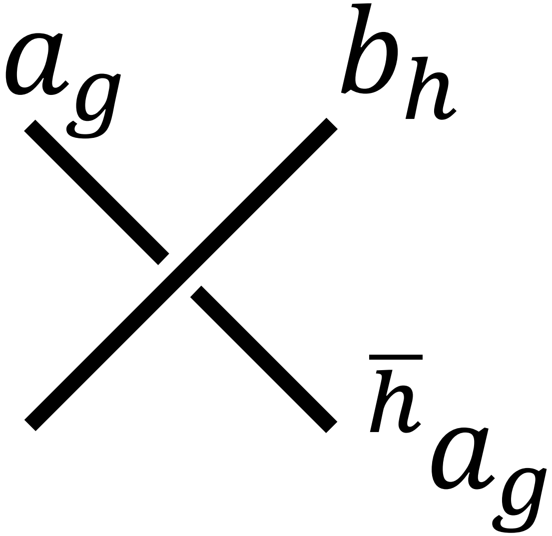


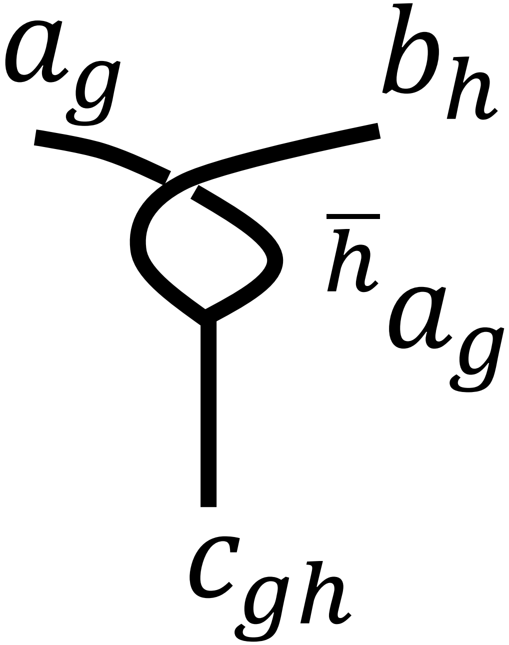


$$= F_{d;rs}^{abc}$$

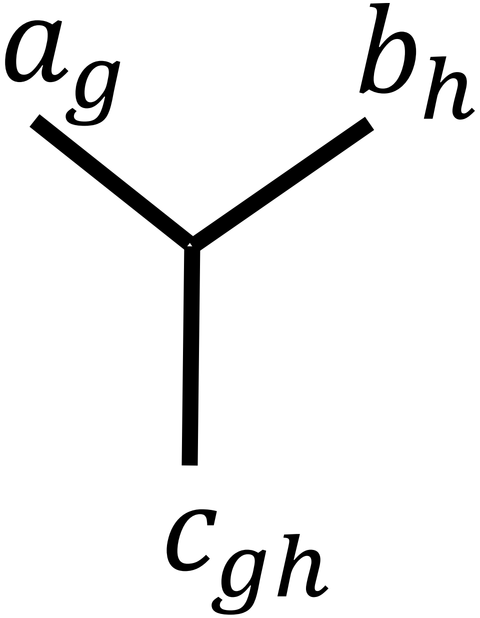


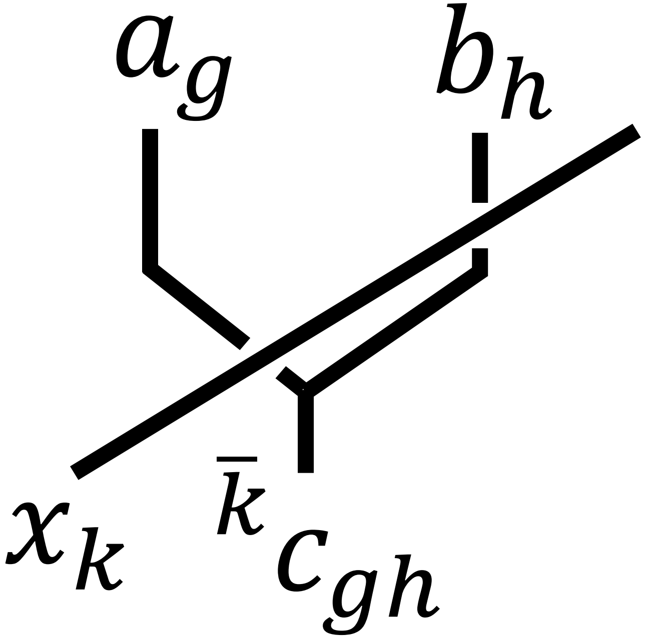
$R^{ab} =$



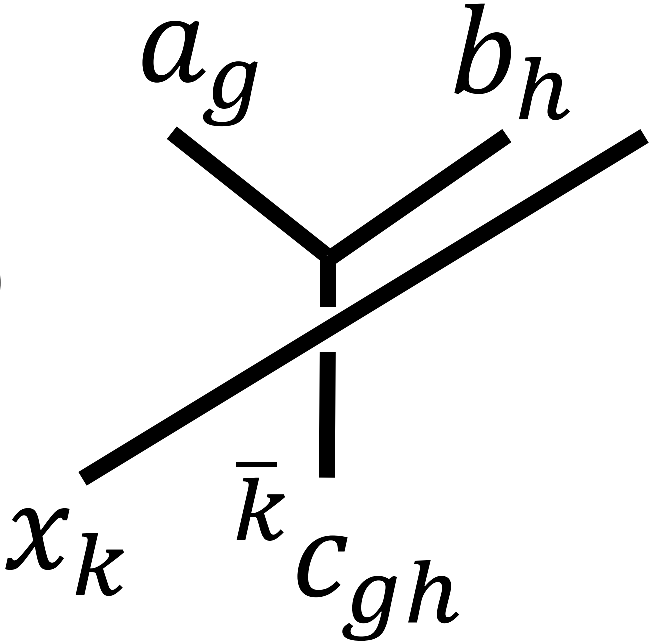


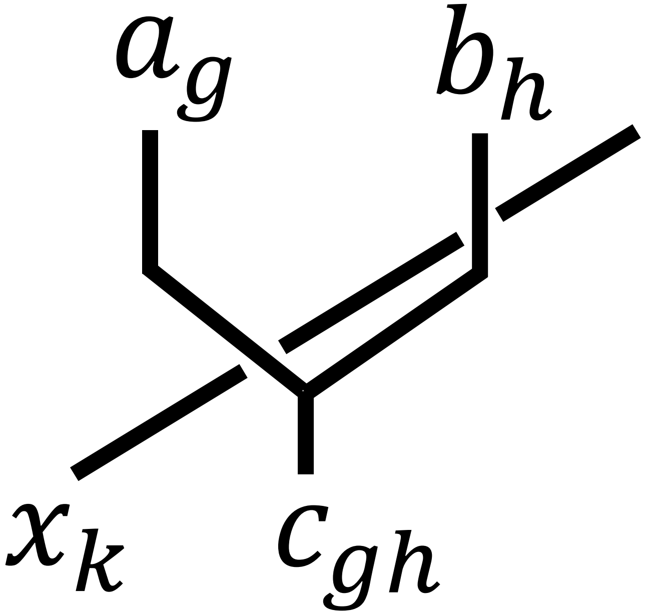
$= R_c^{ab}$



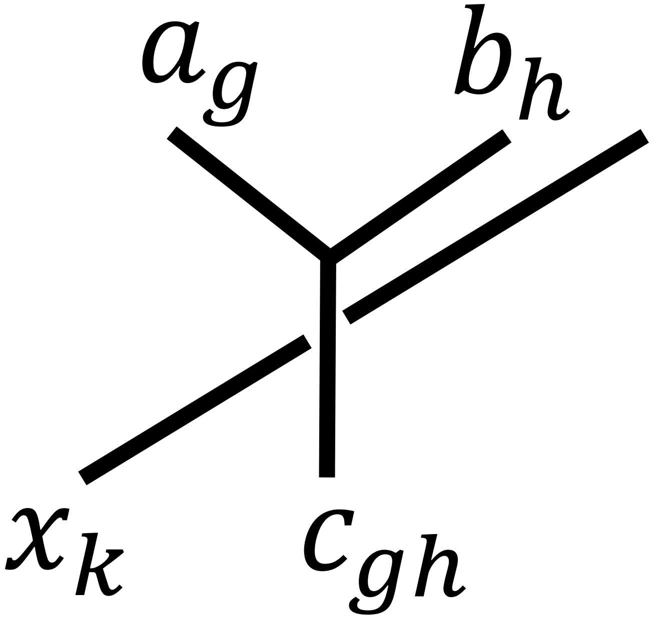


$$= U_k(a, b; c)$$

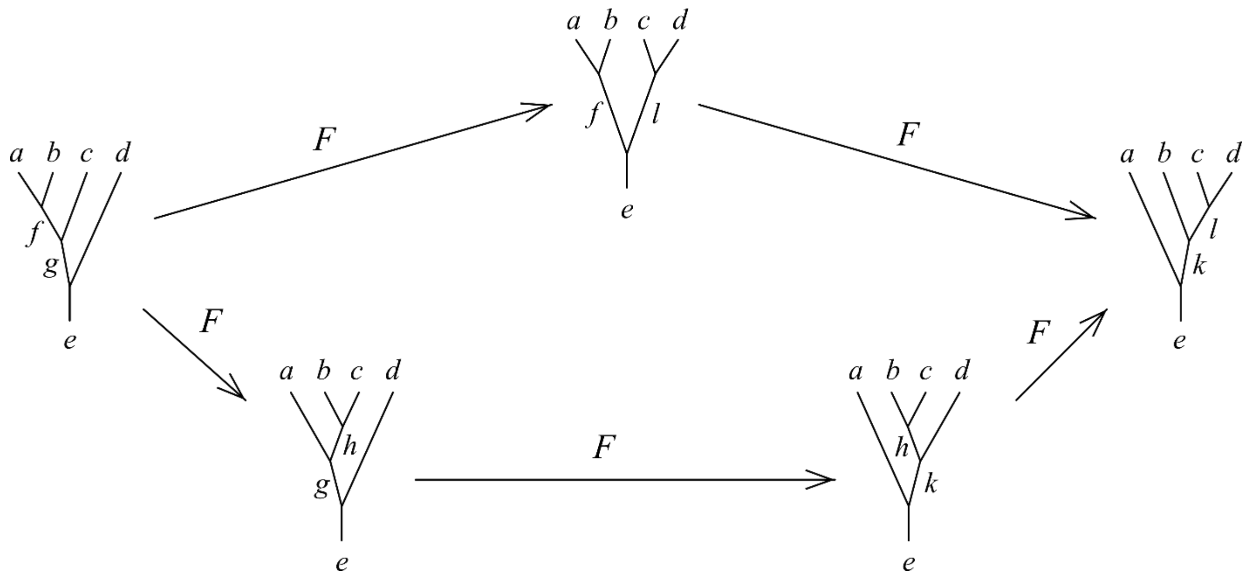




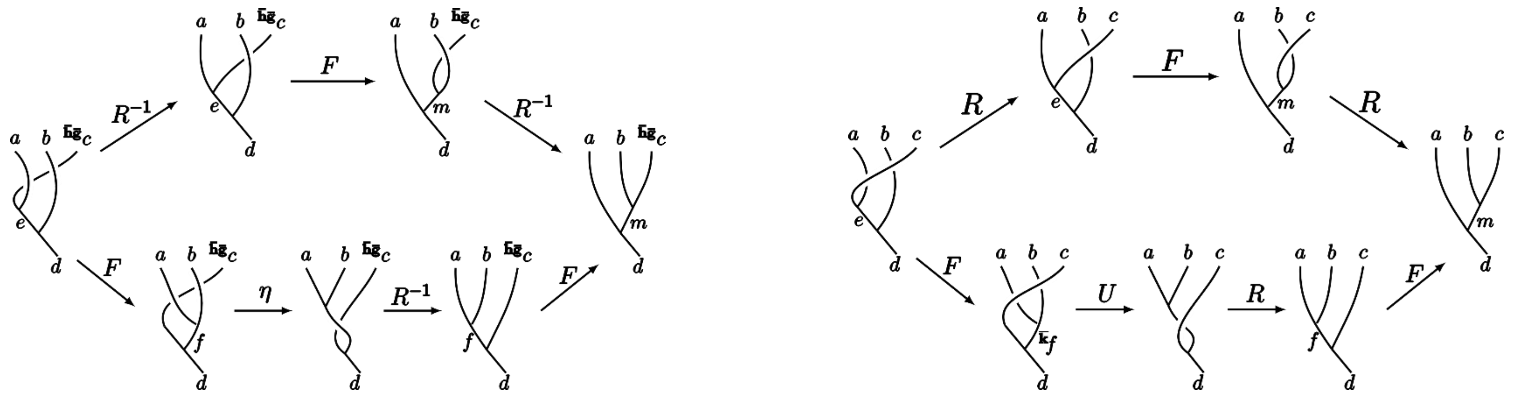
$$= \eta_x(g, h)$$



Pentagon



Heptagons



Toric code with twists $\mathrm{D}(\mathbb{Z}_2)_{\mathbb{Z}_2}$

$$\mathcal{C}_{\mathbb{Z}_2} = \{1, e, m, em\} \bigoplus \{\sigma_+, \sigma_-\}$$

$$\sigma_\pm \times e \; = \sigma_\mp = \sigma_\pm \times m$$

$$\sigma_\pm \times \sigma_\mp = e + m$$

$$\sigma_\pm \times \sigma_\pm = 1 + em$$

$$\sigma_\pm \times em = \sigma_\pm$$

Double Ising

$$\mathcal{C} = \{1, \psi, \sigma\} \otimes \{1, \bar{\psi}, \bar{\sigma}\}$$

$$\psi \times \psi = 1$$

$$\sigma \times \sigma = 1 + \psi$$

$$\sigma \times \psi = \sigma$$

$$\sigma_{\pm} \times \sigma_{\pm} = 1 + em$$

$$\sigma_{\pm} \times em = \sigma_{\pm}$$

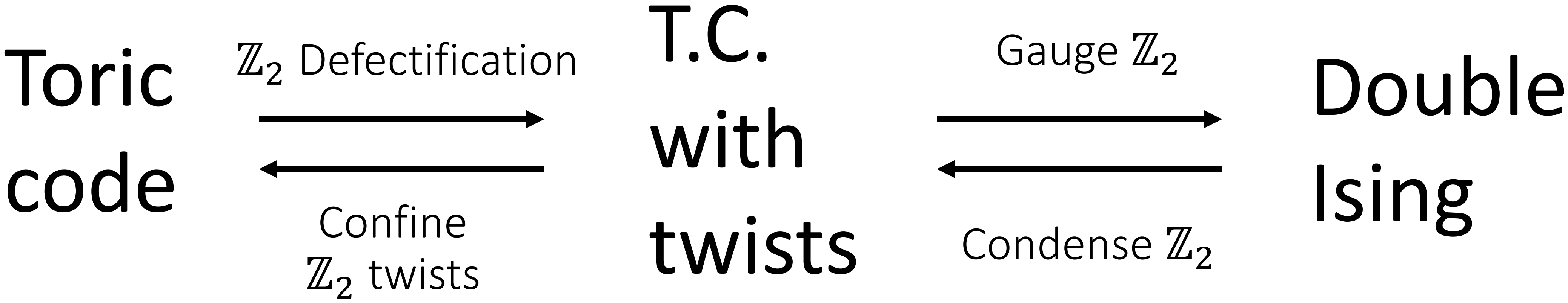
Toric code twists

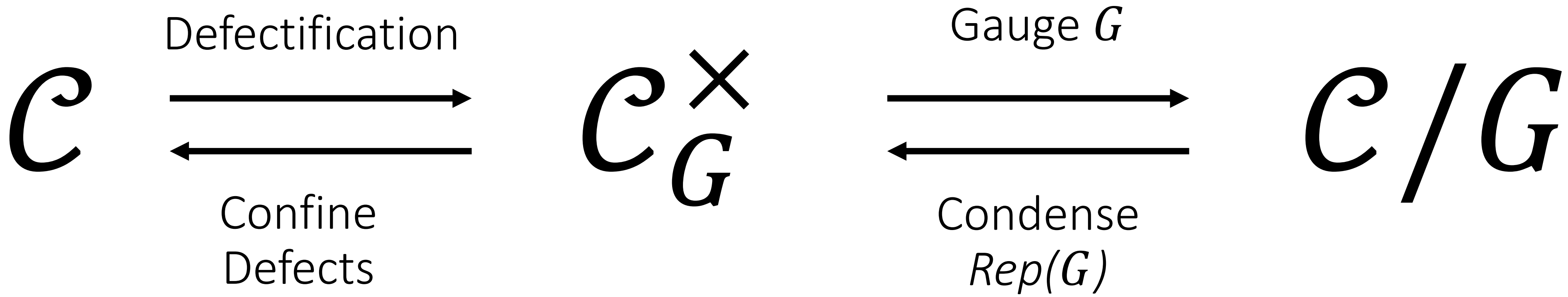
$\updownarrow$?

$$\sigma \times \sigma = 1 + \psi$$

$$\sigma \times \psi = \sigma$$

Double Ising


Toric code
$\mathbb{Z}_2$ Defectification
Confine $\mathbb{Z}_2$ twists
T.C. with twists
Gauge $\mathbb{Z}_2$
Condense $\mathbb{Z}_2$
Double Ising


$\mathcal{C}$
Defectification
Confine Defects
$\mathcal{C}_G^{\times}$
Gauge G
Condense $Rep(G)$
$\mathcal{C}/G$

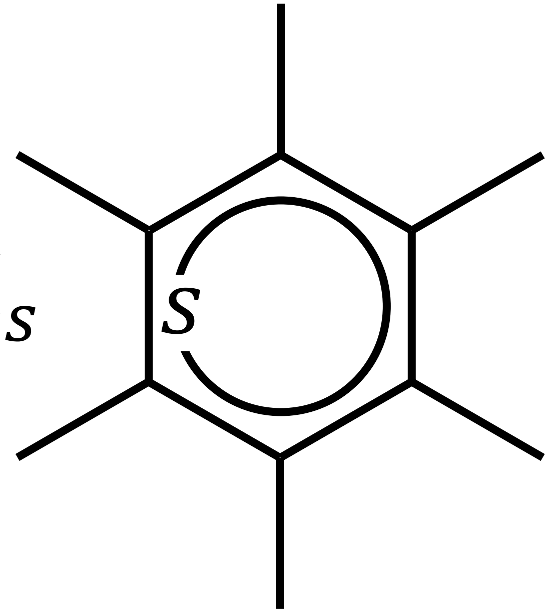
Back to the lattice

String-net models

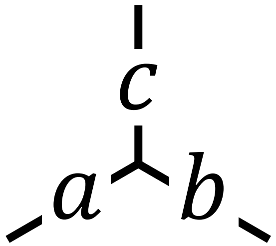
$-s- \;= |s\rangle$ $\qquad s \in \mathcal{C}$ $\qquad$ e.g. $\mathcal{C} = \mathbb{Z}_2$

$$= \{1, \psi, \sigma\}$$

$H = -\sum_{p,s} d_s$



$-\sum_{v} \delta_{ab}^{c}$



arXiv:cond-mat/0404617 Levin Wen

Input:

$$\mathcal{C}$$

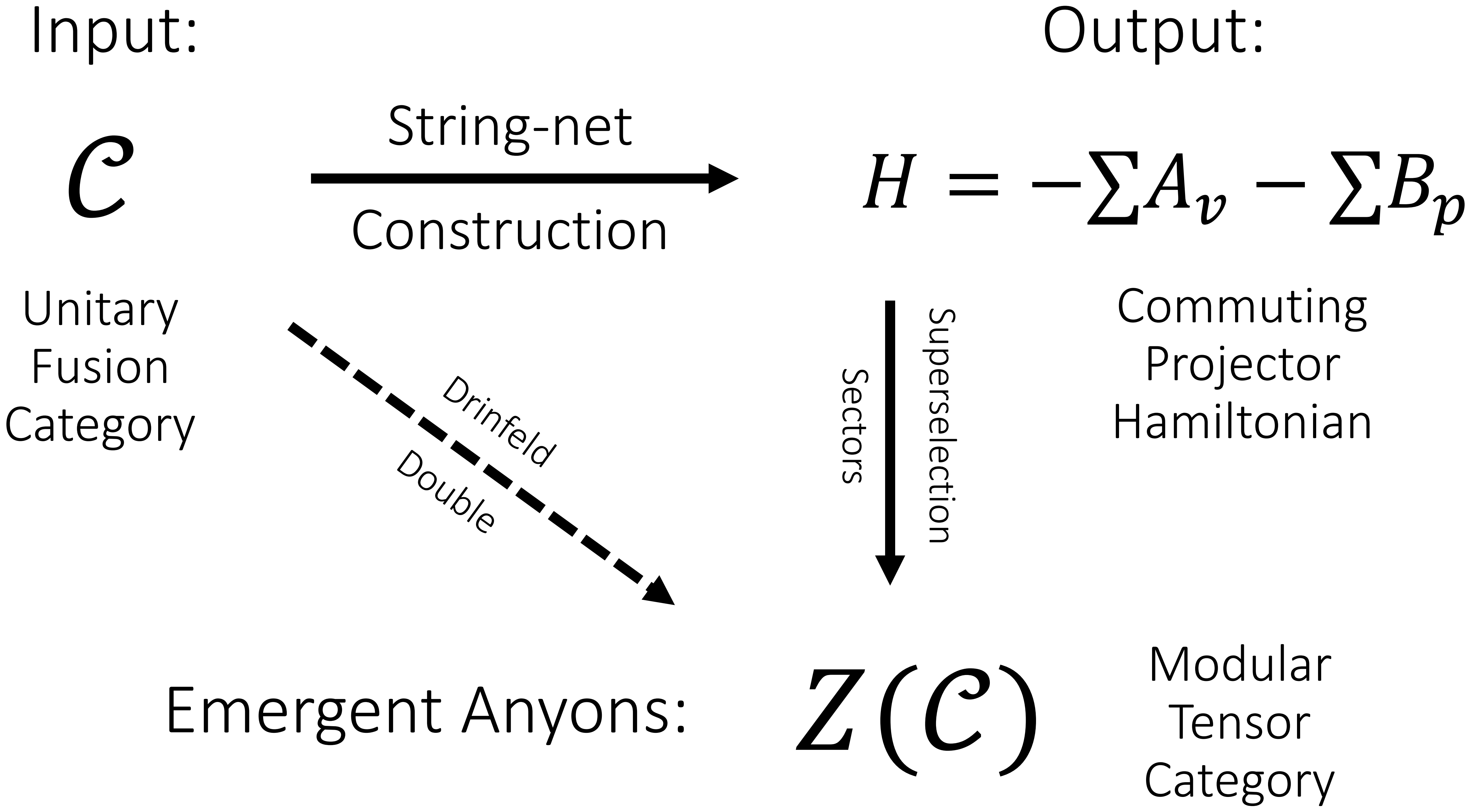
Unitary
Fusion
Category

String-net
→
Construction

Output:

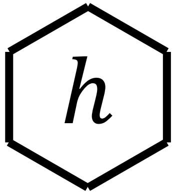
$$H = -\sum A_v - \sum B_p$$

Commuting
Projector
Hamiltonian

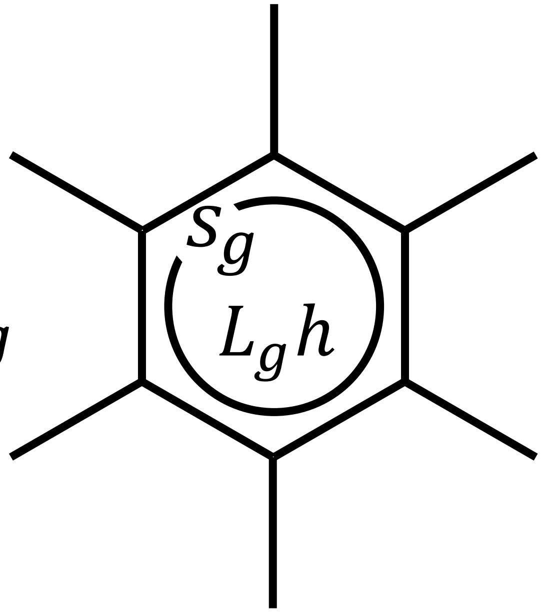

Input:
Output:
$\mathcal{C}$
String-net
Construction
$H = -\sum A_v - \sum B_p$
Unitary Fusion Category
Commuting Projector Hamiltonian
Drinfeld
Double
Superselection
Sectors
Emergent Anyons:
$Z(\mathcal{C})$
Modular Tensor Category

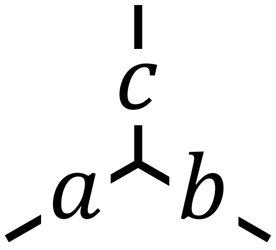
G-graded SET String-net models

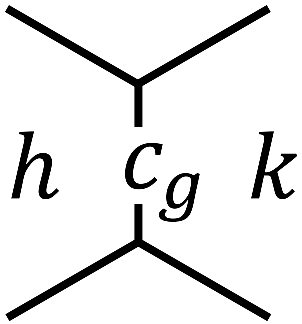
$-s_g- = |s_g\rangle$ $\quad s \in \mathcal{C}_G$ $\quad$ e.g. $\mathcal{C}_{\mathbb{Z}_2} = \{1, \psi\} \oplus \{\sigma\}$



$= |h\rangle$ $\quad h \in G$

$-\sum_{p, s_g} d_{s_g}$


$-\sum_v \delta^c_{ab}$


$-\sum_v \delta^g_{hk^{-1}}$


arXiv:1606.08482, arXiv:1606.07816

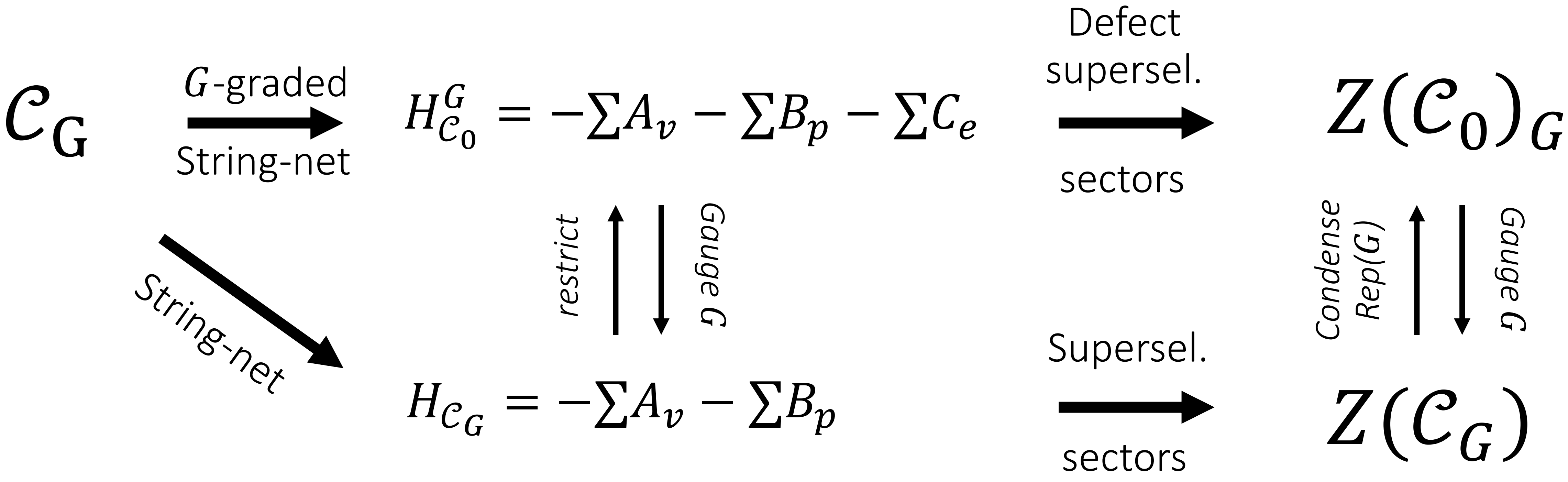
Input:

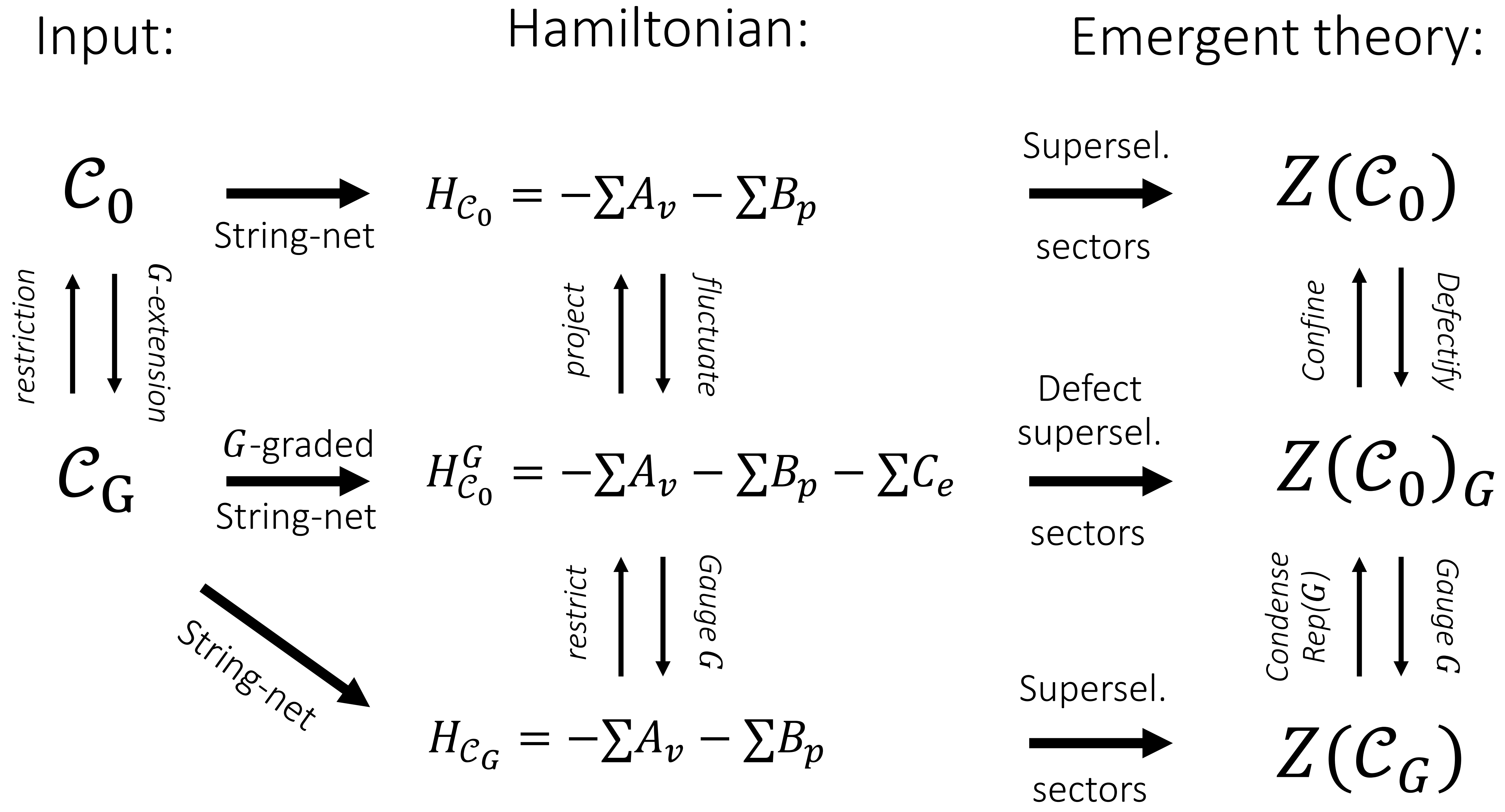
Hamiltonian:

Emergent theory:

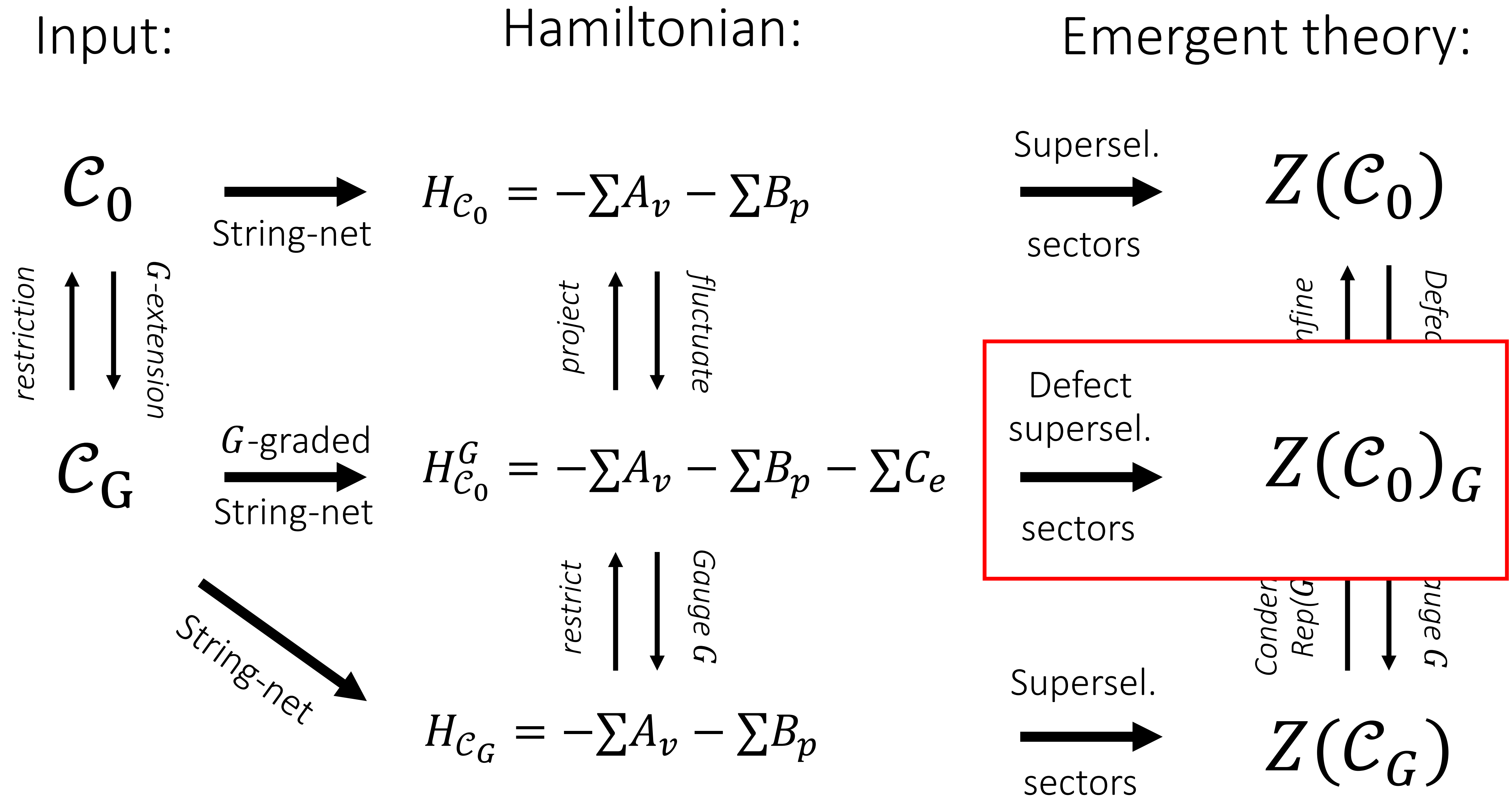
$\mathcal{C}_{\mathbf{G}}$ $\xrightarrow[\text{String-net}]{G\text{-graded}}$ $H^{G}_{\mathcal{C}_0} = -\sum A_v - \sum B_p - \sum C_e$ $\xrightarrow[\text{sectors}]{\text{Defect supersel.}}$ $Z(\mathcal{C}_0)_G$

Input: Hamiltonian: Emergent theory:



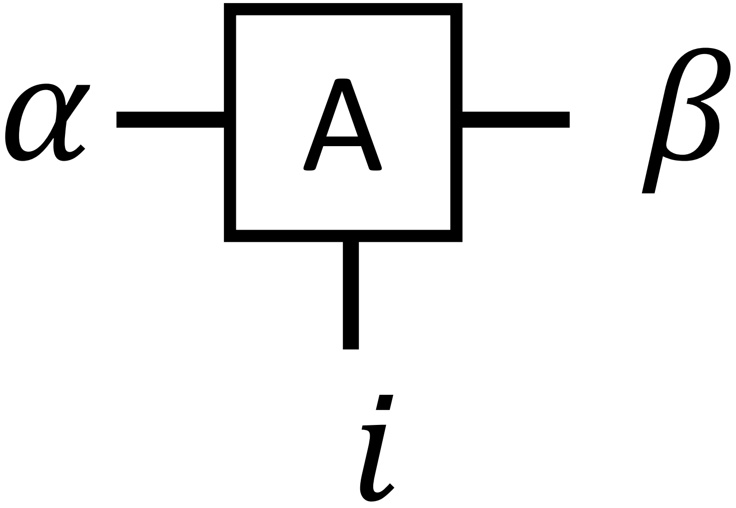


Input:
Hamiltonian:
Emergent theory:
$\mathcal{C}_0$
String-net
$H_{\mathcal{C}_0} = -\sum A_v - \sum B_p$
Supersel.
sectors
$Z(\mathcal{C}_0)$
restriction
G-extension
project
fluctuate
Confine
Defectify
$\mathcal{C}_\mathbf{G}$
G-graded
String-net
$H^G_{\mathcal{C}_0} = -\sum A_v - \sum B_p - \sum C_e$
Defect
supersel.
sectors
$Z(\mathcal{C}_0)_G$
restrict
Gauge G
Condense
Rep(G)
Gauge G
String-net
$H_{\mathcal{C}_G} = -\sum A_v - \sum B_p$
Supersel.
sectors
$Z(\mathcal{C}_G)$

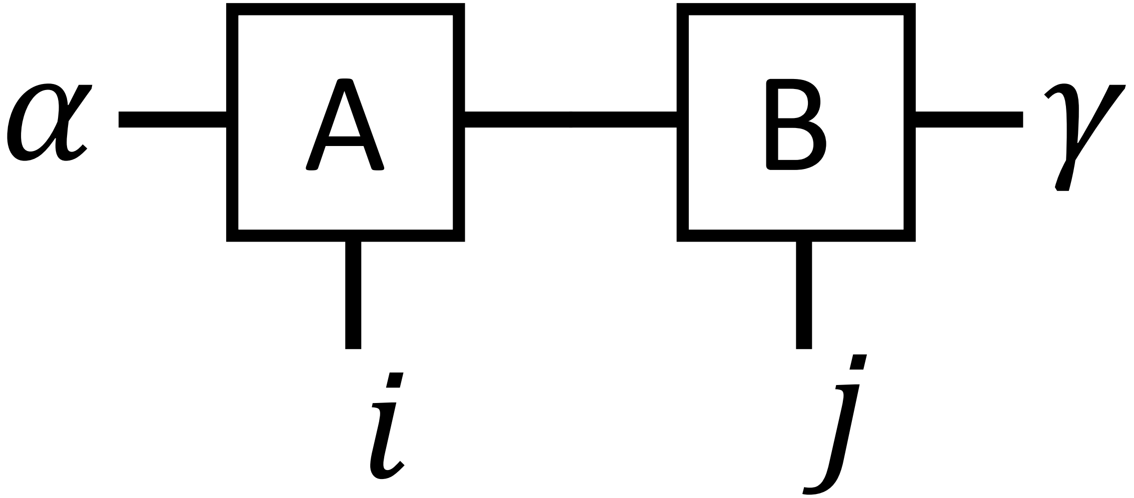


Input:
Hamiltonian:
Emergent theory:
$\mathcal{C}_0$
String-net
$H_{\mathcal{C}_0} = -\sum A_v - \sum B_p$
Supersel.
sectors
$Z(\mathcal{C}_0)$
restriction
G-extension
project
fluctuate
$\mathcal{C}_G$
G-graded
String-net
$H^G_{\mathcal{C}_0} = -\sum A_v - \sum B_p - \sum C_e$
Defect
supersel.
sectors
$Z(\mathcal{C}_0)_G$
restrict
Gauge G
String-net
$H_{\mathcal{C}_G} = -\sum A_v - \sum B_p$
Supersel.
sectors
$Z(\mathcal{C}_G)$

Tensor Networks

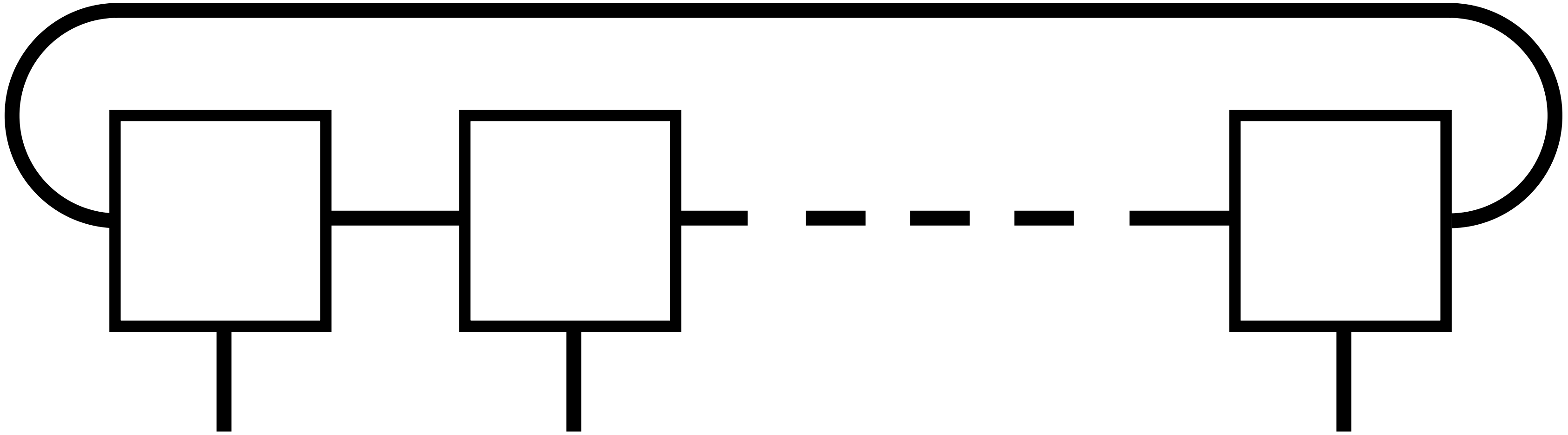


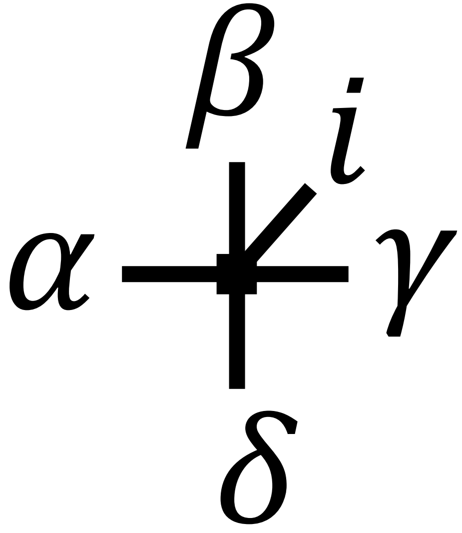
$$= \left(A^{i}\right)_{\alpha\beta} \in \mathbb{C}$$



$$= \sum_{\beta} \left(A^{i}\right)_{\alpha\beta} \left(B^{j}\right)_{\beta\gamma}$$

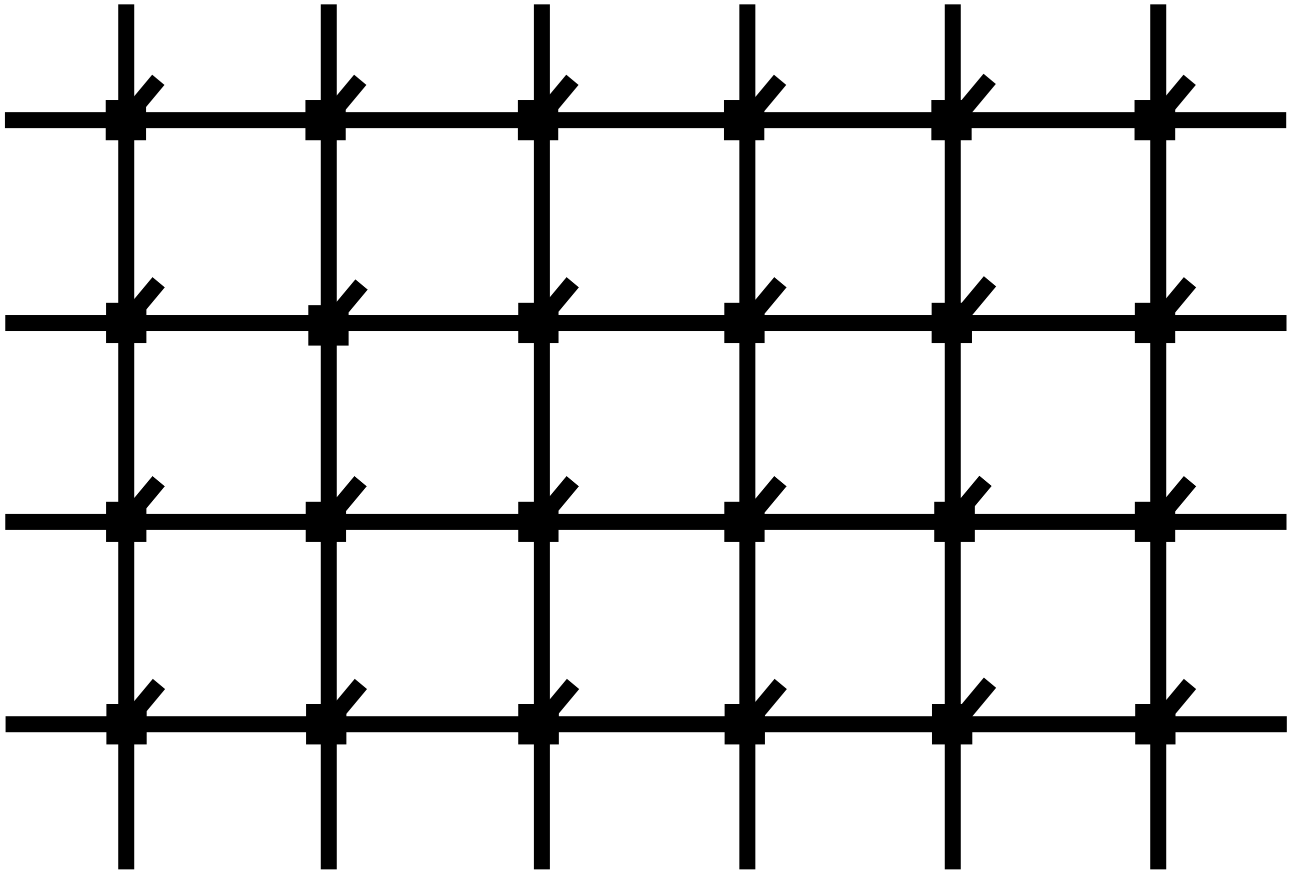
$$|MPS_N(A)\rangle = \sum_{i_1 \ldots i_N} Tr\left[A^{i_1} \ldots A^{i_N}\right]|i_1 \ldots i_N\rangle$$

$=$ 

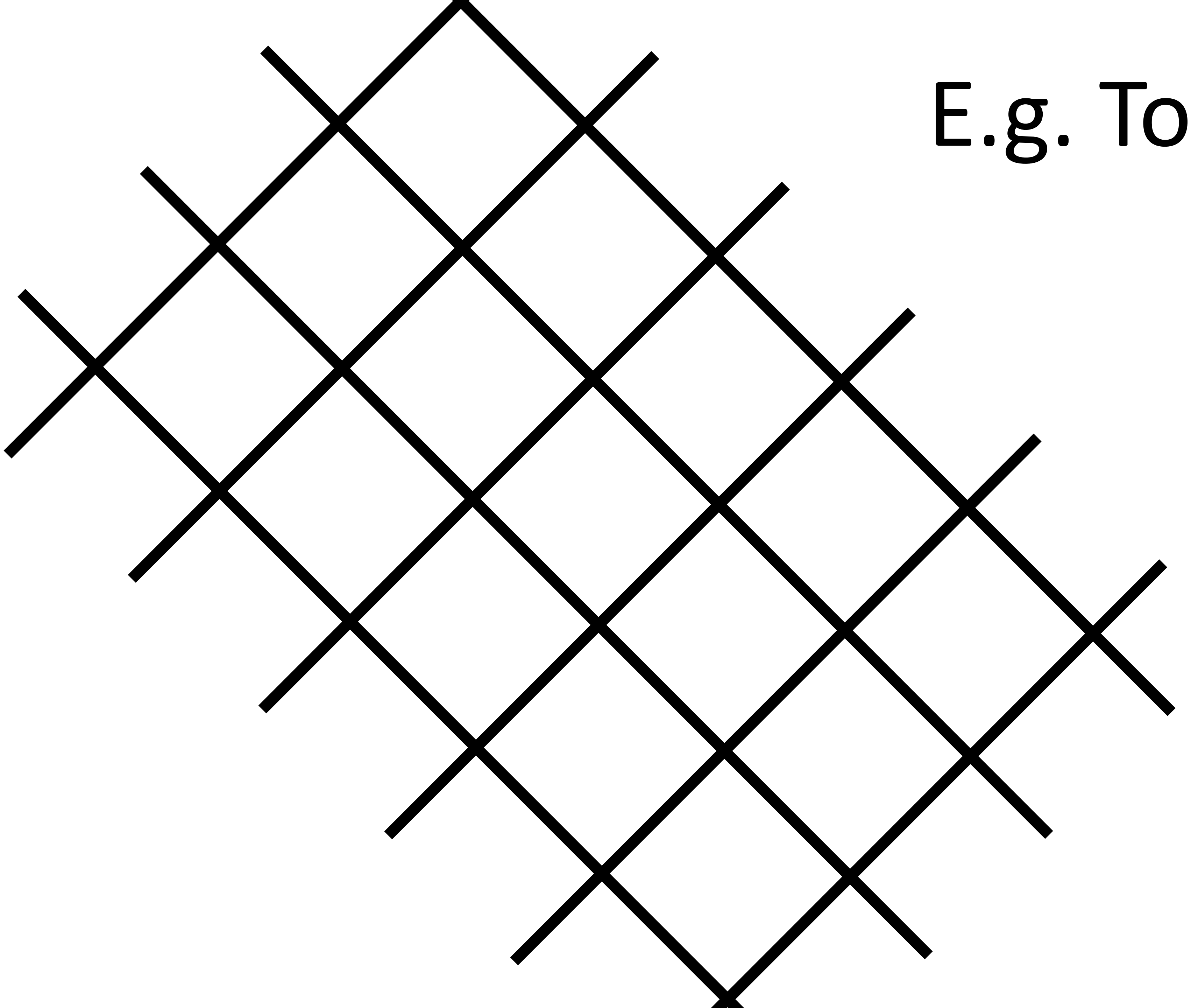


$$= \left(A^{i}\right)_{\alpha\beta\gamma\delta}$$

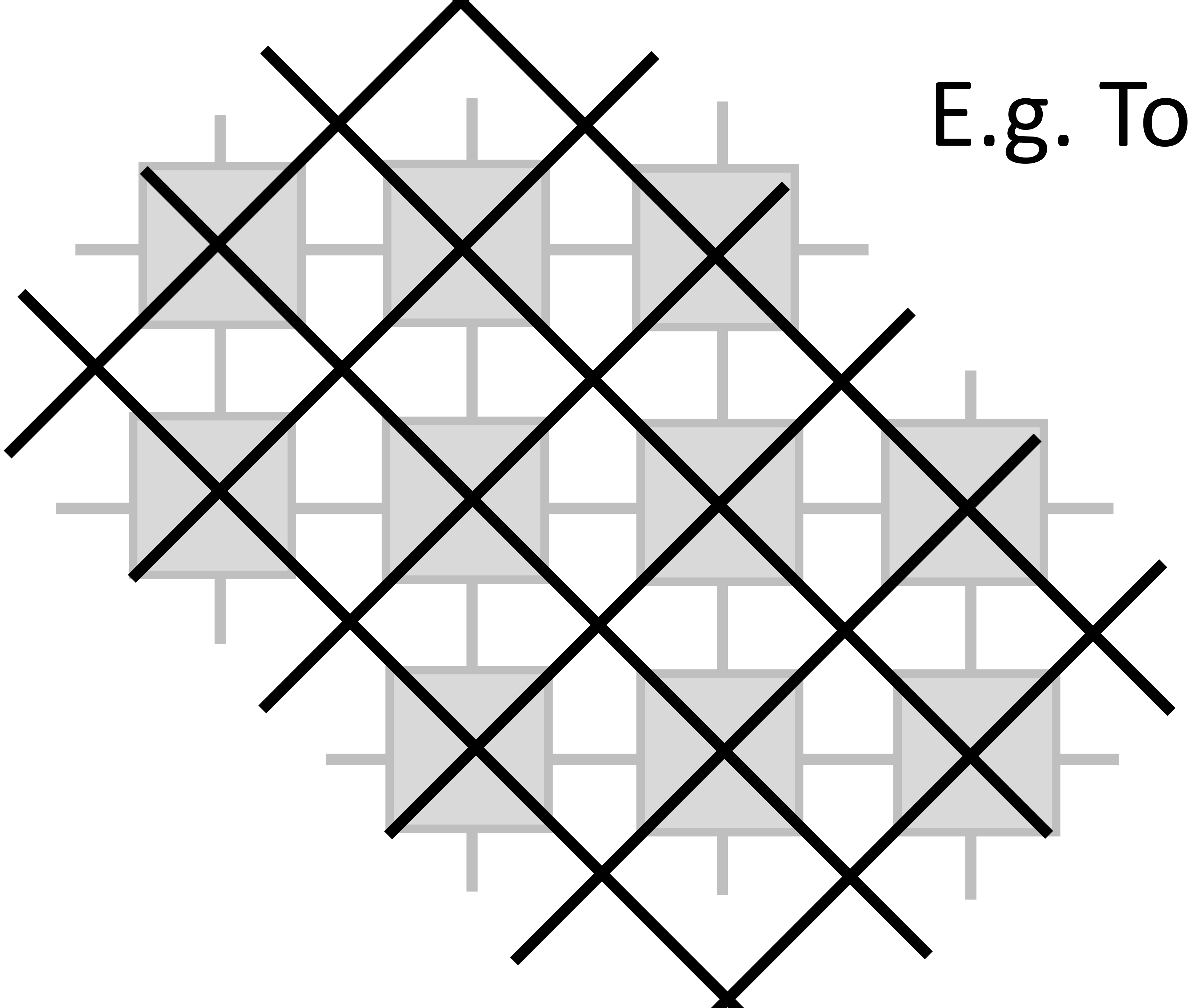
$$|PEPS(A)\rangle =$$



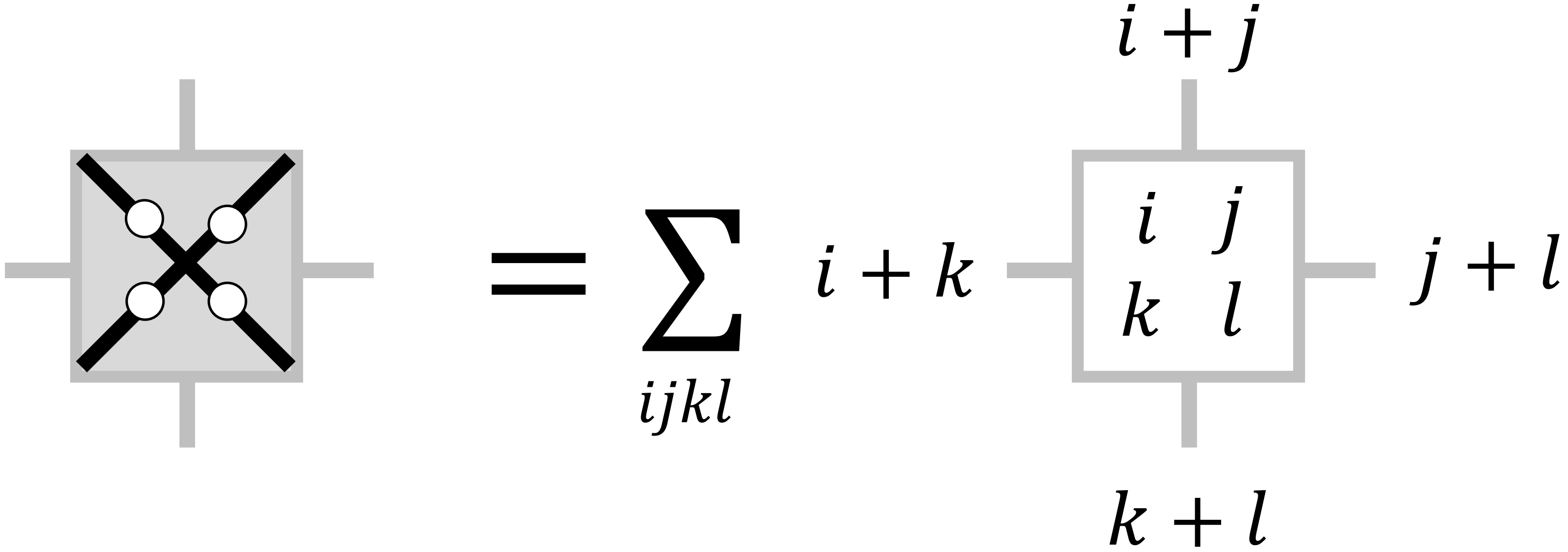
E.g. Toric code

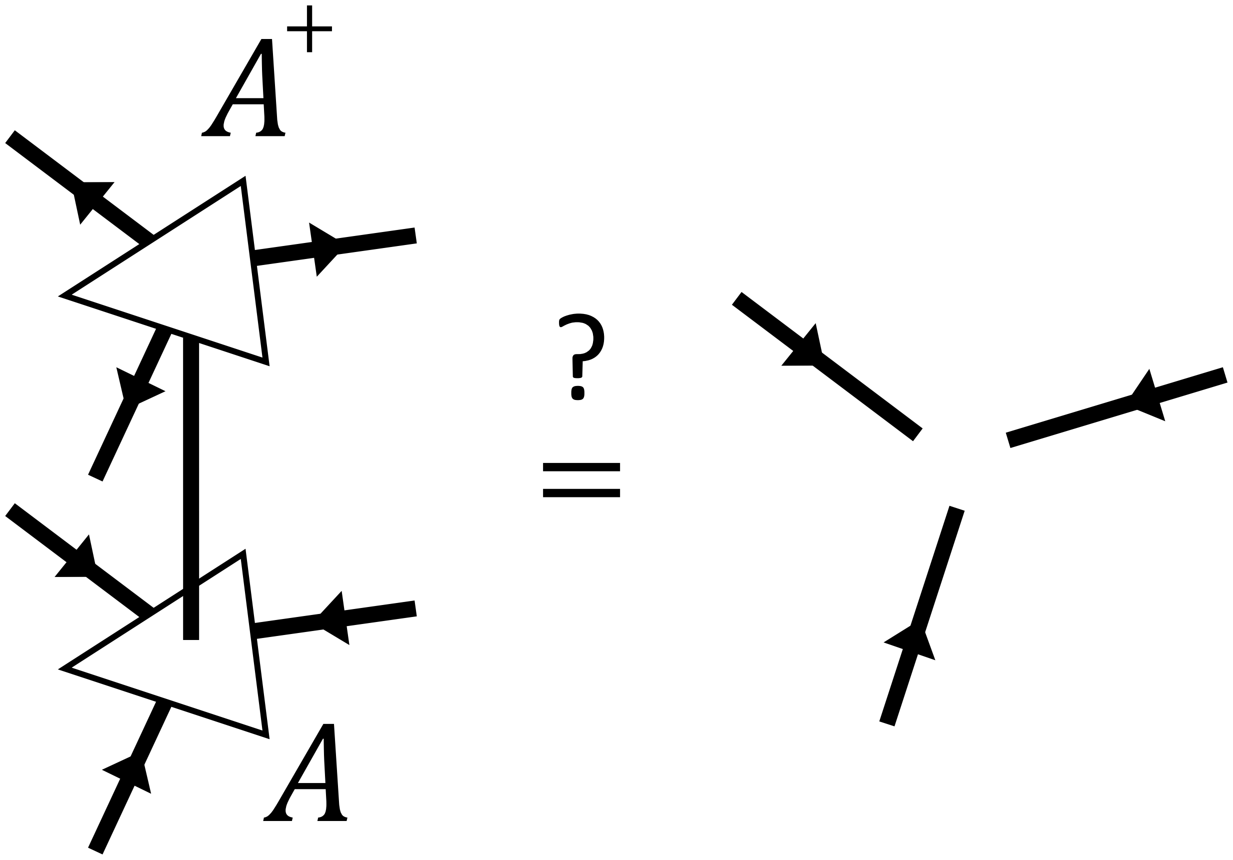


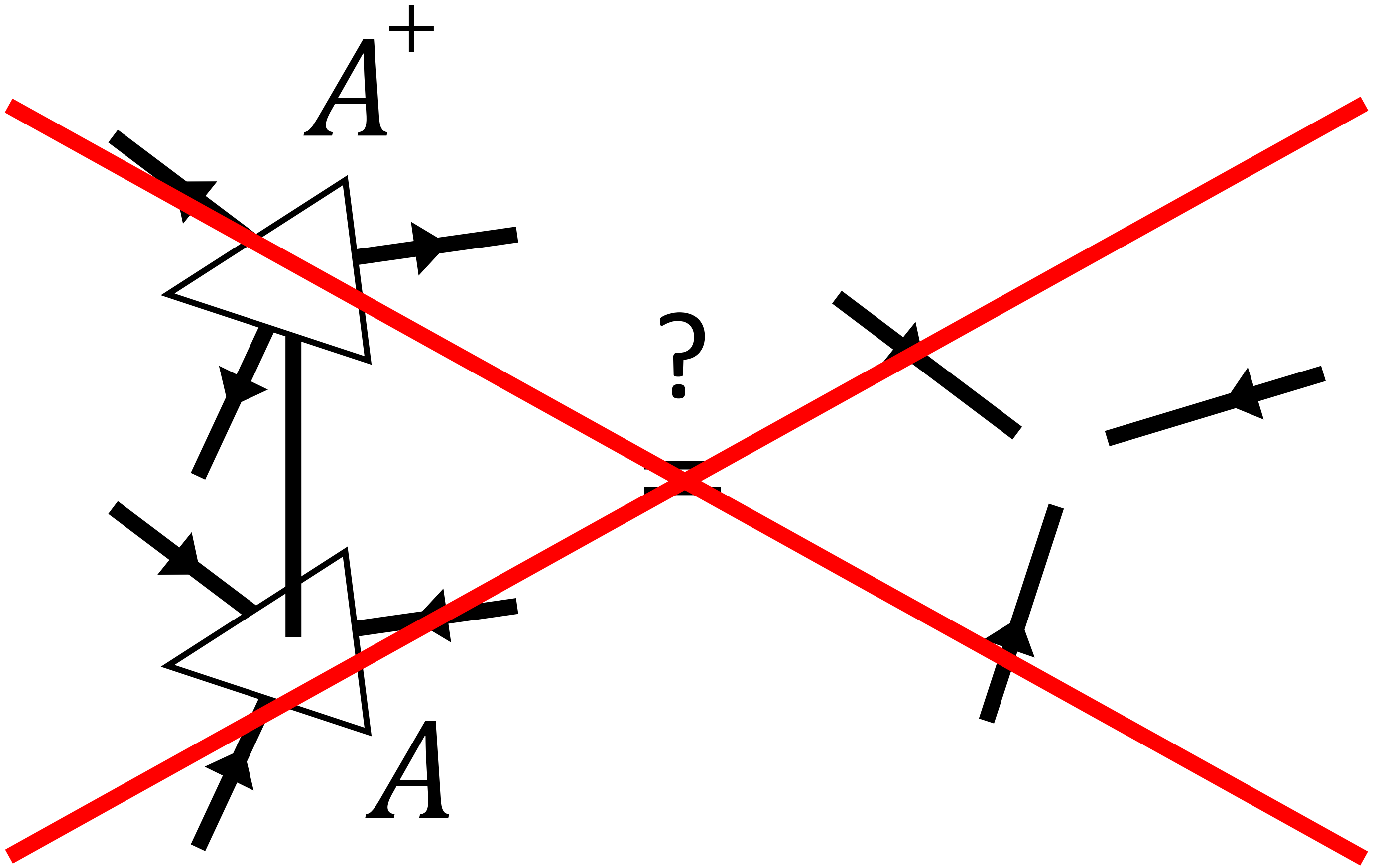
E.g. Toric code



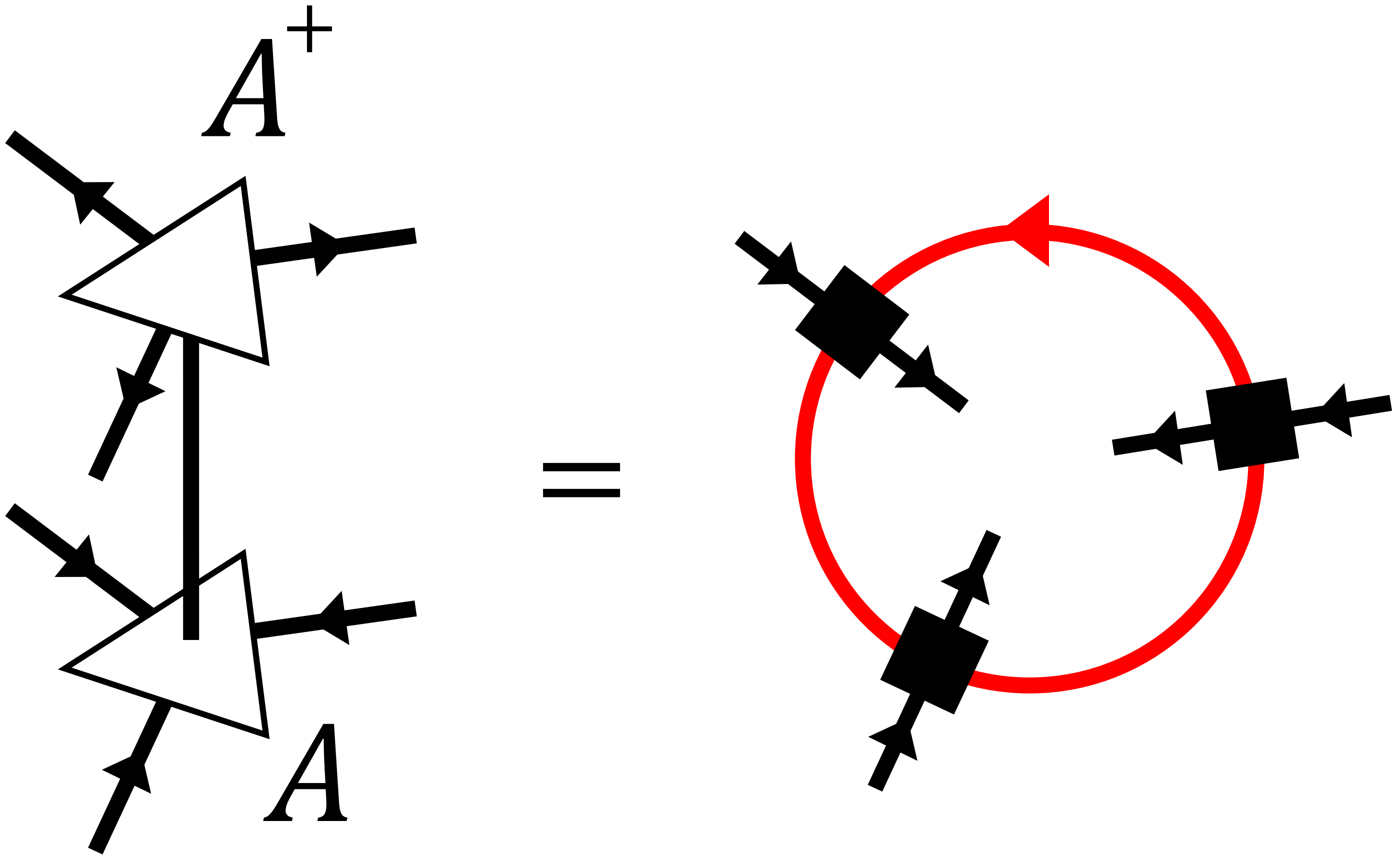
T.C.




A^+
?
=
A



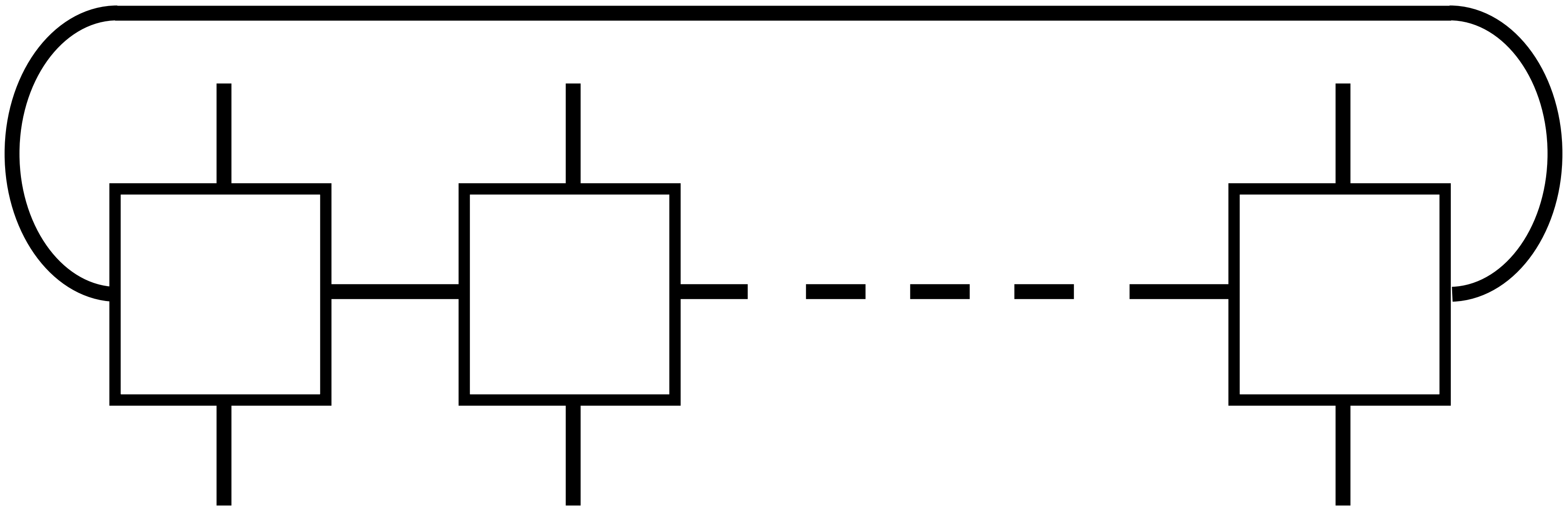
A^+
?
=
A

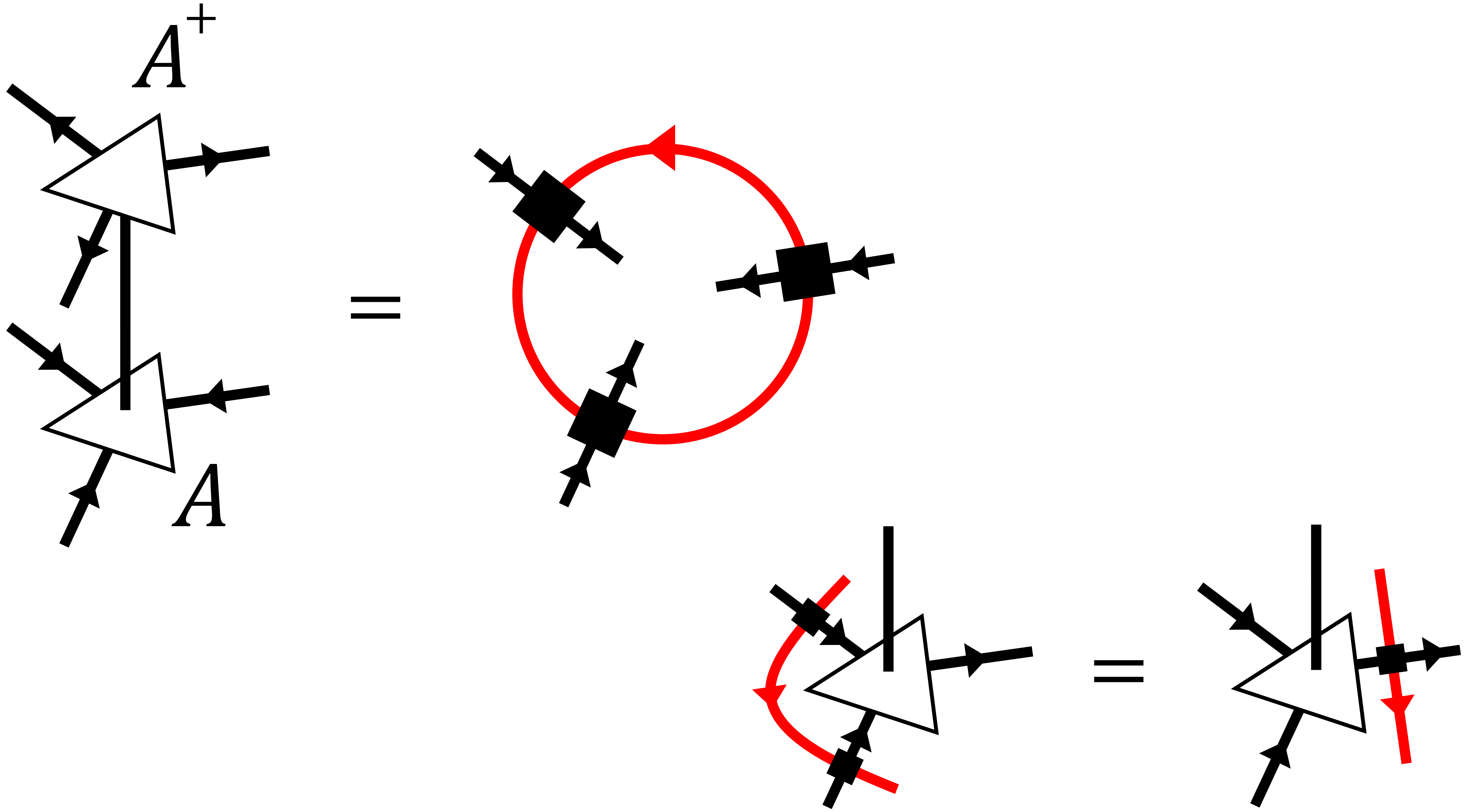


A^+
=
A

$$MPO_N(A) = \sum_{\substack{i_1 \dots i_N \\ j_1 \dots j_N}} Tr\left[A^{i_1 j_1} \dots A^{i_N j_N}\right] |i_1 \dots i_N\rangle\langle j_1 \dots j_N|$$

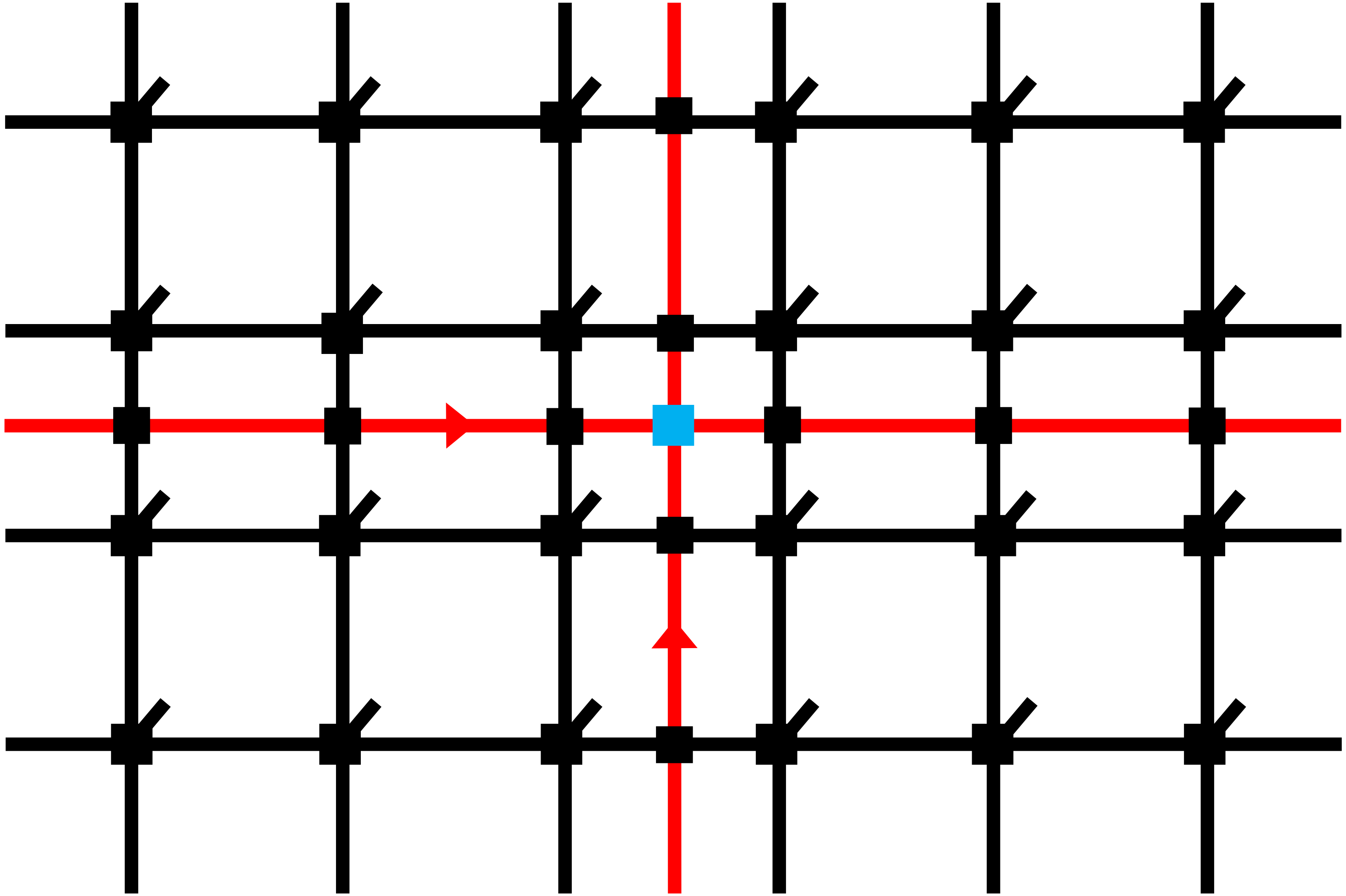
$=$



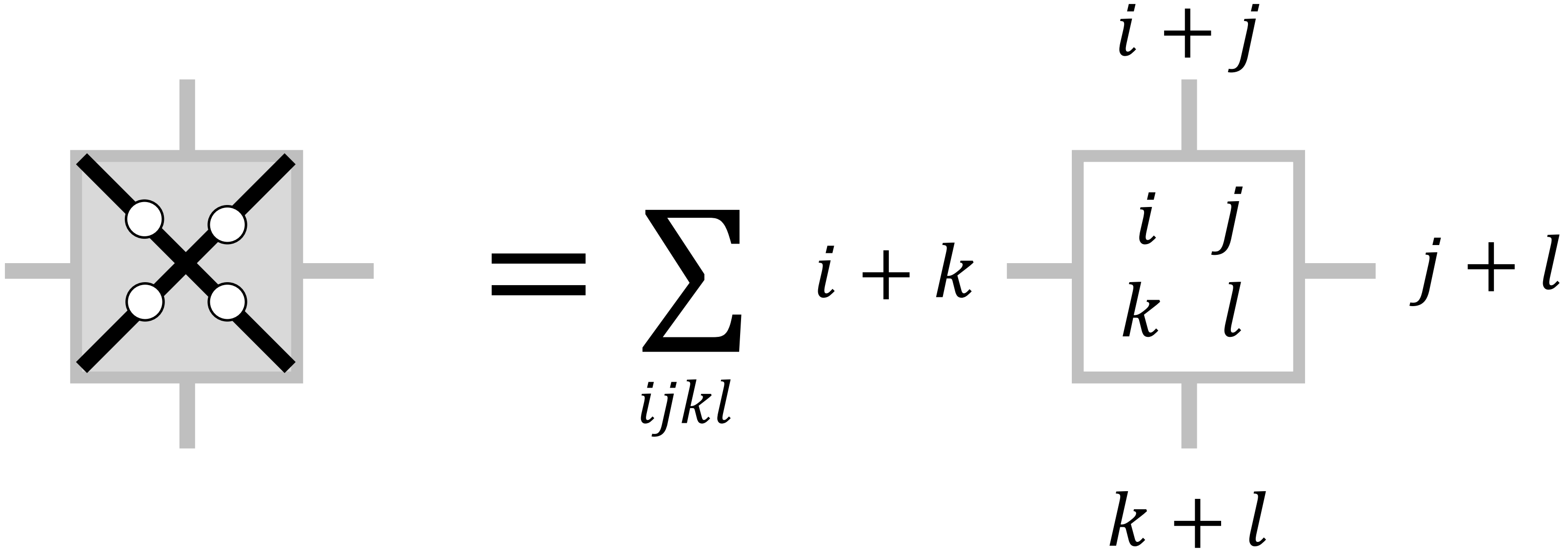


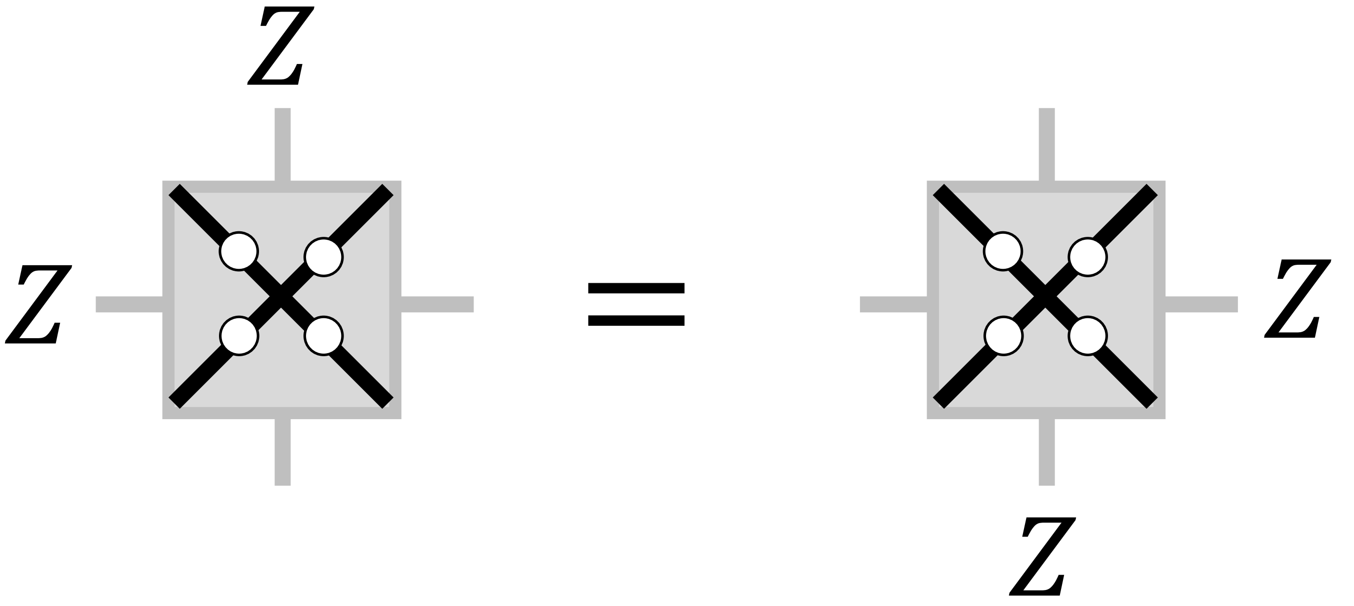
A^+
=
A
=

$$|\Psi_0^{\blacksquare}\rangle =$$



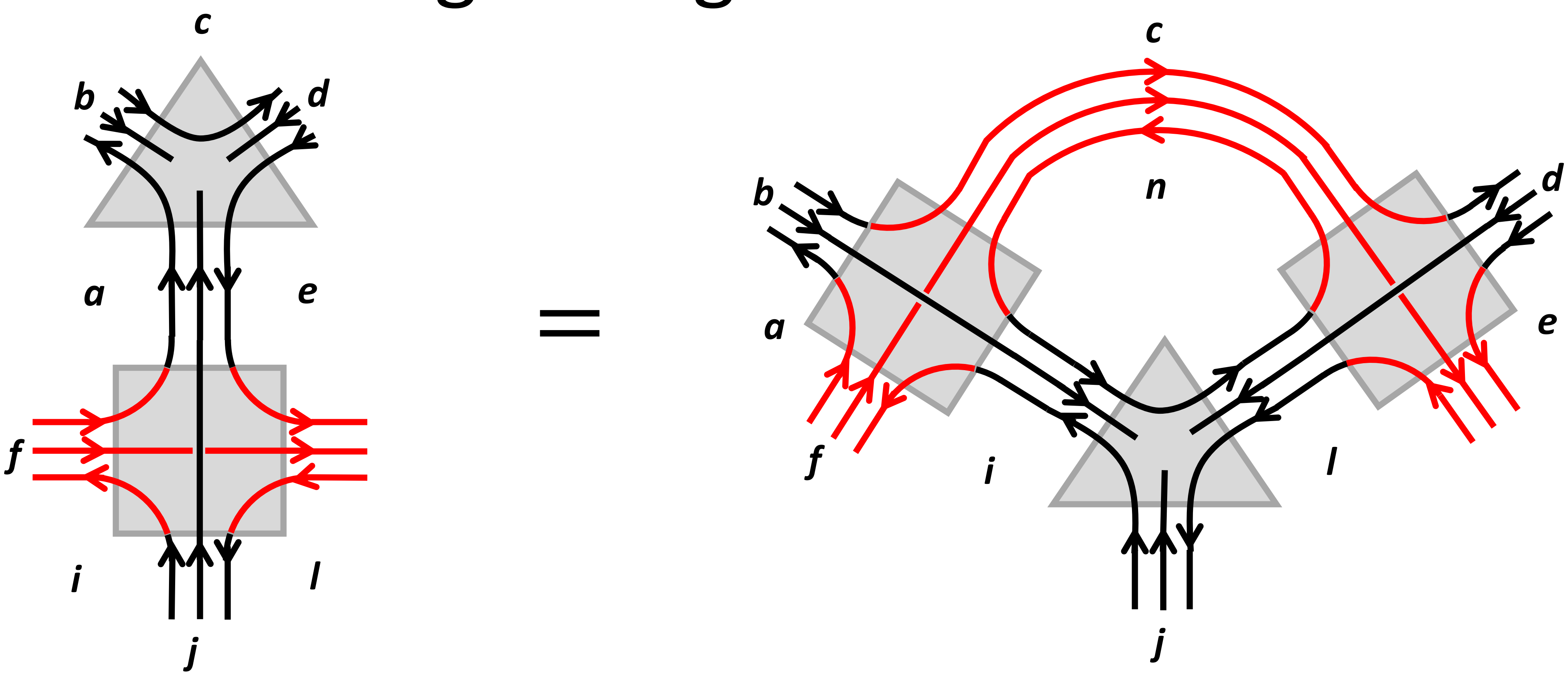
T.C.





E.g. String-net

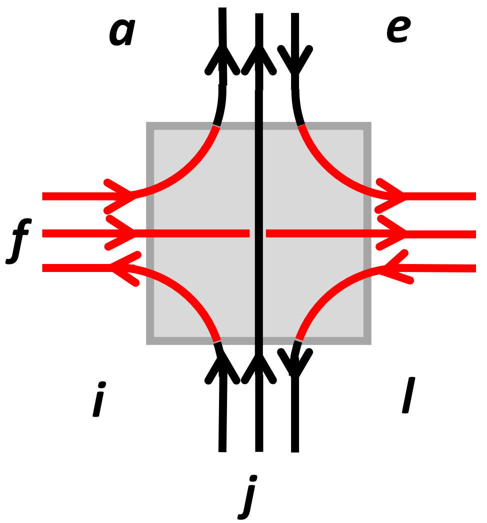
$F^{acd}_{j;be}$



$F^{jaf}_{l;ei}$

$$FF = \sum FFF$$

Double Ising

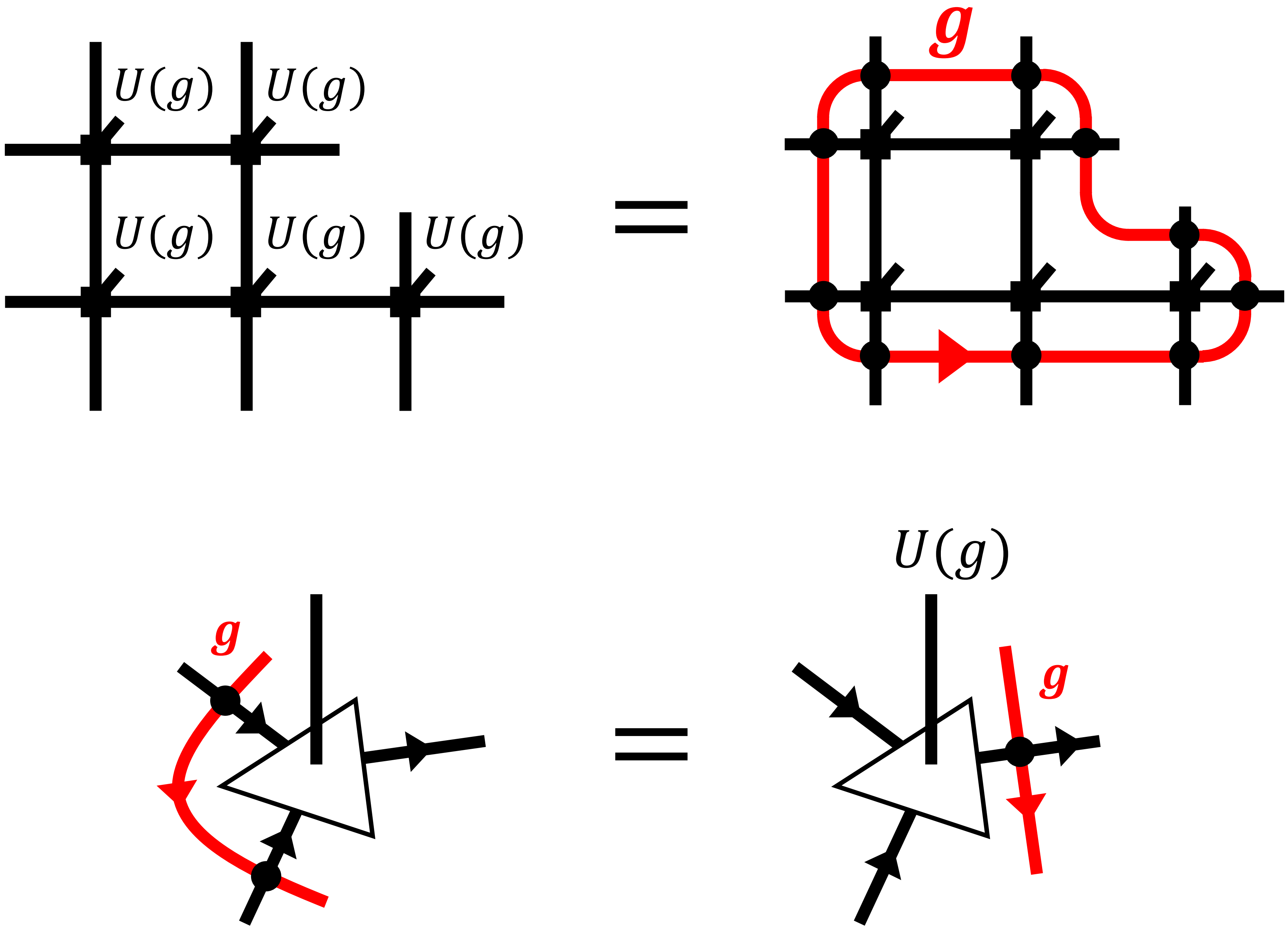


$$\longrightarrow \{1, \psi, \sigma\}$$

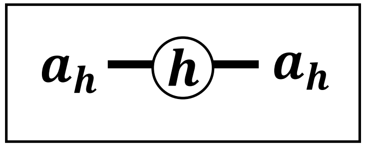
$$F^{jaf}_{l;ei}$$

Form a matrix product operator algebra

With a global symmetry

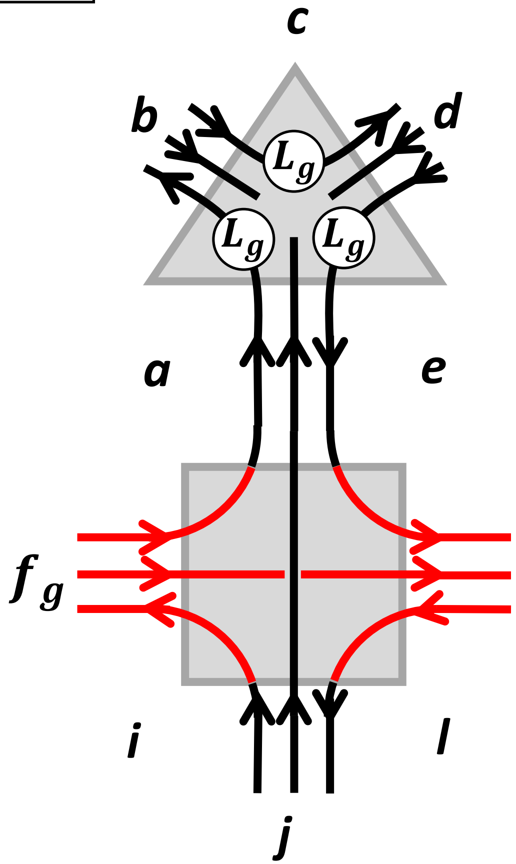


U(g) U(g)
U(g) U(g) U(g)
=
g
g
=
U(g)
g

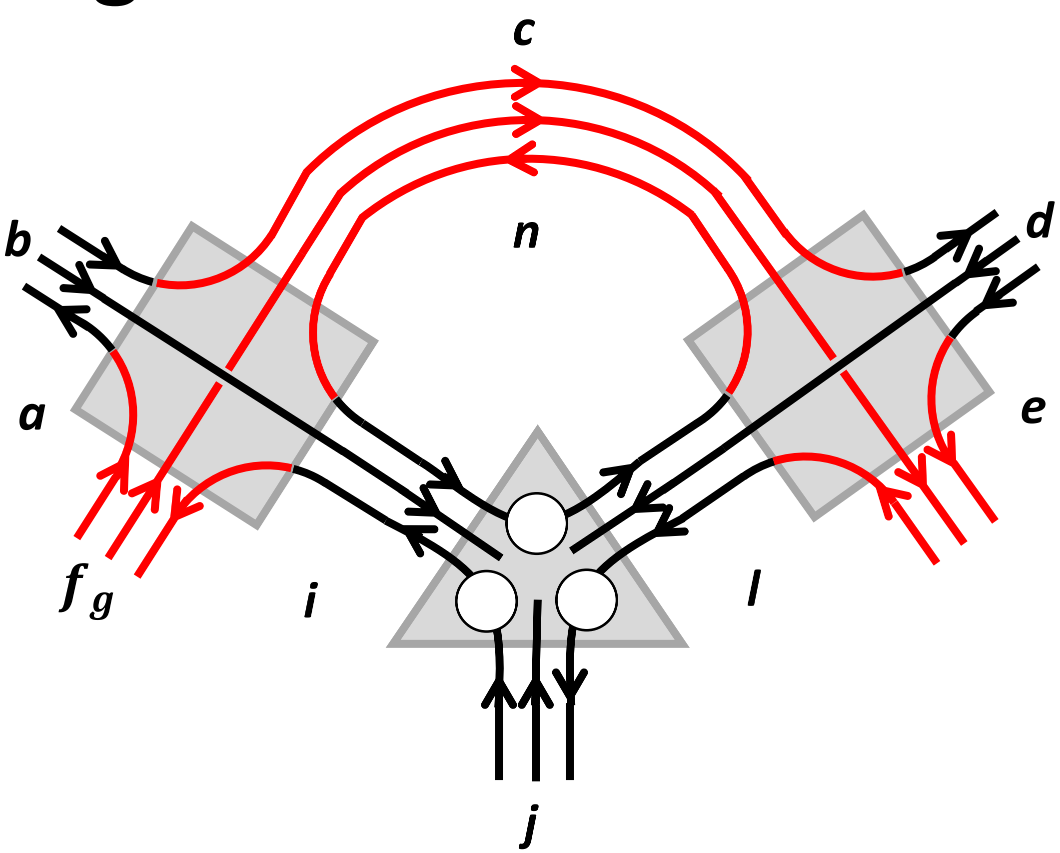


G-Graded String-net

$$F^{acd}_{j;be}$$



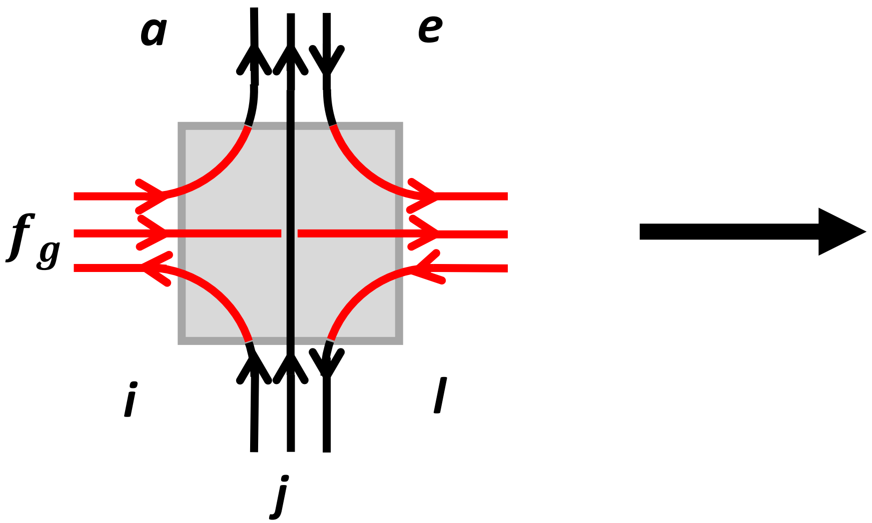
$=$



$$F^{jaf}_{l;ei}$$

$$FF = \sum FFF$$

Toric code w/ twists

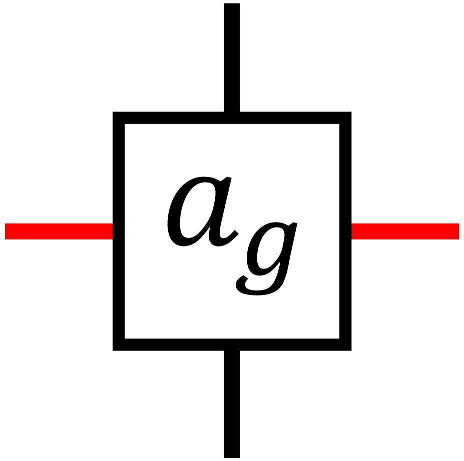


$$\{1, \psi\} \oplus \{\sigma\}$$

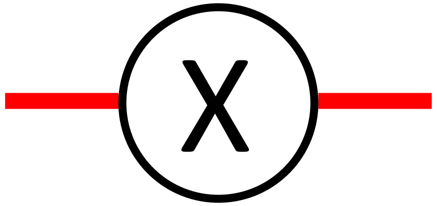
$$F^{jaf}_{l;ei}$$

Form a $\mathbb{Z}_2$-graded matrix product operator algebra

G-graded fusion category $\mathcal{C}_G$



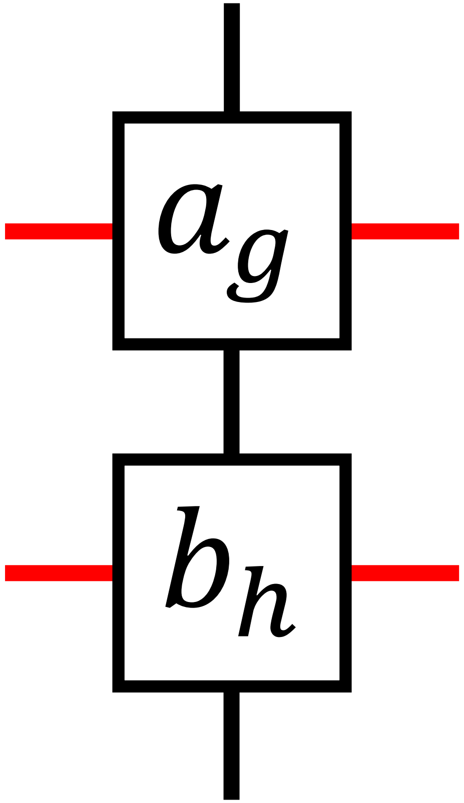
Objects



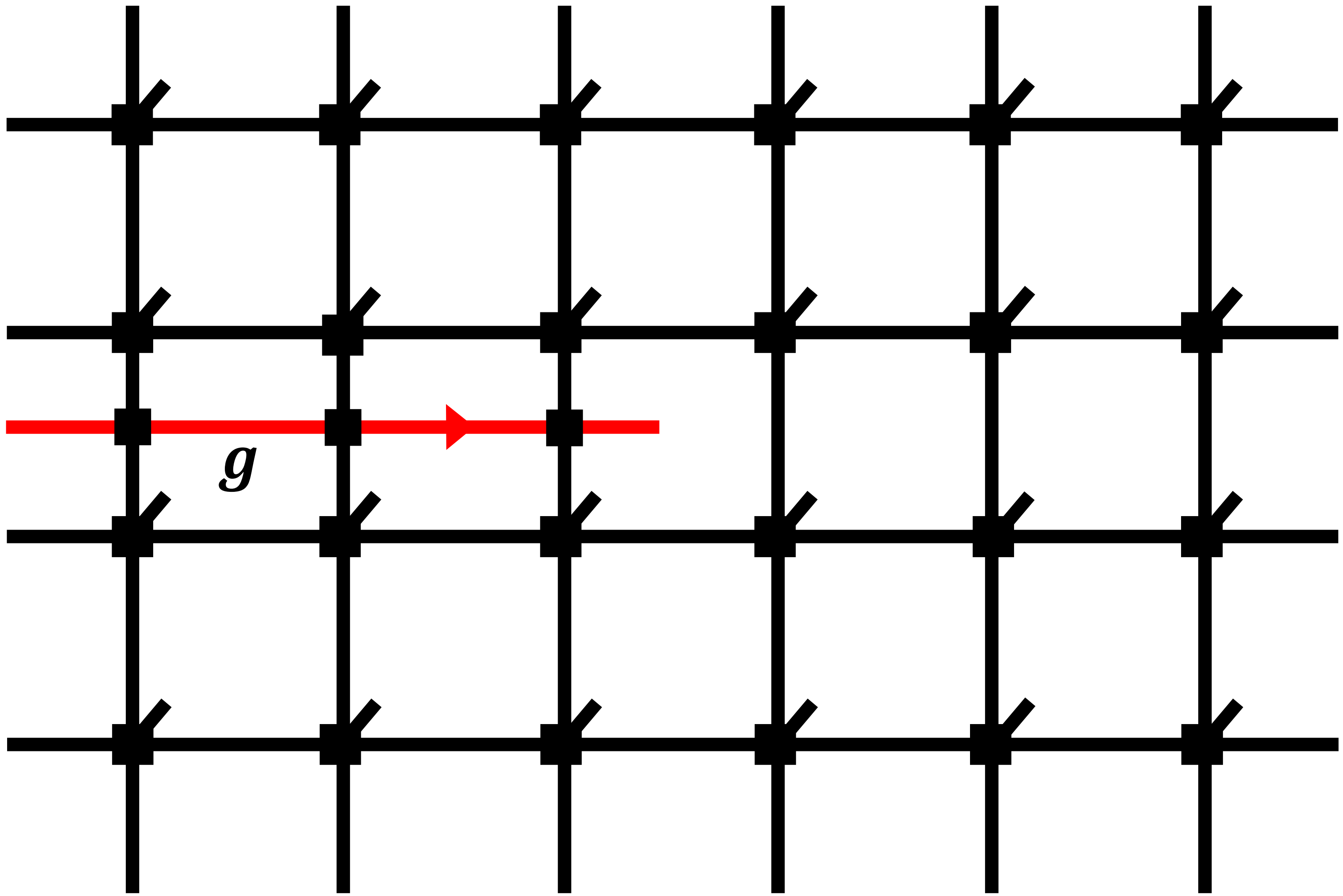
morphisms

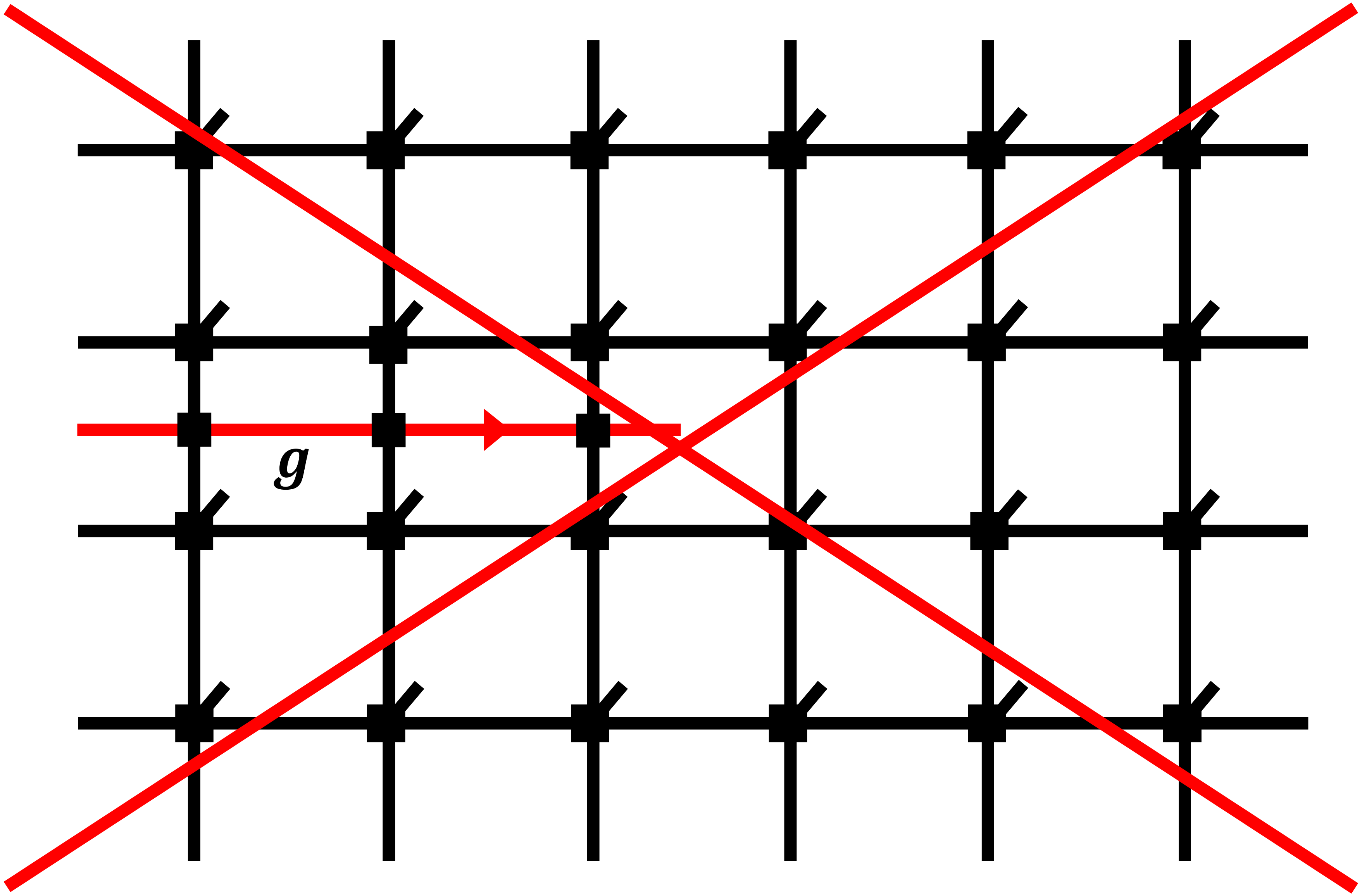
G-graded tensor product

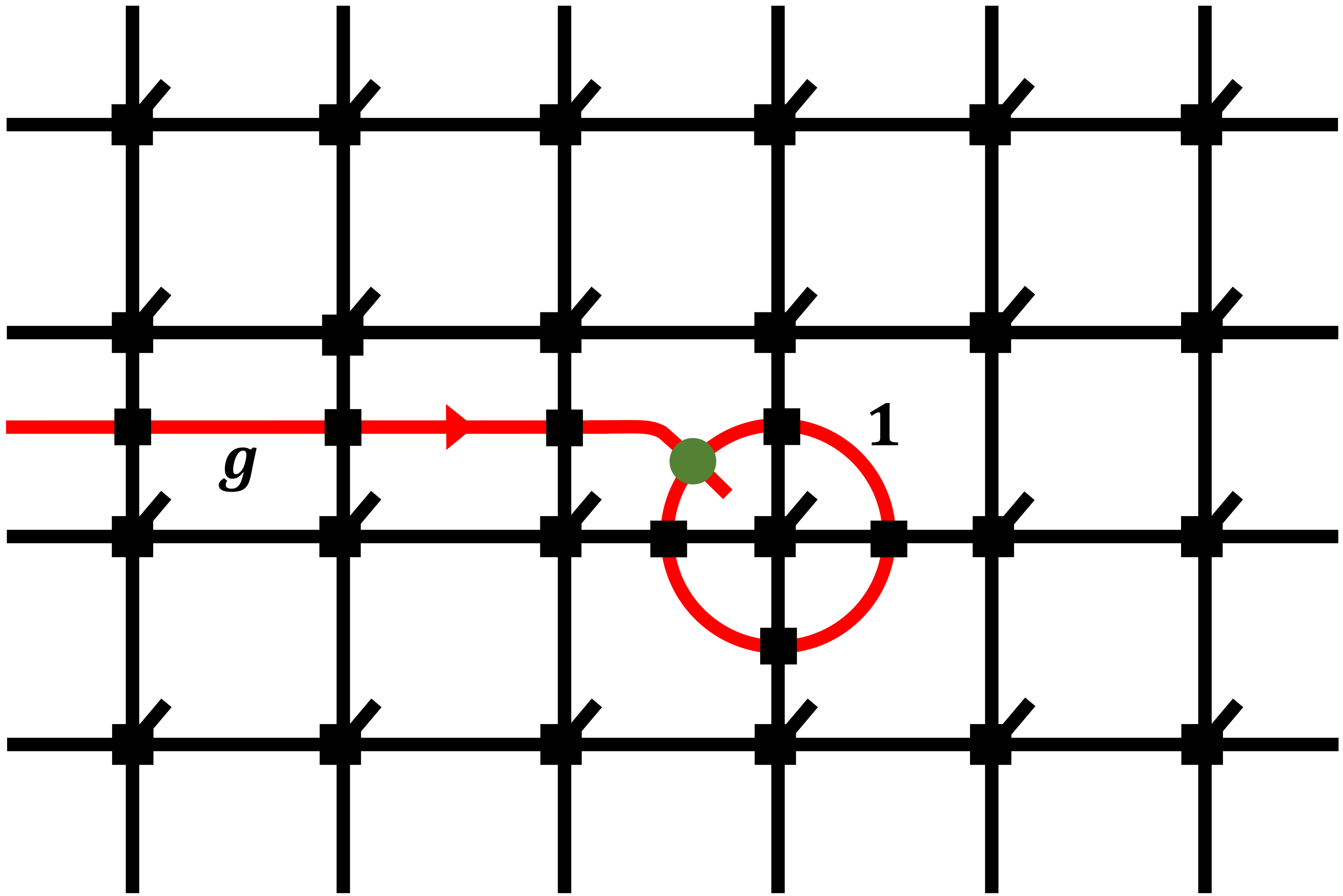
$a_g \otimes b_h$

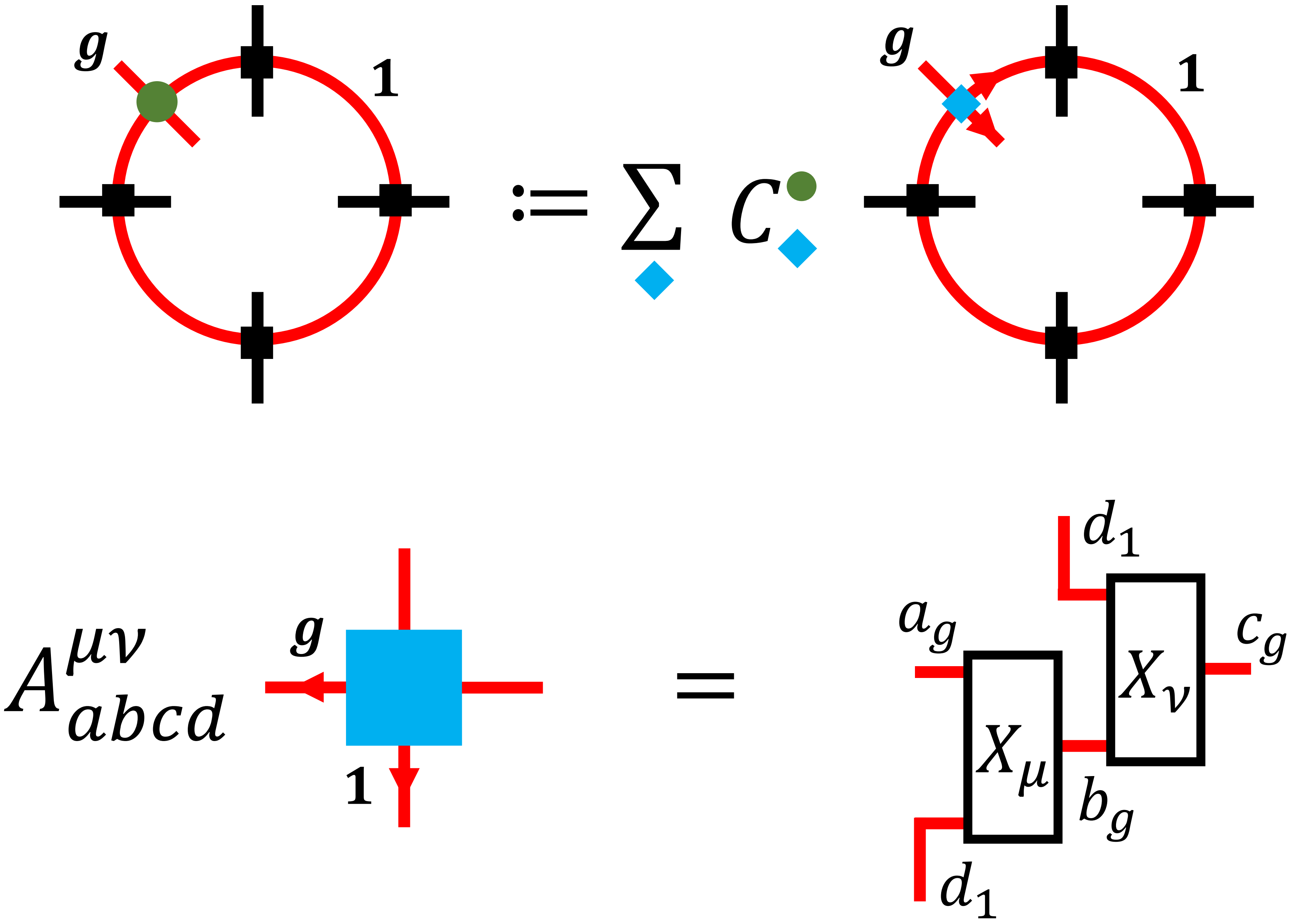


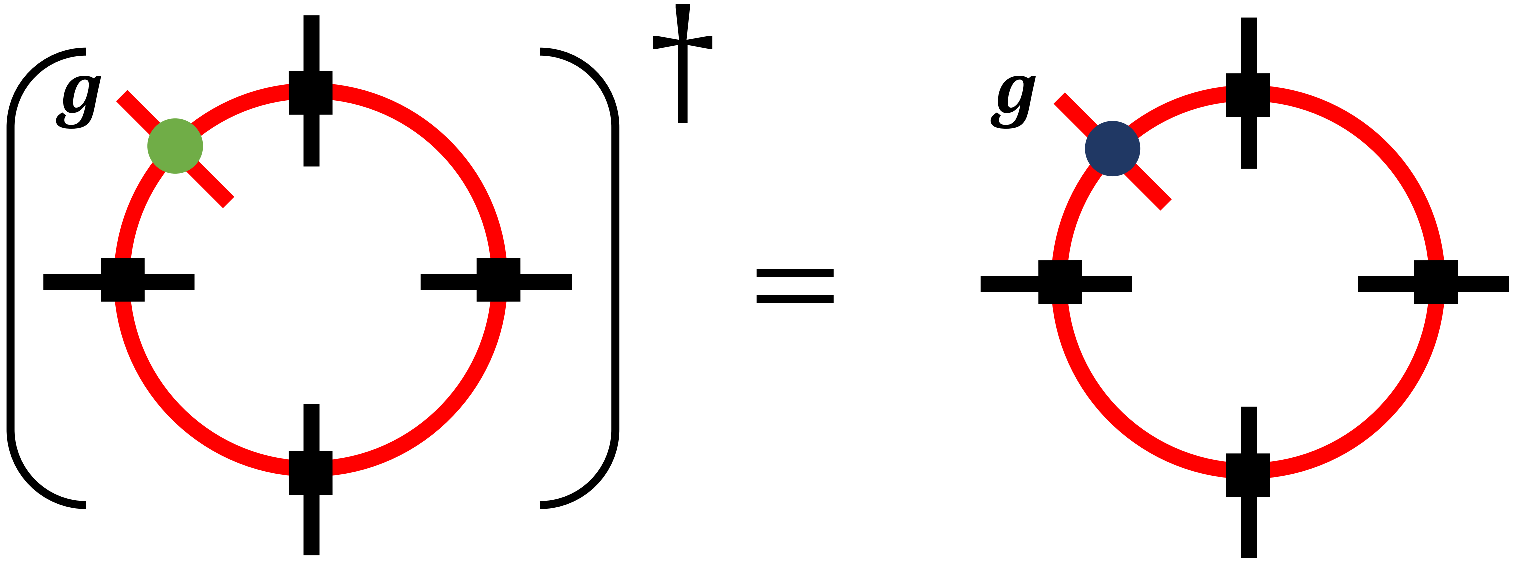
What about anyons and defects?


g

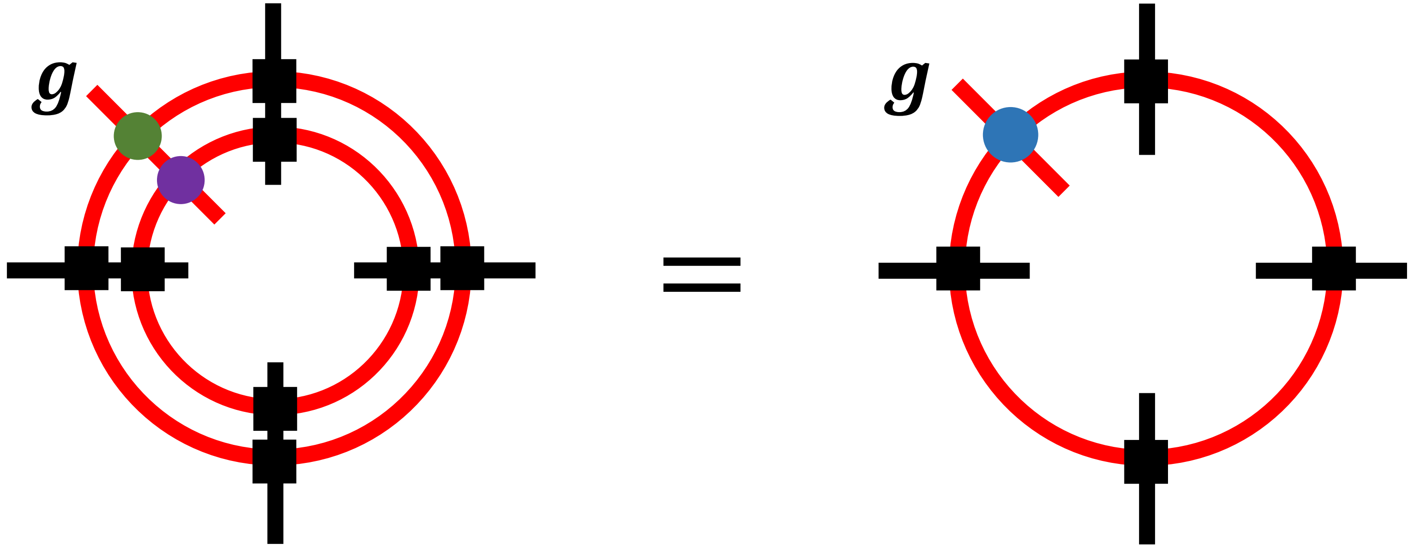

g


g
1

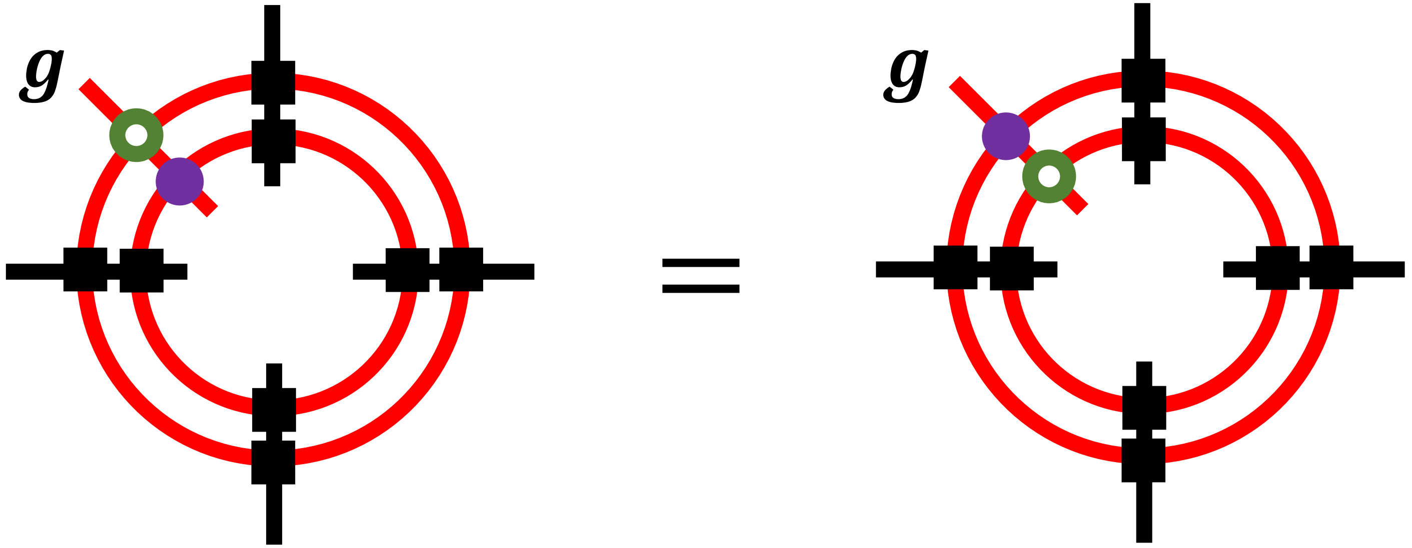

g
1
:= Σ C
g
1
$A^{\mu\nu}_{abcd}$
g
1
=
d_1
a_g
c_g
X_ν
X_μ
b_g
d_1



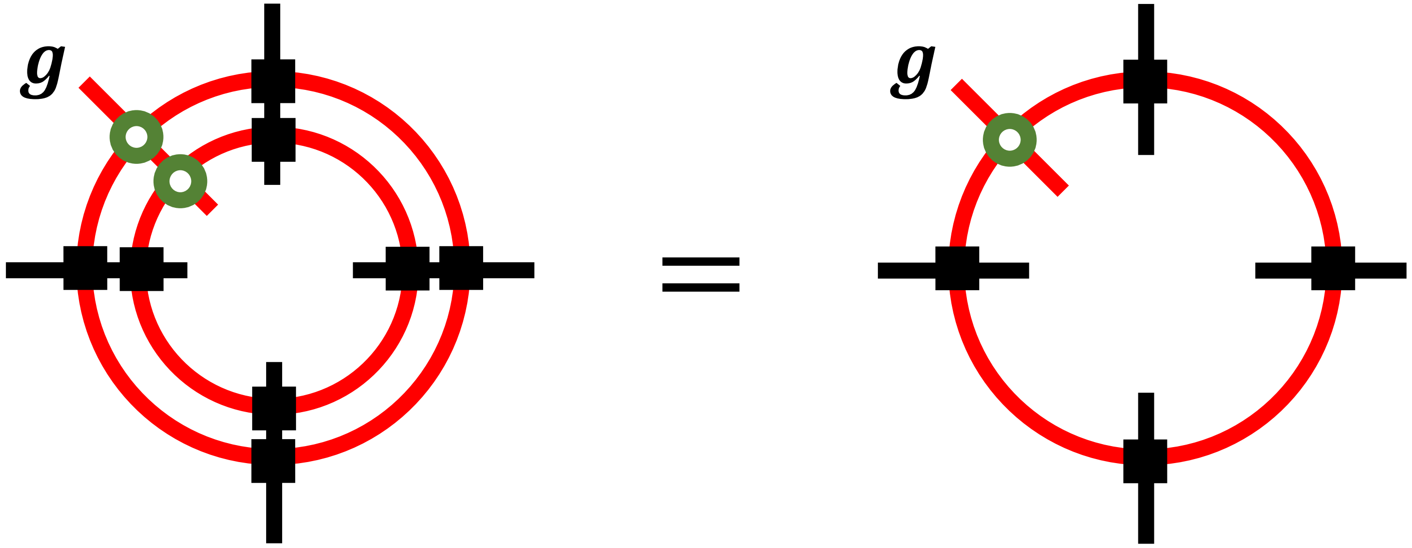
*



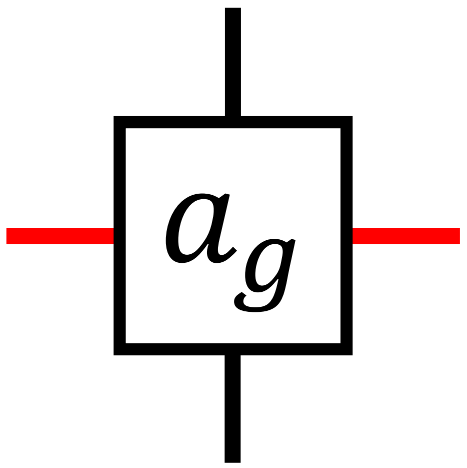
Algebra



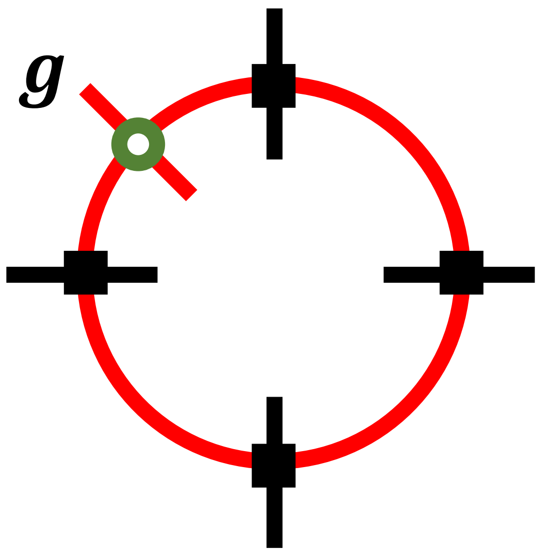
Central



Idempotent



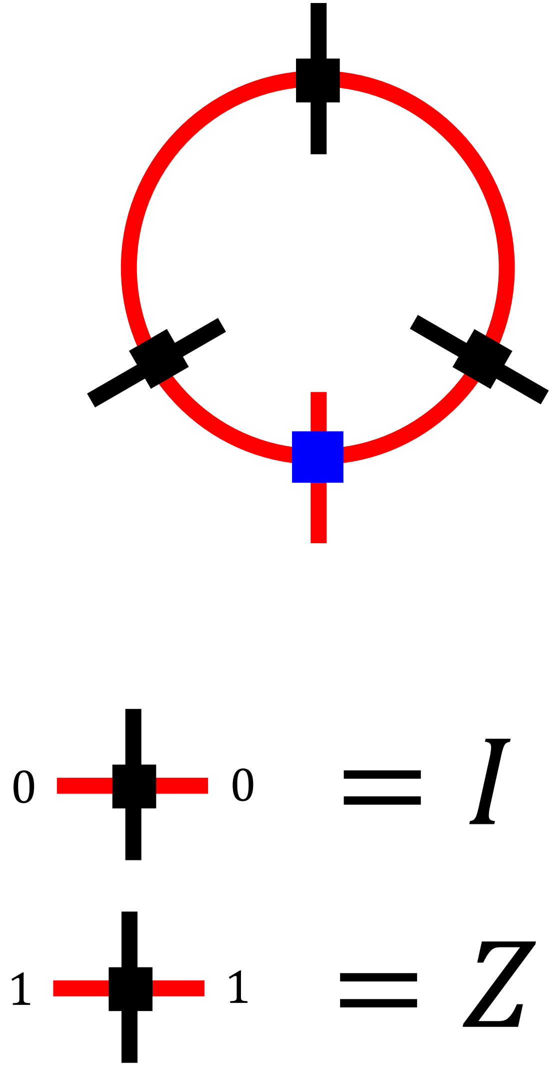
$: \mathcal{C}_G$



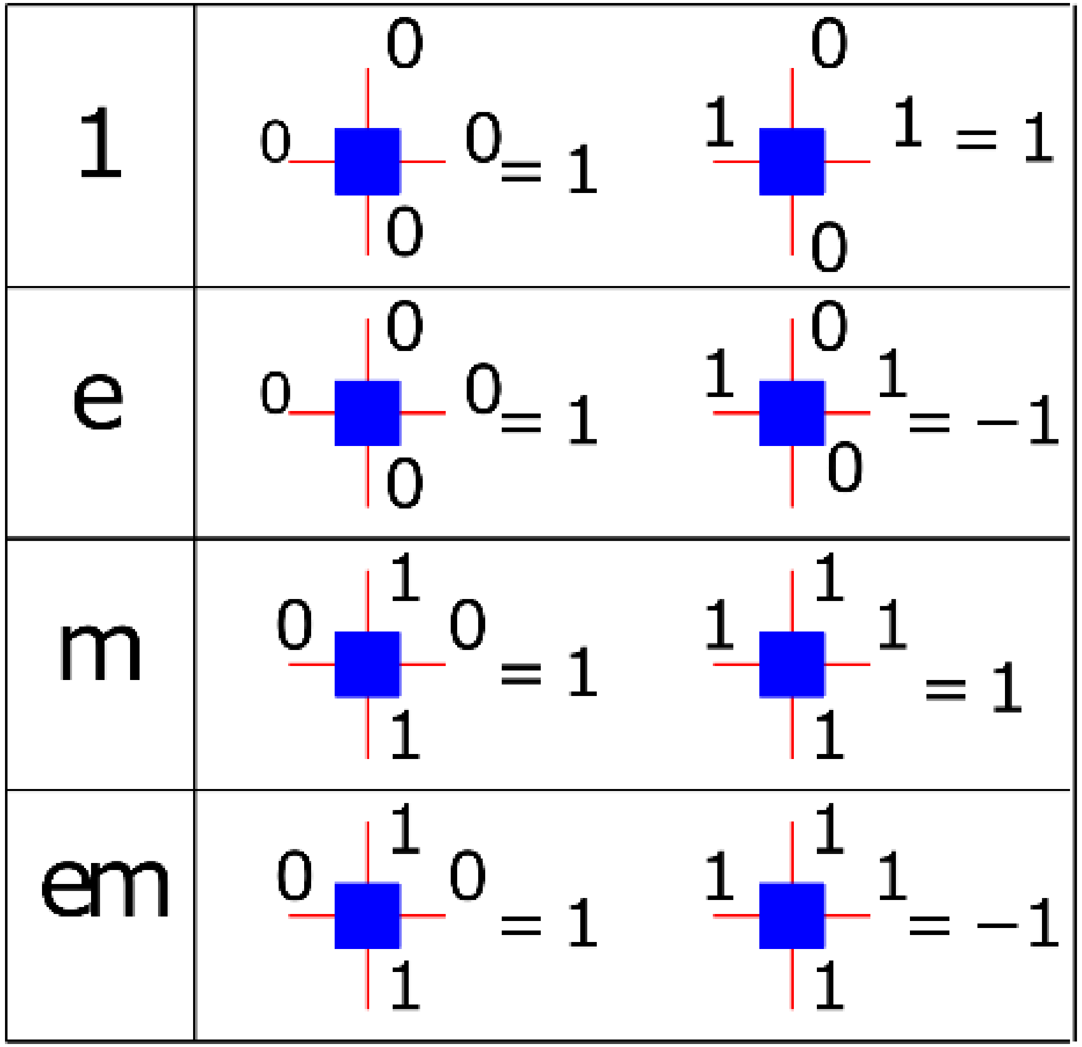
$: Z(\mathcal{C}_0)_G$

Examples

Toric Code

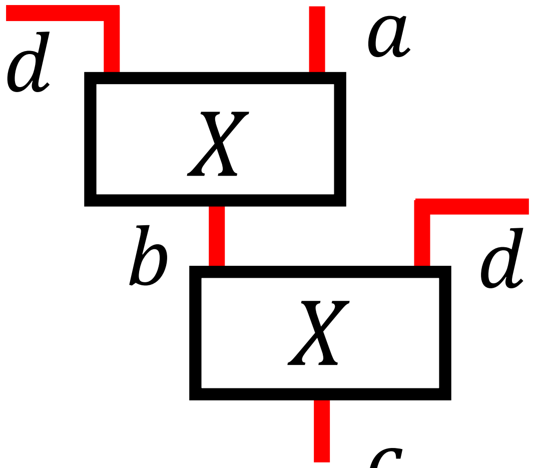


0 0 = I
1 1 = Z



1
0 0 0 0 = 1
1 1 0 0 = 1
e
0 0 0 0 = 1
1 1 0 0 = −1
m
0 0 1 1 = 1
1 1 1 1 = 1
em
0 0 1 1 = 1
1 1 1 1 = −1

Double Ising



A_{abcd}

$$(1,1) = \frac{1}{4}\left(A_{1111} + 2^{3/4}A_{1\sigma1\sigma} + A_{1\psi1\psi}\right)$$

$$(\sigma,1) = \frac{1}{4}\left(A_{\sigma\sigma\sigma1} + 2^{1/4}e^{\frac{\pi i}{8}}A_{\sigma1\sigma\sigma} + 2^{1/4}e^{-\frac{3\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(1,\bar{\sigma}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma1} + 2^{1/4}e^{-\frac{\pi i}{8}}A_{\sigma1\sigma\sigma} + 2^{1/4}e^{\frac{3\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,1) = \frac{1}{4}\left(A_{\psi\psi\psi1} + 2^{3/4}e^{\frac{\pi i}{2}}A_{\psi\sigma\psi\sigma} - A_{\psi1\psi\psi}\right)$$

$$(1,\psi) = \frac{1}{4}\left(A_{\psi\psi\psi1} + 2^{3/4}e^{\frac{-\pi i}{2}}A_{\psi\sigma\psi\sigma} - A_{\psi1\psi\psi}\right)$$

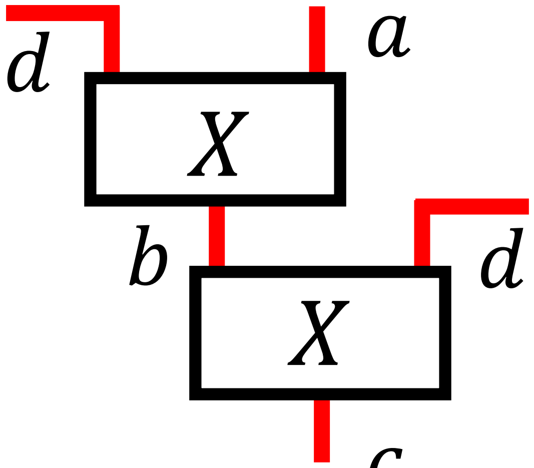
$$(\sigma,\bar{\psi}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma1} + 2^{1/4}e^{-\frac{7\pi i}{8}}A_{\sigma1\sigma\sigma} + 2^{1/4}e^{\frac{5\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar{\sigma}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma1} + 2^{1/4}e^{\frac{7\pi i}{8}}A_{\sigma1\sigma\sigma} + 2^{1/4}e^{-\frac{5\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar{\psi}) = \frac{1}{4}\left(A_{1111} - 2^{3/4}A_{1\sigma1\sigma} + A_{1\psi1\psi}\right)$$

$$(\sigma,\bar{\sigma}) = \frac{1}{2}\left(A_{1111} + A_{\psi\psi\psi1} - A_{1\psi1\psi} + A_{\psi1\psi\psi}\right)$$

Condensing $\mathbb{Z}_2$



A_{abcd}

$$(1,1) = \frac{1}{4}\left(A_{1111} + 2^{3/4}\cancel{A_{1\sigma 1\sigma}} + A_{1\psi 1\psi}\right)$$

$$(\sigma,1) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{\frac{\pi i}{8}}\cancel{A_{\sigma 1\sigma\sigma}} + 2^{1/4}e^{-\frac{3\pi i}{8}}\cancel{A_{\sigma\psi\sigma\sigma}} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(1,\bar{\sigma}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{-\frac{\pi i}{8}}\cancel{A_{\sigma 1\sigma\sigma}} + 2^{1/4}e^{\frac{3\pi i}{8}}\cancel{A_{\sigma\psi\sigma\sigma}} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,1) = \frac{1}{4}\left(A_{\psi\psi\psi 1} + 2^{3/4}e^{\frac{\pi i}{2}}\cancel{A_{\psi\sigma\psi\sigma}} - A_{\psi 1\psi\psi}\right)$$

$$(1,\psi) = \frac{1}{4}\left(A_{\psi\psi\psi 1} + 2^{3/4}e^{\frac{-\pi i}{2}}\cancel{A_{\psi\sigma\psi\sigma}} - A_{\psi 1\psi\psi}\right)$$

$$(\sigma,\bar{\psi}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{-\frac{7\pi i}{8}}\cancel{A_{\sigma 1\sigma\sigma}} + 2^{1/4}e^{\frac{5\pi i}{8}}\cancel{A_{\sigma\psi\sigma\sigma}} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar{\sigma}) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{\frac{7\pi i}{8}}\cancel{A_{\sigma 1\sigma\sigma}} + 2^{1/4}e^{-\frac{5\pi i}{8}}\cancel{A_{\sigma\psi\sigma\sigma}} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar{\psi}) = \frac{1}{4}\left(A_{1111} - 2^{3/4}\cancel{A_{1\sigma 1\sigma}} + A_{1\psi 1\psi}\right)$$

$$(\sigma,\bar{\sigma}) = \frac{1}{2}\left(A_{1111} + A_{\psi\psi\psi 1} - A_{1\psi 1\psi} + A_{\psi 1\psi\psi}\right)$$

Condensing $\mathbb{Z}_2$

1	$(1,1)$	$\sim$	$(\psi,\bar\psi)$
e,m	$(\sigma,\bar\sigma)$	$\rightarrow$	Splits
em	$(\psi,1)$	$\sim$	$(1,\psi)$
σ_+	$(\sigma,1)$	$\sim$	$(\sigma,\bar\psi)$
σ_-	$(1,\bar\sigma)$	$\sim$	$(\psi,\bar\sigma)$

$\Leftarrow$

$$(1,1) = \frac{1}{4}\left(A_{1111} + 2^{3/4}A_{1\sigma 1\sigma} + A_{1\psi 1\psi}\right)$$

$$(\sigma,1) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{\frac{\pi i}{8}}A_{\sigma 1\sigma\sigma} + 2^{1/4}e^{-\frac{3\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(1,\bar\sigma) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{-\frac{\pi i}{8}}A_{\sigma 1\sigma\sigma} + 2^{1/4}e^{\frac{3\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,1) = \frac{1}{4}\left(A_{\psi\psi\psi 1} + 2^{3/4}e^{\frac{\pi i}{2}}A_{\psi\sigma\psi\sigma} - A_{\psi 1\psi\psi}\right)$$

$$(1,\psi) = \frac{1}{4}\left(A_{\psi\psi\psi 1} + 2^{3/4}e^{\frac{-\pi i}{2}}A_{\psi\sigma\psi\sigma} - A_{\psi 1\psi\psi}\right)$$

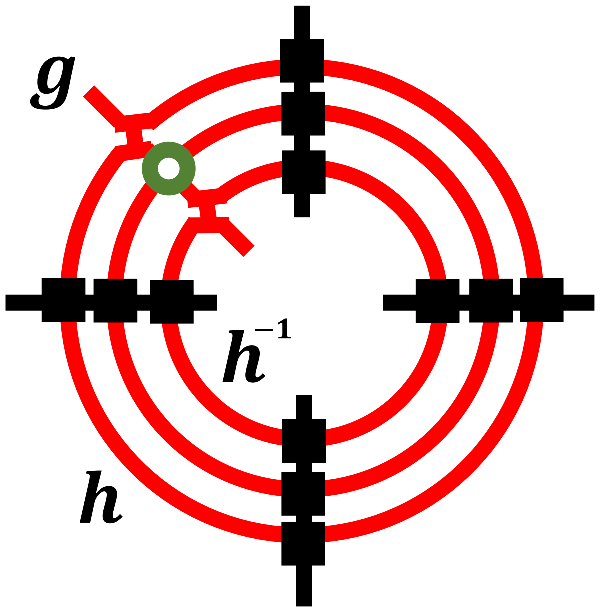
$$(\sigma,\bar\psi) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{-\frac{7\pi i}{8}}A_{\sigma 1\sigma\sigma} + 2^{1/4}e^{\frac{5\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar\sigma) = \frac{1}{4}\left(A_{\sigma\sigma\sigma 1} + 2^{1/4}e^{\frac{7\pi i}{8}}A_{\sigma 1\sigma\sigma} + 2^{1/4}e^{-\frac{5\pi i}{8}}A_{\sigma\psi\sigma\sigma} + e^{-\frac{\pi i}{2}}A_{\sigma\sigma\sigma\psi}\right)$$

$$(\psi,\bar\psi) = \frac{1}{4}\left(A_{1111} - 2^{3/4}A_{1\sigma 1\sigma} + A_{1\psi 1\psi}\right)$$

$$(\sigma,\bar\sigma) = \frac{1}{2}\left(A_{1111} + A_{\psi\psi\psi 1} - A_{1\psi 1\psi} + A_{\psi 1\psi\psi}\right)$$

U_g:



A_{1111} $\longrightarrow$ $A_{1111}+A_{\psi\psi\psi1}+A_{1\psi1\psi}+A_{\psi1\psi\psi}$

$A_{\psi\psi\psi1}$ $\longrightarrow$ $A_{1111}+A_{\psi\psi\psi1}-A_{1\psi1\psi}-A_{\psi1\psi\psi}$

$A_{1\psi1\psi}$ $\longrightarrow$ $A_{1111}-A_{\psi\psi\psi1}+A_{1\psi1\psi}-A_{\psi1\psi\psi}$

$A_{\psi1\psi\psi}$ $\longrightarrow$ $A_{1111}-A_{\psi\psi\psi1}-A_{1\psi1\psi}+A_{\psi1\psi\psi}$

U:

e		m
$A_{1111} - A_{1\psi 1\psi}$	$\longleftrightarrow$	$A_{\psi\psi\psi 1} + A_{\psi 1\psi\psi}$

$$\begin{array}{l} A_{1111} \\ A_{\psi\psi\psi 1} \\ A_{1\psi 1\psi} \\ A_{\psi 1\psi\psi} \end{array} \longrightarrow \begin{array}{l} A_{1111} + A_{\psi\psi\psi 1} + A_{1\psi 1\psi} + A_{\psi 1\psi\psi} \\ A_{1111} + A_{\psi\psi\psi 1} - A_{1\psi 1\psi} - A_{\psi 1\psi\psi} \\ A_{1111} - A_{\psi\psi\psi 1} + A_{1\psi 1\psi} - A_{\psi 1\psi\psi} \\ A_{1111} - A_{\psi\psi\psi 1} - A_{1\psi 1\psi} + A_{\psi 1\psi\psi} \end{array}$$

- Tensor networks describing SET states have matrix product operator symmetries
- By studying these MPOs we extract a graded fusion algebra
- From this we can construct a dube algebra to extract (all) information about the SET order in the system

Questions?

Questions?

Normal form

$$A^i \sim \begin{pmatrix} A_0^i & M_{01}^i & \cdots \\ 0 & A_1^i & \\ \vdots & & \ddots \end{pmatrix}$$

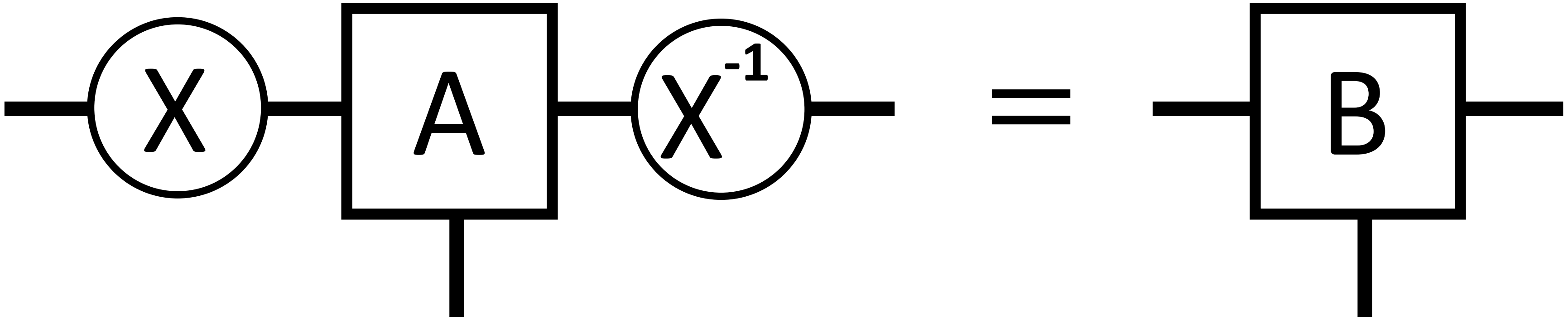
s.t. $\{A_k^i\}_i$ generates an irred. algebra

$$|MPS_N(A)\rangle = \sum_k |MPS_N(A_k)\rangle$$

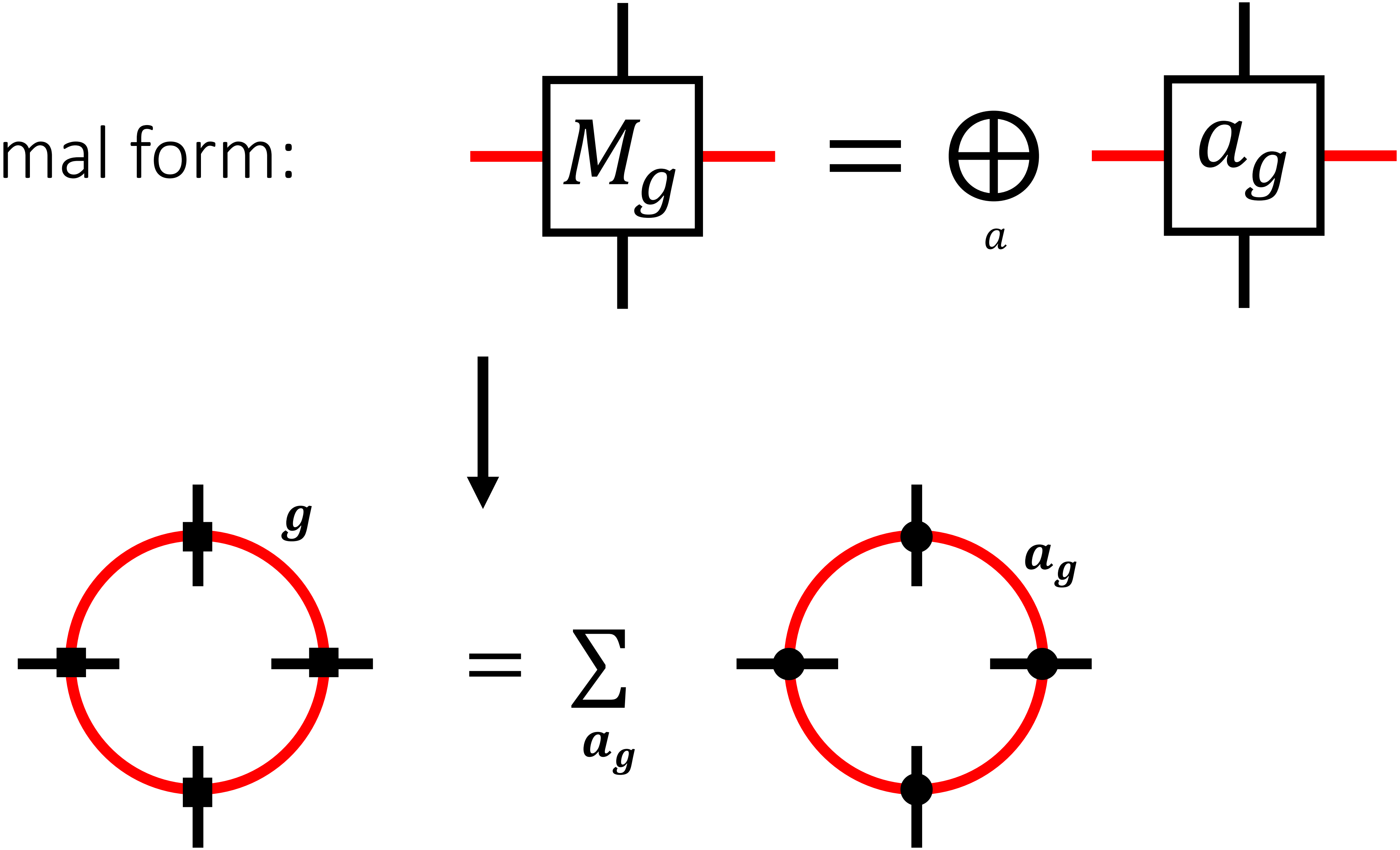
Fundamental Thm. of MPS

$$|MPS_N(A)\rangle = |MPS_N(B)\rangle \;\; \forall N$$

$\Rightarrow \exists\, X$ s.t.

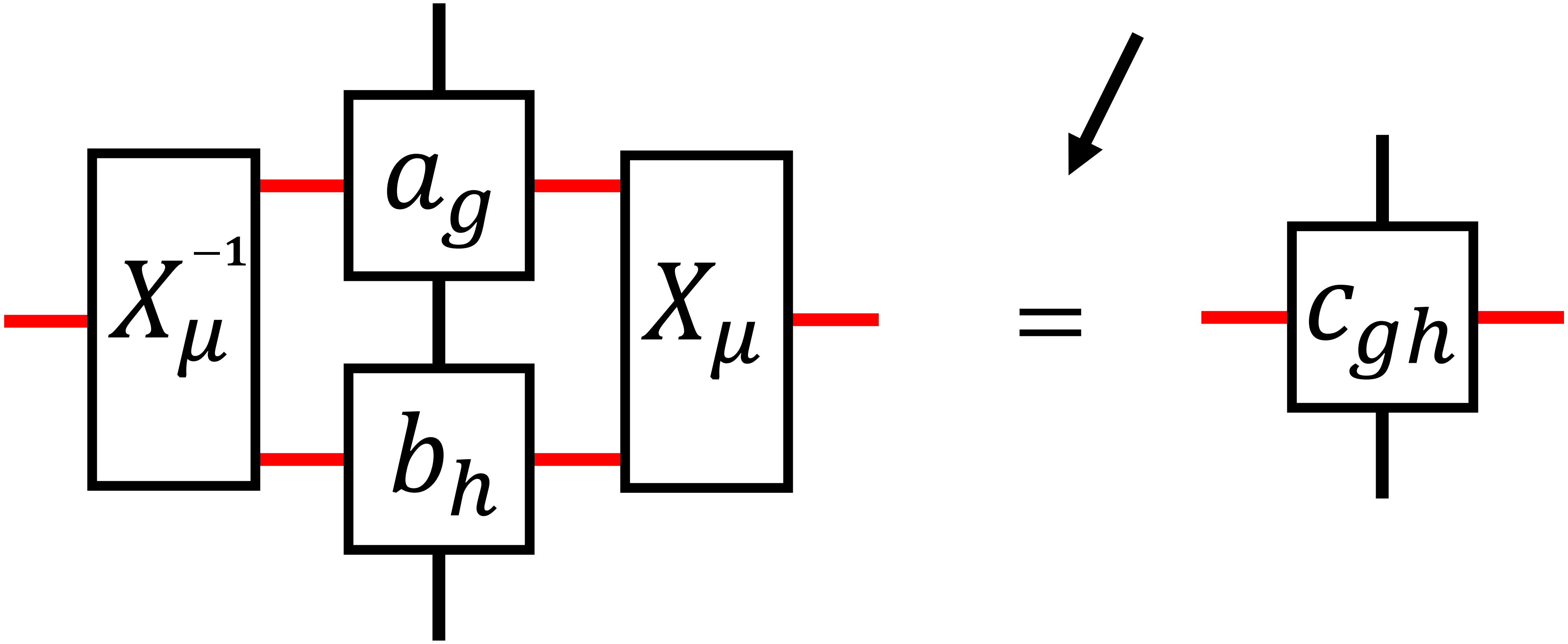


Normal form:

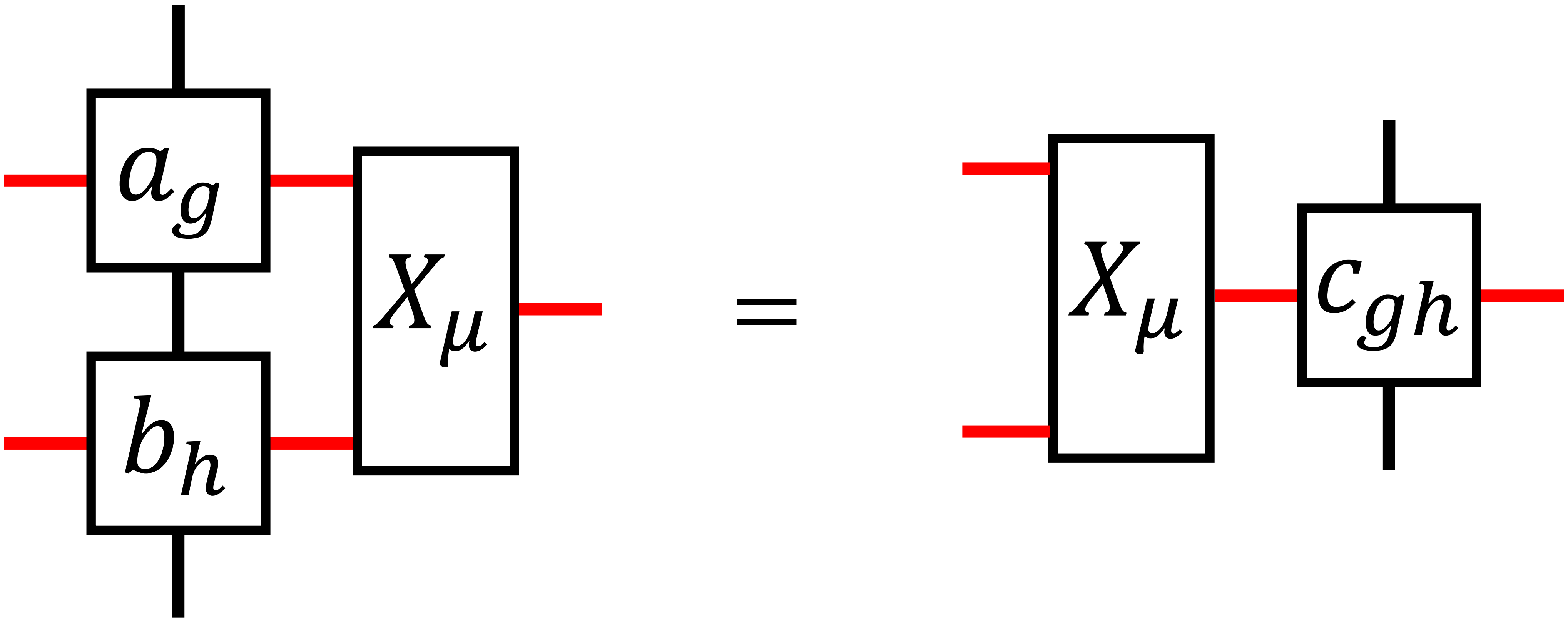


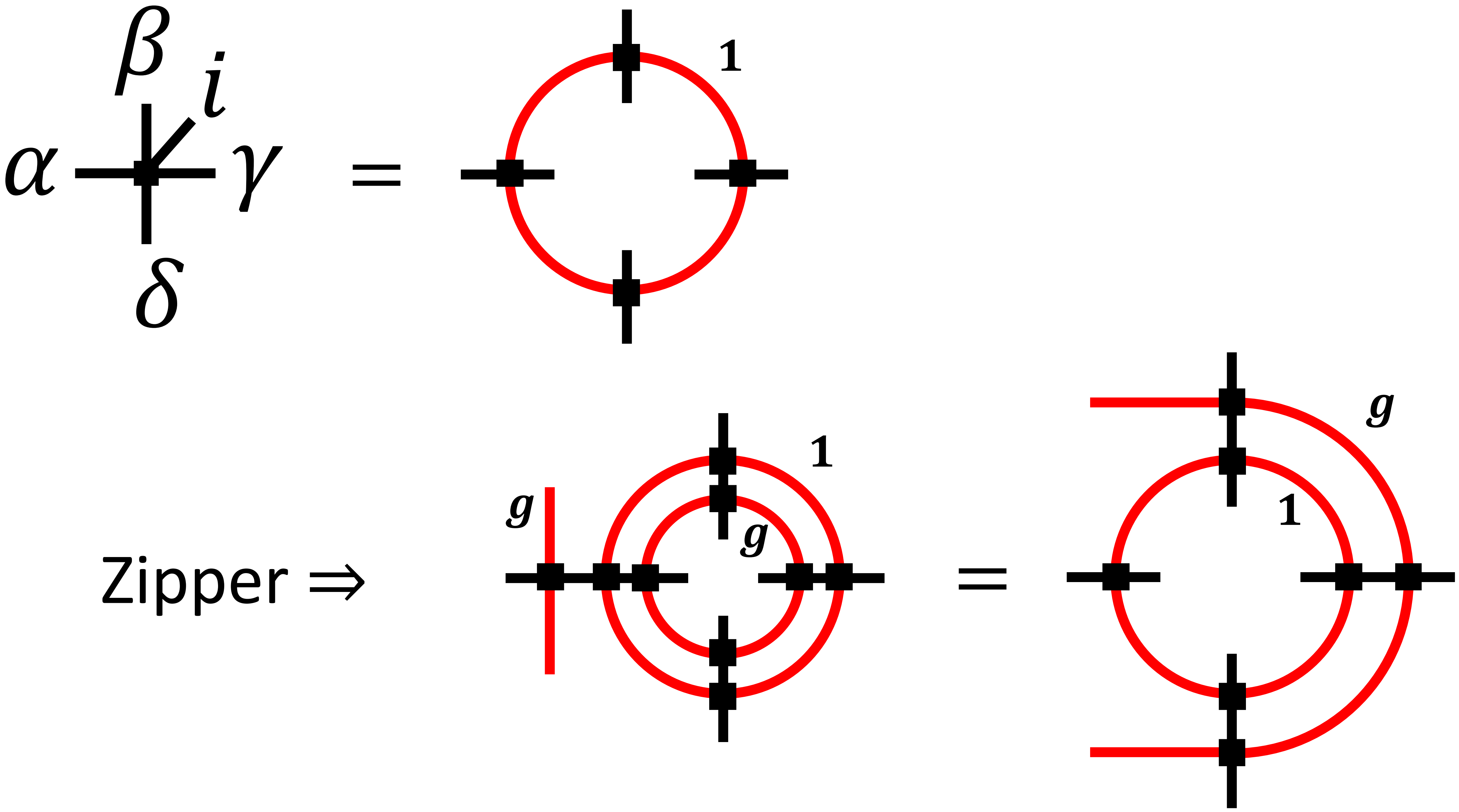
$$M_g(L)M_h(L) = M_{gh}(\mathrm{L})$$

$$M_{a_g}\mathrm{M}_{\mathrm{b_h}} = \sum_c N_{ab}^c M_{c_{gh}}$$

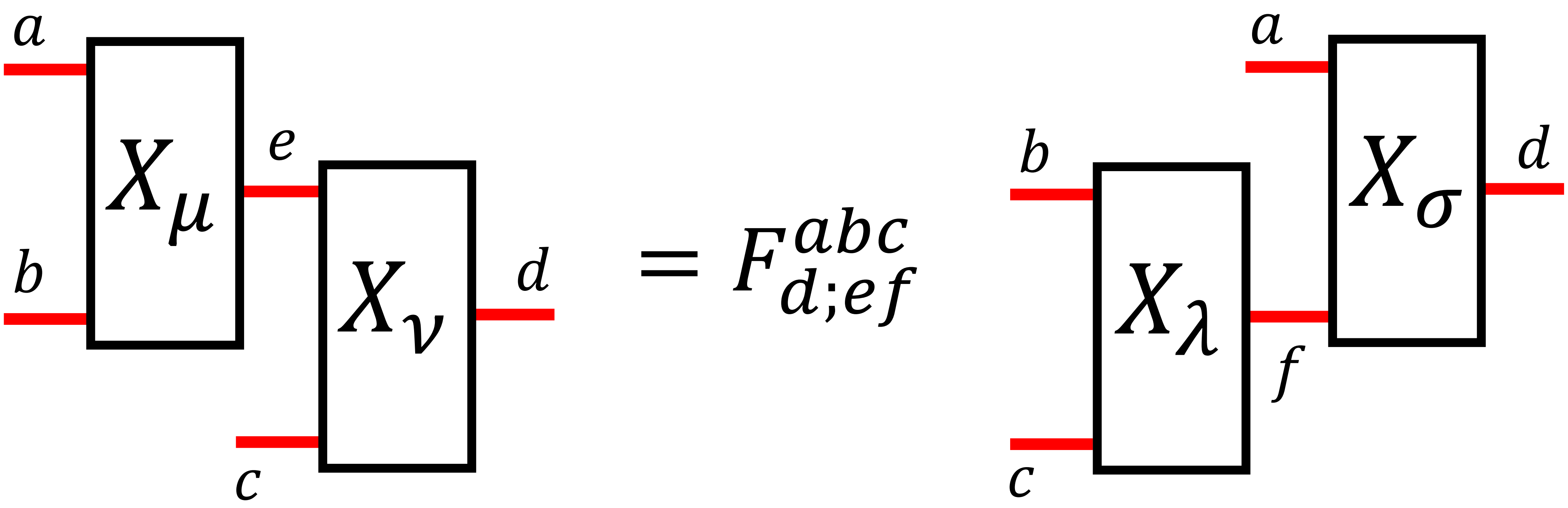


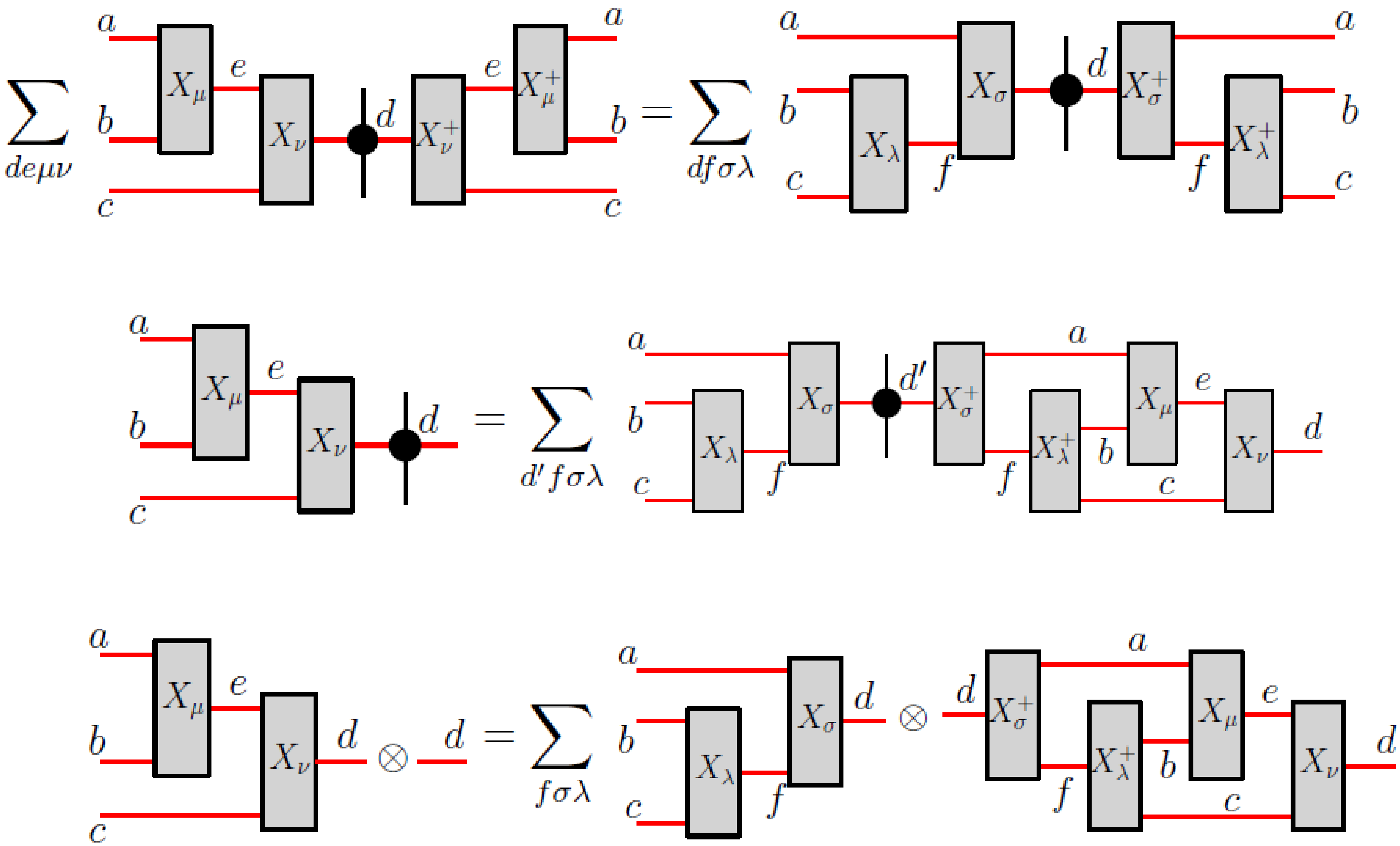
The Zipper condition





β
i
α
γ
δ
=
1
Zipper ⇒
g
1
g
=
g
1


a
b
c
e
d
X_μ
X_ν
$= F^{abc}_{d;ef}$
a
b
c
f
d
X_λ
X_σ



$\sum_{de\mu\nu}$
a b c e d
X_μ X_ν X_ν^+ X_μ^+
=
$\sum_{df\sigma\lambda}$
a b c f d
X_λ X_σ X_σ^+ X_λ^+
a b c e d
X_μ X_ν
=
$\sum_{d'f\sigma\lambda}$
a b c f d' e d
X_λ X_σ X_σ^+ X_λ^+ X_μ X_ν
a b c e d
X_μ X_ν
$\otimes$
=
$\sum_{f\sigma\lambda}$
a b c f d
X_λ X_σ X_σ^+ X_λ^+ X_μ X_ν