Gauge

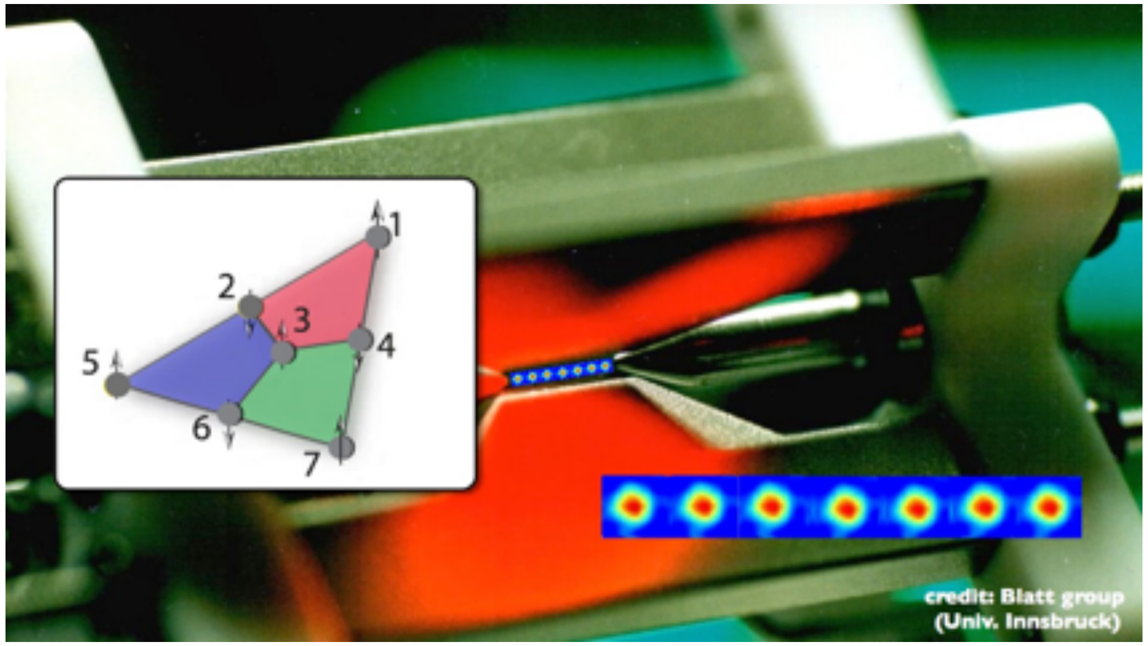
Robin Blume-Kohout

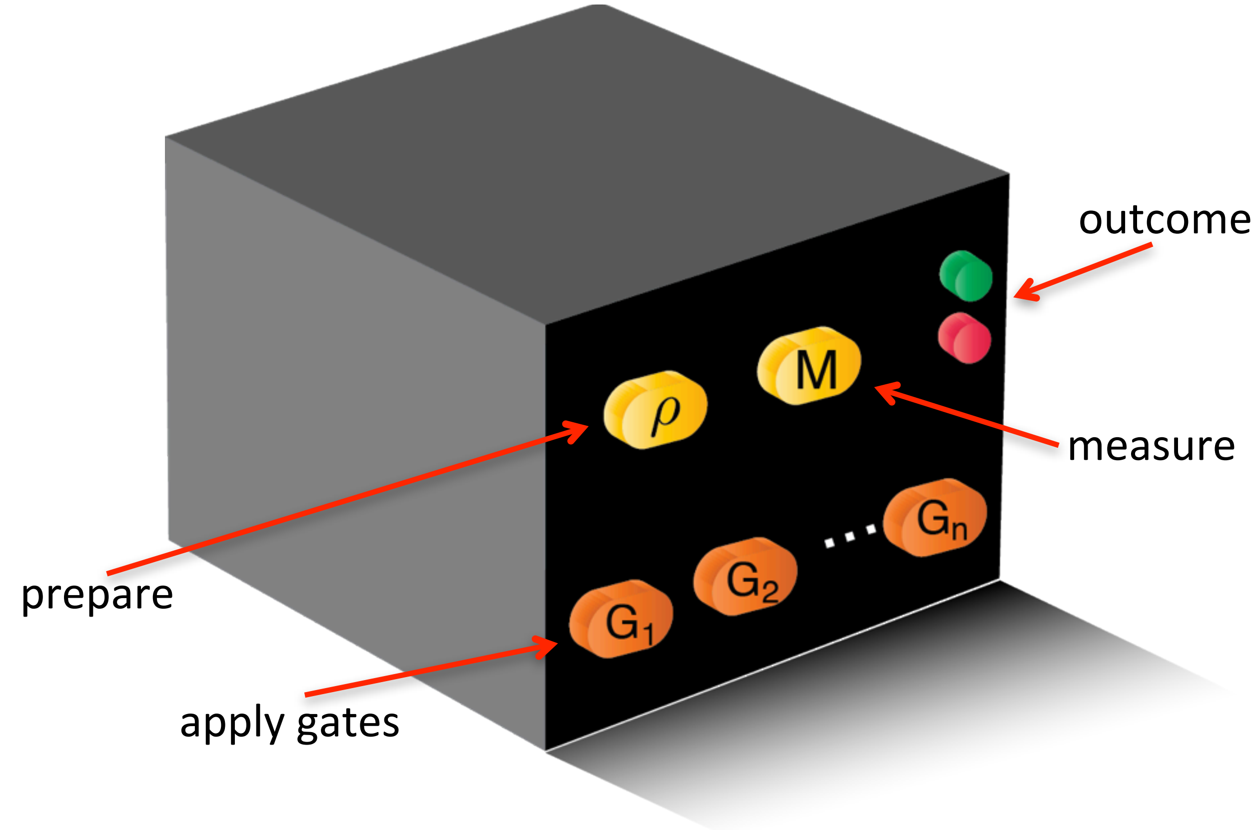
Center for Computing Research
Sandia National Labs

Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

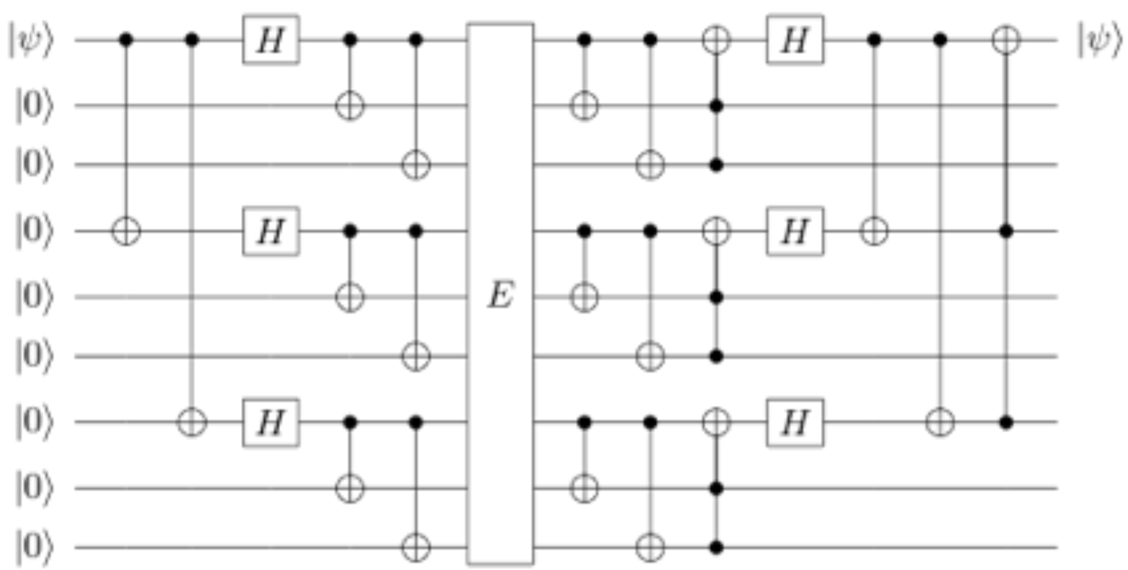
Abstract: Quantum computation is a gauge theory. As-built quantum information processors (QIPs) are described by "gate sets" that associate a quantum process matrix to each logic gate that can appear in a quantum circuit. But these descriptions are not unique. For any given QIP, there is an infinite set of equivalent gatesets that look quite different, but are experimentally indistinguishable. This is surprisingly inconvenient for characterizing QIPs — i.e., for tomography, randomized benchmarking, and any other attempt to infer properties of the gateset from experimental data. I will present what is known about the gauge freedom, survey the problems that it presents, and issue a challenge to the audience to slay this dragon by developing a gauge-free theory of QIPs

What gauge are we talking about?





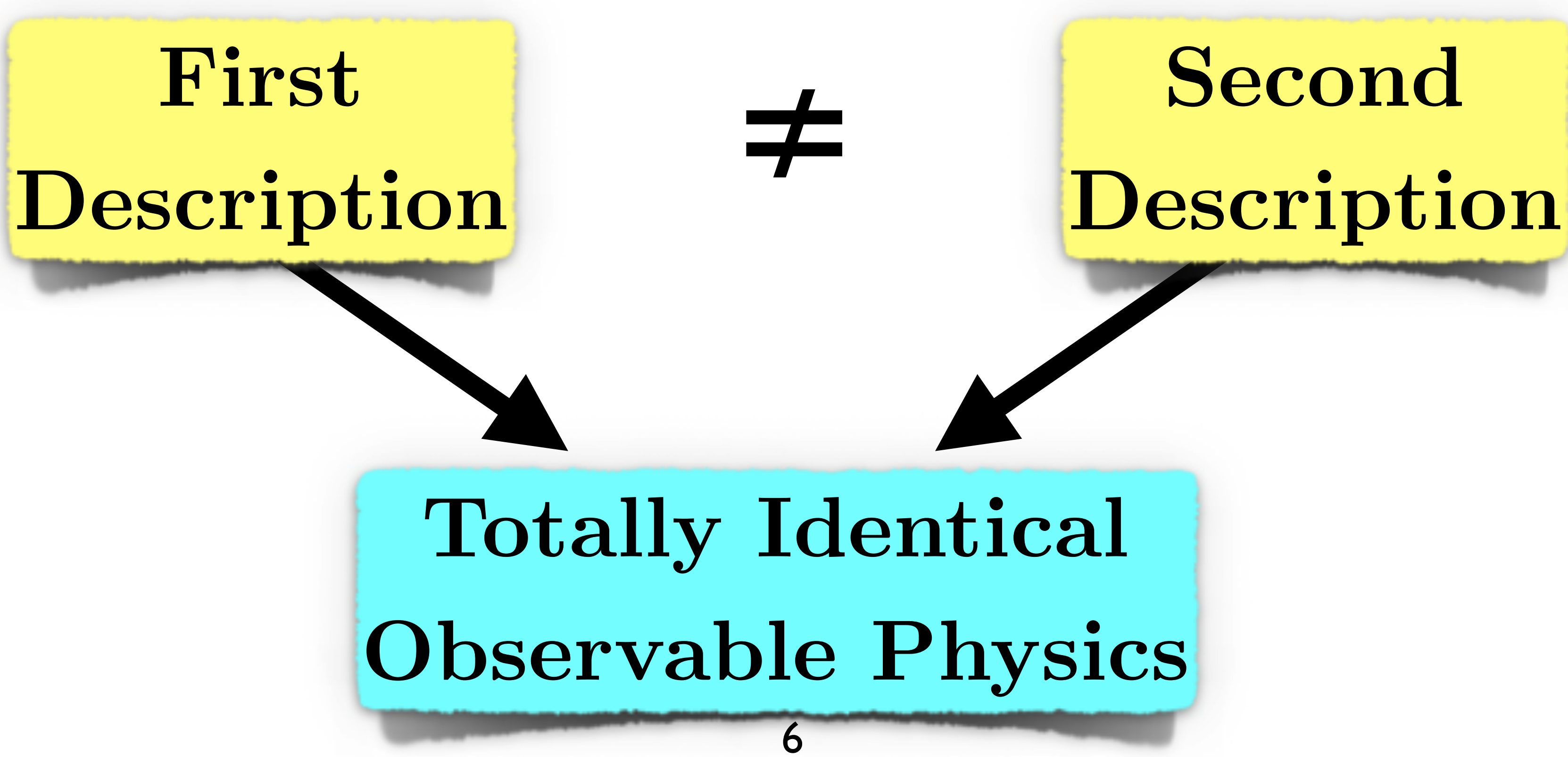
What gauge are we talking about?



$$\{\rho, E, \{G_i\}\}$$

What is a gauge?

What is a gauge?

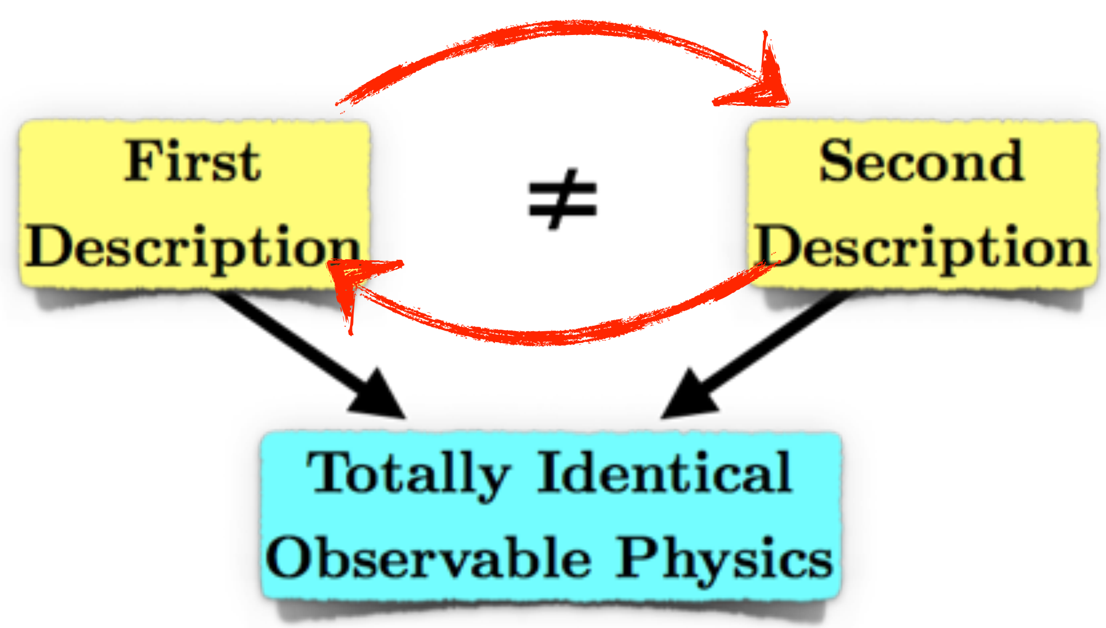


What is a gauge transformation?

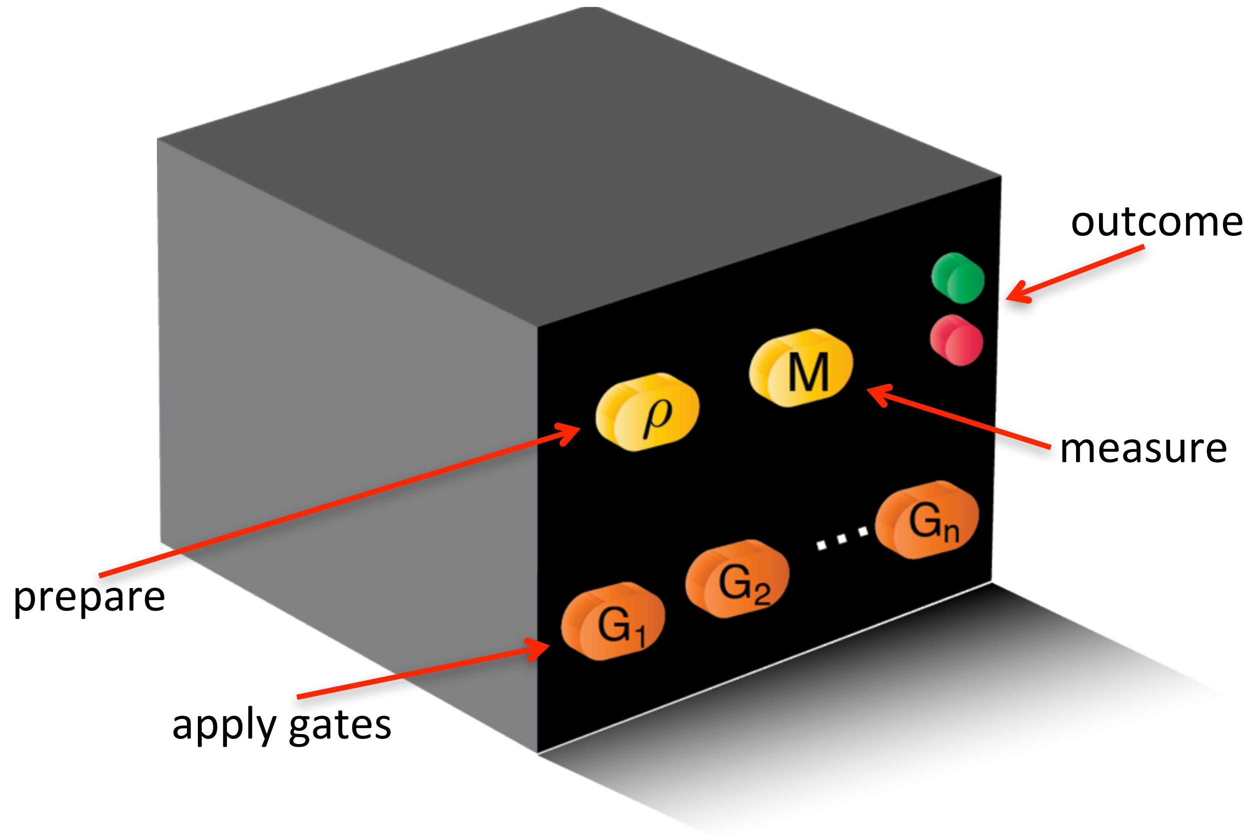
What is a gauge transformation?

A mathematical mapping that turns one description of a system into another (equivalent) description of the same system.

They typically form a group.



What does this gauge act on?



What does this gauge act on?

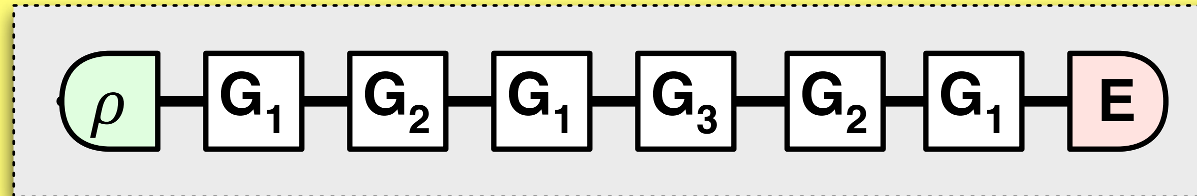
Gatesets that describe a black-box QIP.

$$\{\rho, E, \{G_i\}\}$$

$$\begin{pmatrix}1\\x\\y\\z\end{pmatrix} \qquad (w,\ x',\ y',\ z') \qquad \begin{pmatrix}1 & 0 & 0 & 0\\0 & 0.8 & 0 & 0\\0 & 0 & 0 & 0.8\\0.1 & 0 & -0.8 & 0\end{pmatrix}$$

How does it act?

Note: Every experiment on a QIP looks like this:



$$Pr(E) = \langle\!\langle E|\, G_1 G_2 \ldots G_1 \,|\rho\rangle\!\rangle$$

How does it act?

$$\langle\!\langle E'| = \langle\!\langle E|\, T$$

Let T be an invertible trace-preserving superoperator (*not* necessarily CP)

$$|\rho'\rangle\!\rangle = T^{-1}\,|\rho\rangle\!\rangle$$

$$G'_k = T^{-1} G_k T$$

Is that *all* the gauge transformations?

(i.e., are two gatesets *not* thus related necessarily distinguishable?)

Is that *all* the gauge transformations?

(i.e., are two gatesets *not* thus related necessarily distinguishable?)

Yes, except possibly on a set of measure zero.
Tomography can reveal the entire gateset
***up to* similarity transformations of that kind.**

Does this gauge apply to individual gates (process matrices)?

Does this gauge apply to individual gates (process matrices)?

No! It's a property of the whole *gate set*.

Gates have individual & relational properties.

Does every process matrix transform that way?

Does every process matrix transform that way?

No, only ones representing *gates*.

Counterexample: “error processes”

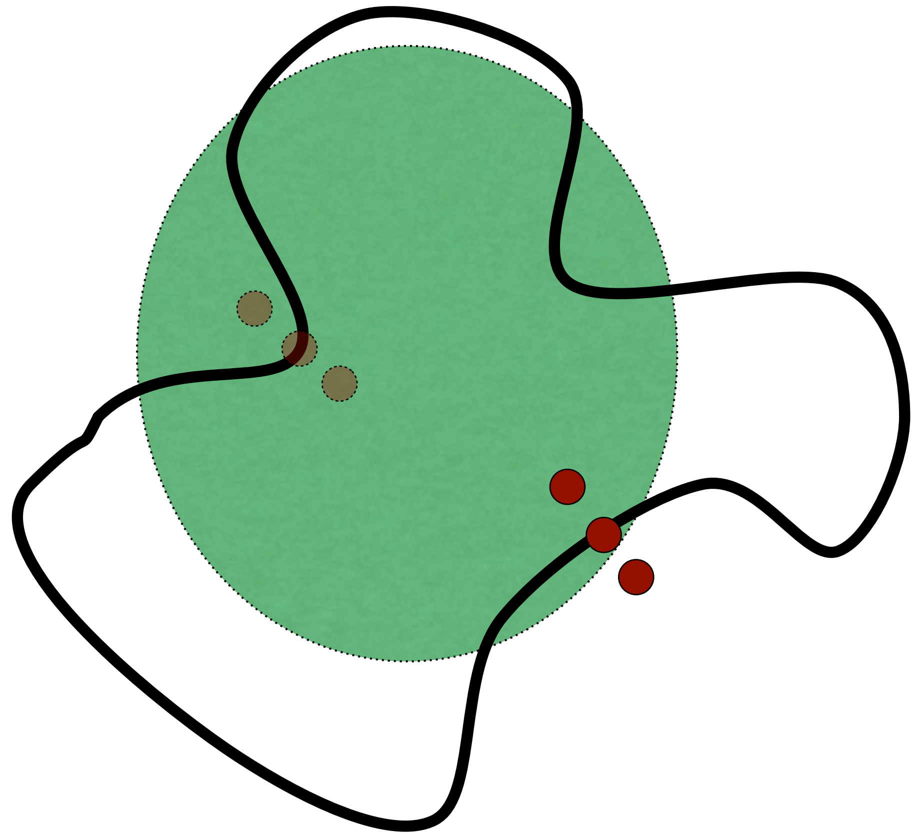
$$\Lambda_i \equiv G_i \circ \left(G_i^{(ideal)} \right)^{-1}$$

What is the gauge group?

What is the gauge group?

Good question. I think it's basically $GL(d^2)$, but not a faithful representation...

Is it really a group?

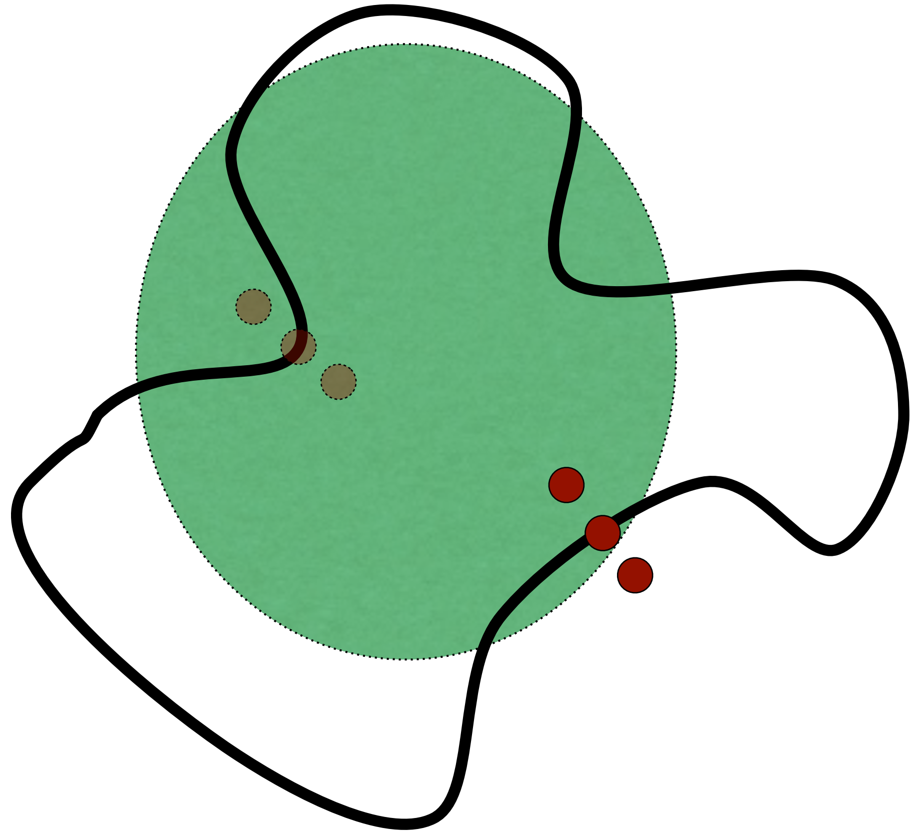


Is it really a group?

Yes, if you ignore complete positivity.

If you only allow gauge transformations that preserve positivity, then it's restricted

Do gauge transformations preserve [complete] positivity?



Do gauge transformations preserve [complete] positivity?

No.

- There's a unitary subgroup that *always* does.
- If gateset is extremal, then *only* that subgroup does.
- If all the gates have full-rank Choi matrices (e.g. some depolarizing noise), then *any sufficiently small* gauge transformation preserves CP.

Do gauge transformations preserve distances between things?

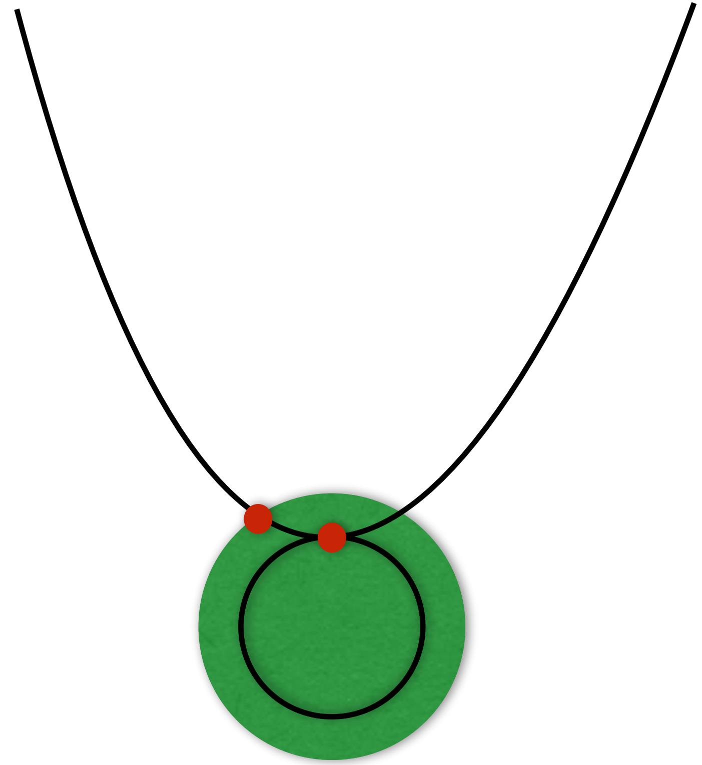
Do gauge transformations preserve distances between things?

Yes — for some metrics (2-norm, for example).

No — for most QI metrics (diamond norm, fidelity)

No **for distances to ideal reference gates!...**
...except ***Yes*** **for fidelity with the identity operation.**

Can’t I just consider unitary gauge transformations?

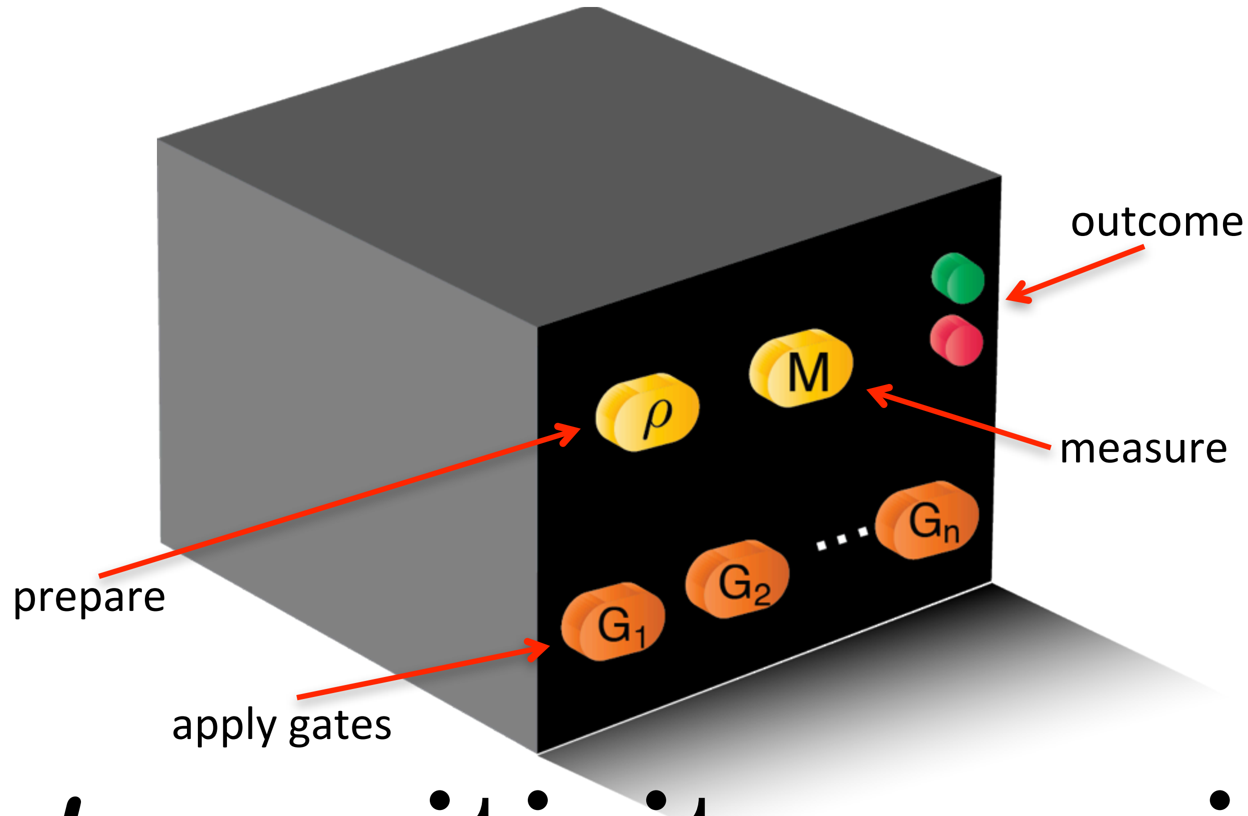


Can't I just consider unitary gauge transformations?

Well, yeah. But it will mislead you.

***Example*: I give you two noisy gatesets related by a non-unitary gauge transformation. Do you see that they are the same?**

Is *complete* positivity meaningful in a black box context?



Is *complete* positivity meaningful in a black box context?

???

Can I tell whether two gatesets are gauge-equivalent?

Can I tell whether two gatesets are gauge-equivalent?

Yes. There are efficient algorithms for this.

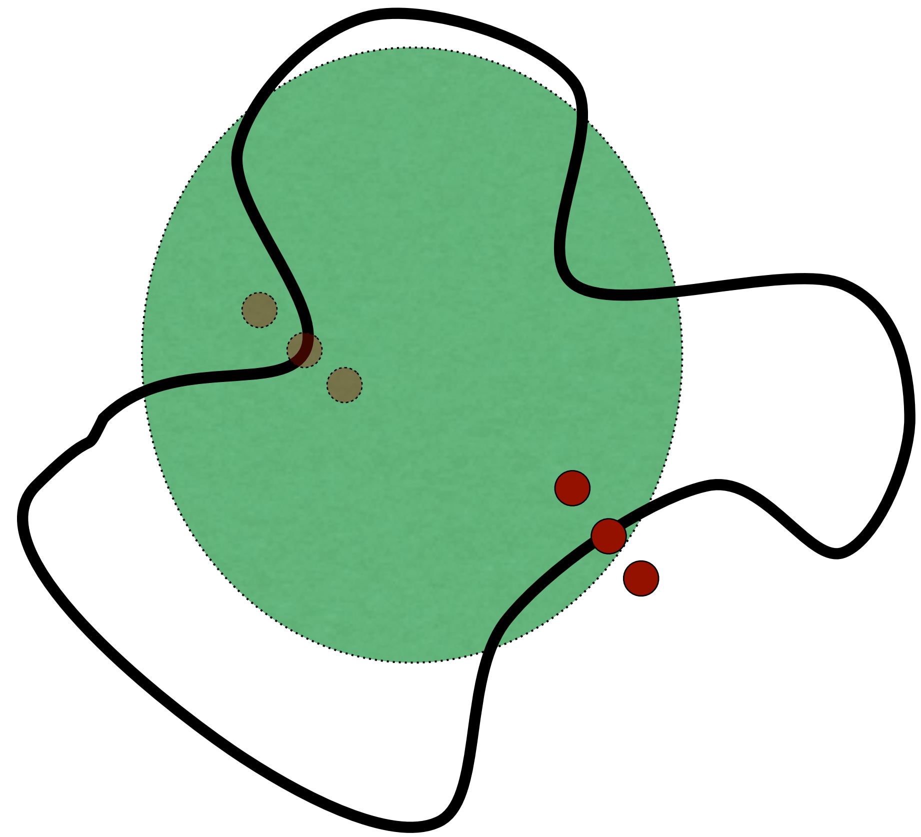
- **simulate linear GST**
- **or reduce both gatesets to a canonical form.**

Can I tell whether two gatesets are *close* to gauge-equivalent?

Can I tell whether two gatesets are *close* to gauge-equivalent?

Probably, but we don't really know what the right definition of "close" is. So... no, not now.

Can I tell whether a gateset is [gauge-equivalent to] CP?



Can I tell whether a gateset is [gauge-equivalent to] CP?

We don't even know how hard this is.

Best guess is that it's NP-hard.

See also "Completely Positive Realization Problem"

What things are gauge-invariant?

What things are gauge-invariant?

- **Eigenvalues of each gate G_i**
- **Inner products between:**

  (1) $\{$left eigenvectors of any G_i + effects $E\}$.

  (2) $\{$right eigenvectors of any G_i + state $\rho\}$.

Decay rates, rotation angles, fidelity w/$\mathbb{1}$, SPAM...

...but not cooling (T1), gate fidelities, rotation *axes*,...

What's the gauge-invariant version of:

- **Process fidelity?**
- **State fidelity?**
- **Diamond norm distance?**

What's the gauge-invariant version of:

- **Process fidelity?**
- **State fidelity?**
- **Diamond norm distance?**

???

In all cases, it depends on *between what*.

Gate properties are often *relative* to other gates.

What properties are gauge-*variant*?

(i.e., not gauge-invariant)

What properties are gauge-*variant*?

(i.e., not gauge-invariant)

- Almost anything you want to know, probably.

- Generally, anything that compares a particular gate to an external reference frame is *highly* gauge-variant.
- Example: the "error maps" that appear in RB.

- Most properties of *individual* gates.

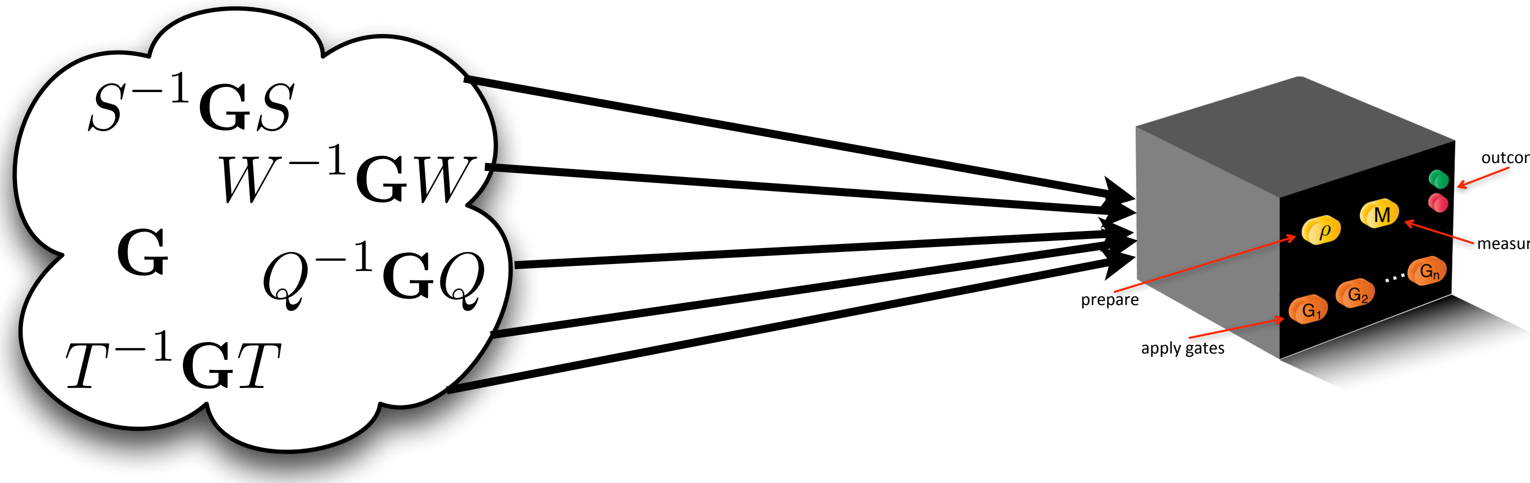
How much do they vary with gauge?

How much do they vary with gauge?

- **Some properties vary a lot. Examples:**
  - Elements of process matrices
  - Amount of non-unitality (amplitude-damping)

- **Others can be pretty stable under normal conditions:**
  - Distances between gates *in the same gateset*
  - Decoherence rates.

What consequences does gauge have for *forward* (prediction) problems?



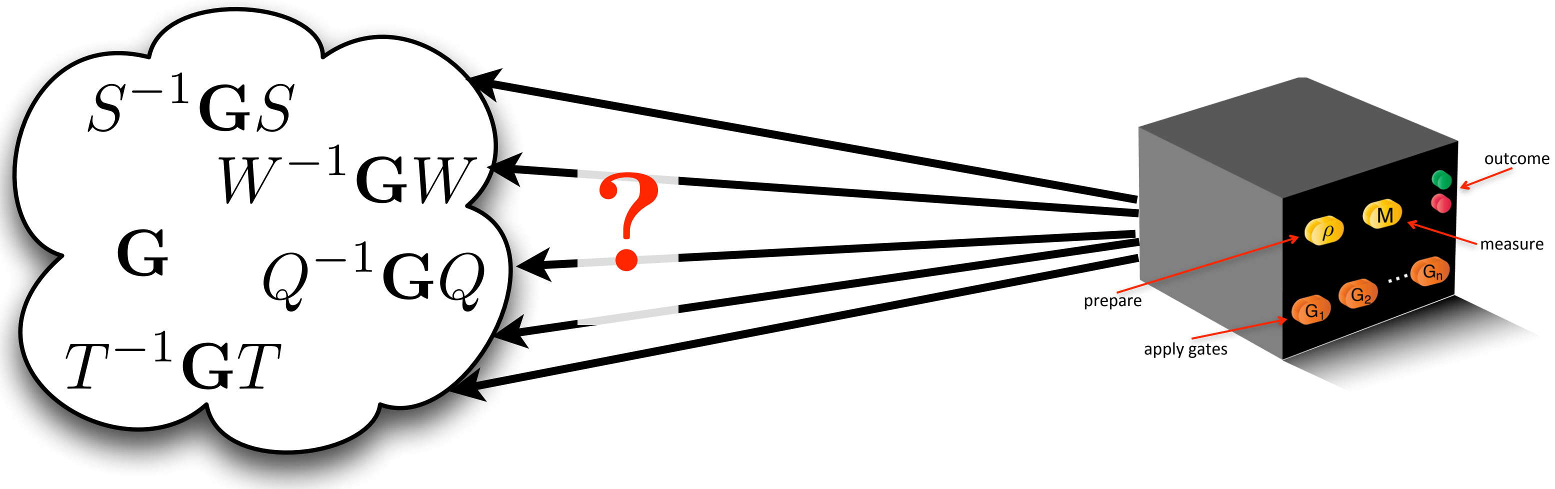
What consequences does gauge have for *forward* (prediction) problems?

- **None, if you stick to predicting circuit probabilities:**

$$Pr(E) = \langle\langle E| G_1 G_2 \ldots G_1 |\rho\rangle\rangle$$

- **Significant, if you try to *extrapolate* results from popular properties of gates that aren't gauge-invariant:**
  - Randomized benchmarking <==> fidelity
  - Worst-case error <==> diamond norm

What consequences does gauge have for *backward* (inference) problems?



What consequences does gauge have for *backward* (inference) problems?

Pretty severe

- None, as long as you:
  1. Only infer/estimate gauge-invariant quantities, OR
  2. Make sure to use your estimates correctly.

- In practice, today, everybody wants to infer/estimate/use *non-gauge-invariant* quantities.

Do I care about this?

Why should I care about this?

"Historically, the search for logically consistent and computationally tractable gauge fixing procedures, and efforts to demonstrate their equivalence in the face of a bewildering variety of technical difficulties, has been a major driver of mathematical physics from the late nineteenth century to the present."

- https://en.wikipedia.org/wiki/Gauge_fixing

Do I care about this?

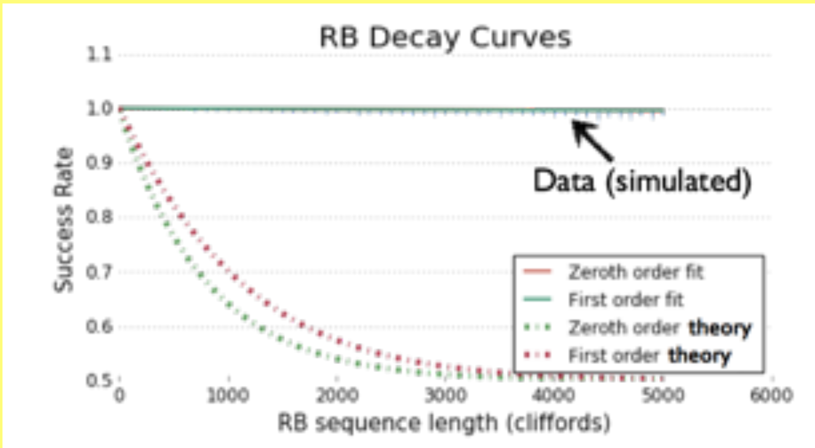
Why should I care about this?

Only if you have some connection to experimental quantum information processing.

Or care about deep and tricky problems in mathematical/foundational physics.

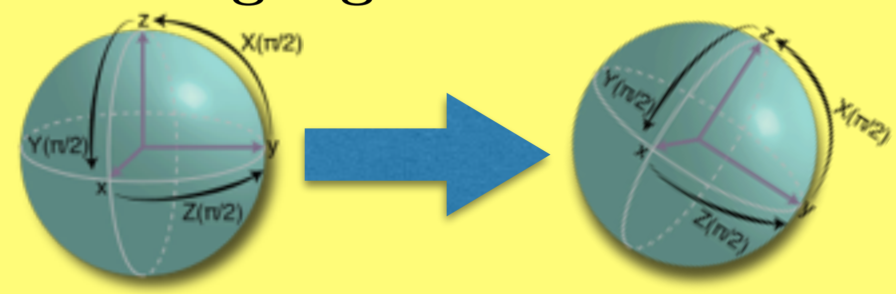
What are some particularly nasty examples where gauge crops up?

Unitary changes of gauge can increase *average gate infidelity* (related to RB) a whole lot.



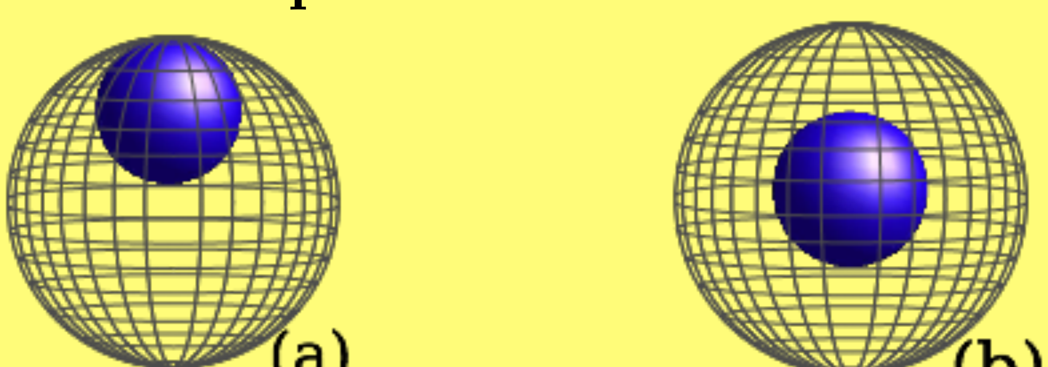
Detuning on a single qubit, with $X_{\pi/2}$ and $Y_{\pi/2}$ gates only, is [almost] undetectable by any method...

...because the "noise" is [almost] equal to a gauge transformation.



What are some particularly nasty examples where gauge crops up?

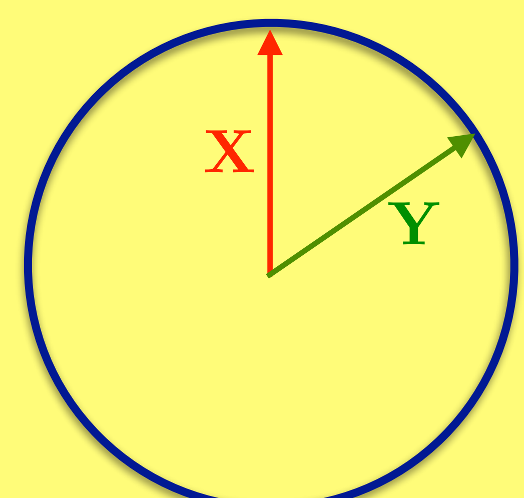
Qubit gates with T1 decay to |0> are gauge-equivalent to gates with unital noise ... unless you look carefully at the SPAM operations.



A qubit with $X_{\pi/2}$ and $Y_{\pi/2}$ gates... except their rotation axes are not orthogonal.

Which gate is erroneous?

How much error does each of the two gates have?



Stop doing gauge transformations!
They're annoying.

Stop doing gauge transformations!

They're annoying.

It wasn't my idea. Really.

A gauge "transformation" is just a handy way of describing a fact:

*** Different gatesets produce identical physics. ***

Why don't you just pick a gauge and stick with it?

Why don't you just pick a gauge and stick with it?

Because "choosing a gauge" — like Coulomb or Lorenz — actually means "define a gauge-fixing procedure". And we don't have any satisfactory general procedures yet.

(There's no well-defined notion of "What gauge is this gate-set *in*?")

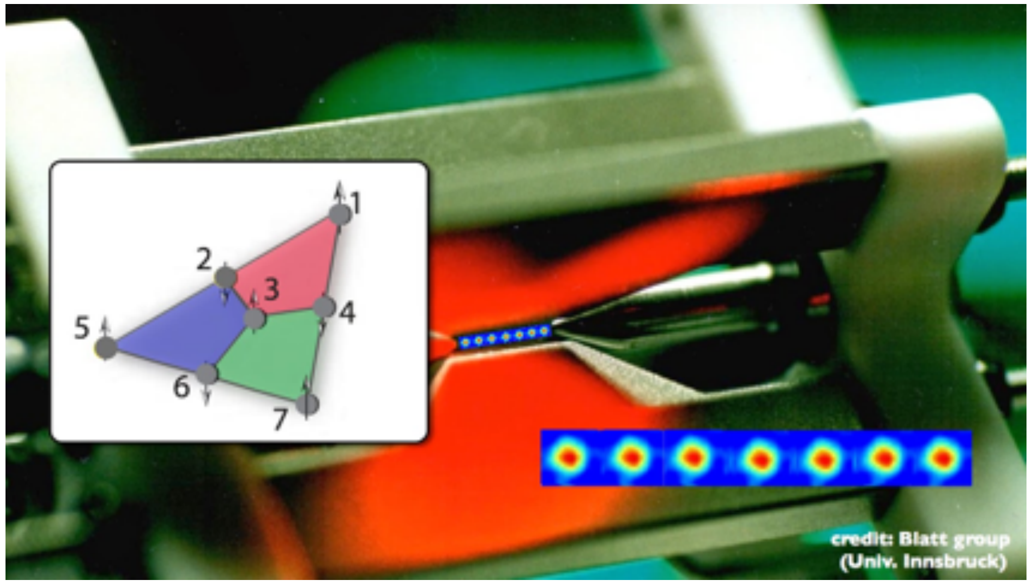
I'm okay as long as I only measure gauge-invariant things, right?

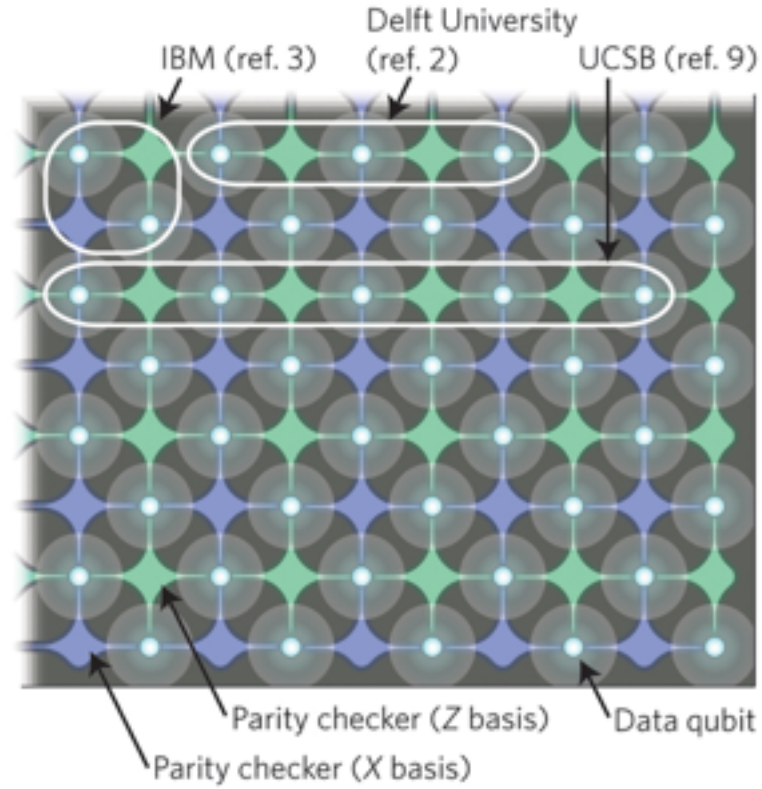
I'm okay as long as I only measure gauge-invariant things, right?

Only gauge-invariant things *can* be measured!
Gauge-*variant* quantities aren't real/observable/measurable.

You're okay as long as you only ever *think* about gauge-invariant things. Good luck with that.

If I probe two parts of a QIP, do I have to glue the gauges together?





If I probe two parts of a QIP, do I have to glue the gauges together?

Yes. Probably. We aren't totally sure what this means yet.

What *can* I learn about the gateset describing a QIP?

What *can* I learn about the gateset describing a QIP?

1) **All the gauge-invariant properties.**
2) **The entire gateset, *up to gauge transformations*.**
3) **Everything needed to predict all possible circuits.**

(Sorry if that's not the answer you were looking for...)

Gauge is just a GST thing, right?

Gauge is just a GST thing, right?

No.

This doesn't matter for randomized benchmarking, right?

This doesn't matter for randomized benchmarking, right?

Yes, it does.

RB is an experiment — what it measures is gauge-invariant.

But we don't know what that *is*, theoretically. It's not any popular "fidelity" because they aren't gauge-invariant.

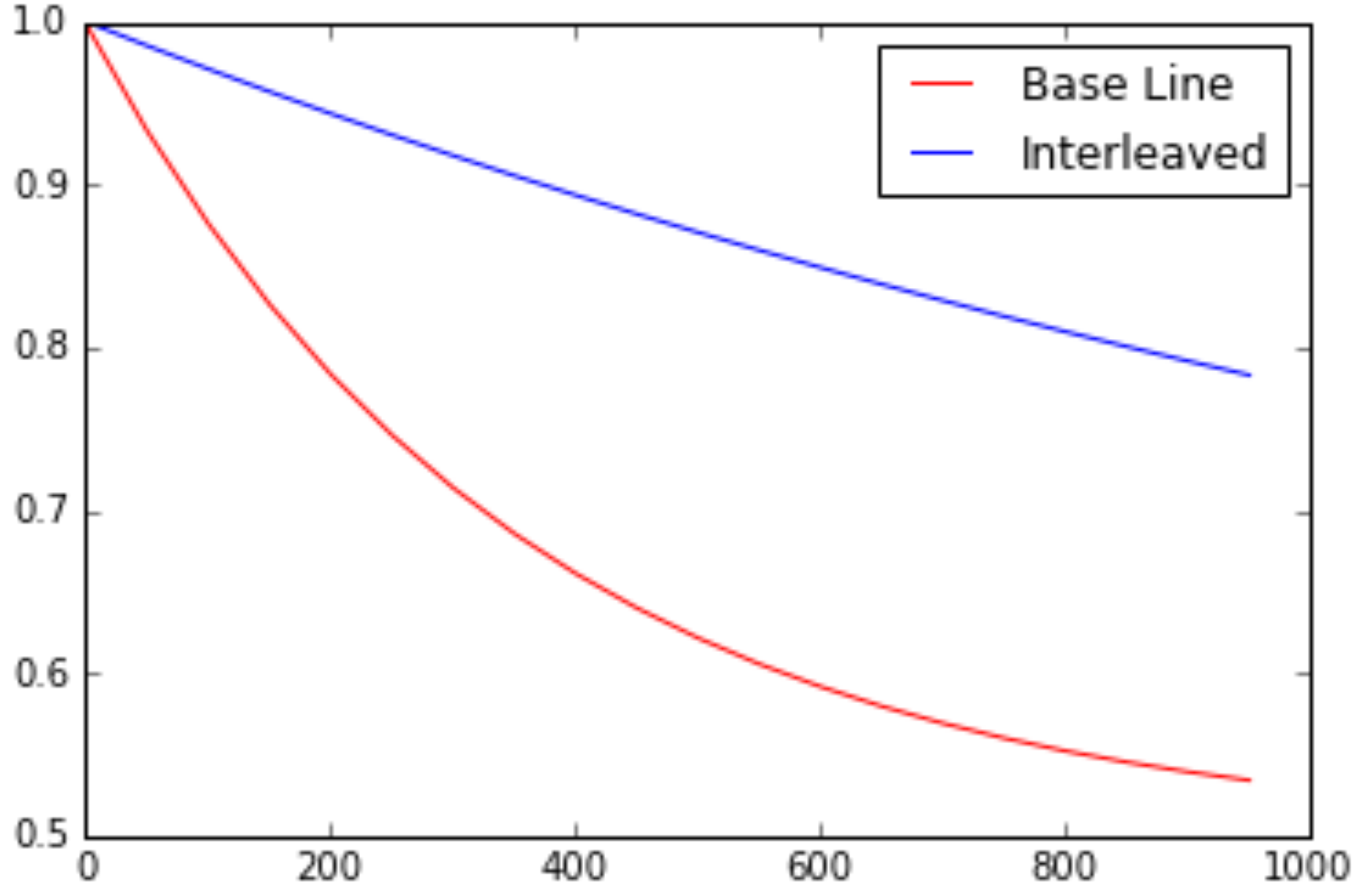
This doesn't matter for fault tolerant QEC, right?

This doesn't matter for fault tolerant QEC, right?

It matters if you want to understand how observables (like logical failure rates) depend on features of the noise model.

==> we (probably) need to make sure the "error metrics" we use to describe as-built qubits are gauge-invariant.

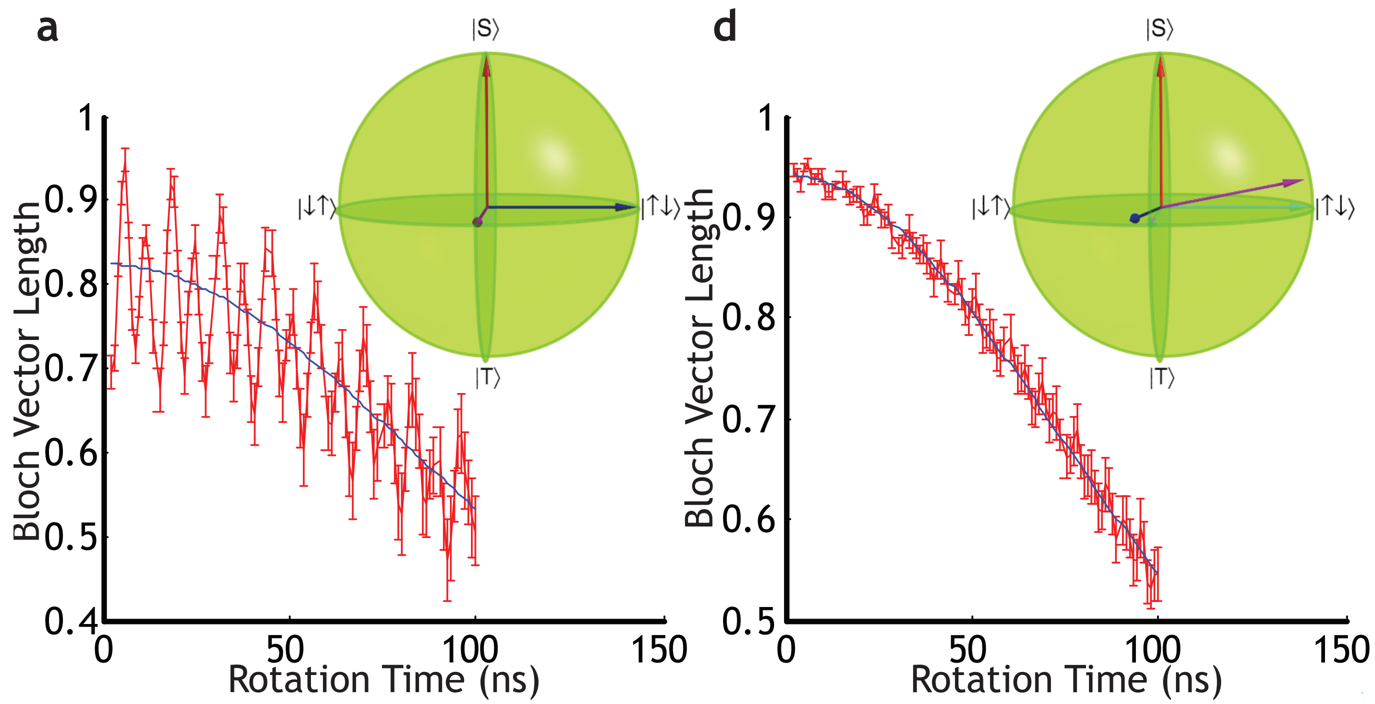
Does interleaved RB work?



Does interleaved RB work?

Not always.

What if I just do state tomography? Does gauge show up there?



M.D. Shulman *et al*, **Science** 336, 202-205 (2012)

What if I just do state tomography? Does gauge show up there?

State tomography estimates $\boldsymbol{\rho}$ *relative* to a fixed reference frame defined by X, Y, Z (or their counterparts).

It *assumes* that you can measure in these bases.

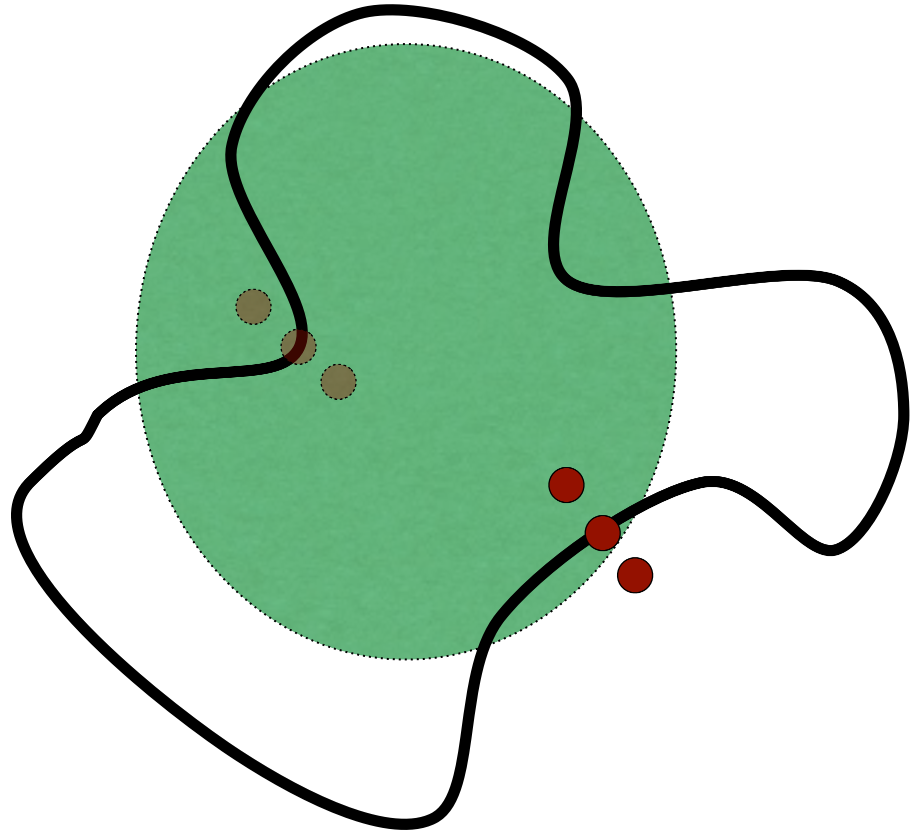
If you test/prove that assumption, you get gauge.

Why not just do process tomography, and get the whole process matrix?

Why not just do process tomography, and get the whole process matrix?

Same problem as state tomography: you're eliminating the gauge by making an assumption that's not generally true.

Is the gauge relevant to maximum likelihood estimation (MLE)?



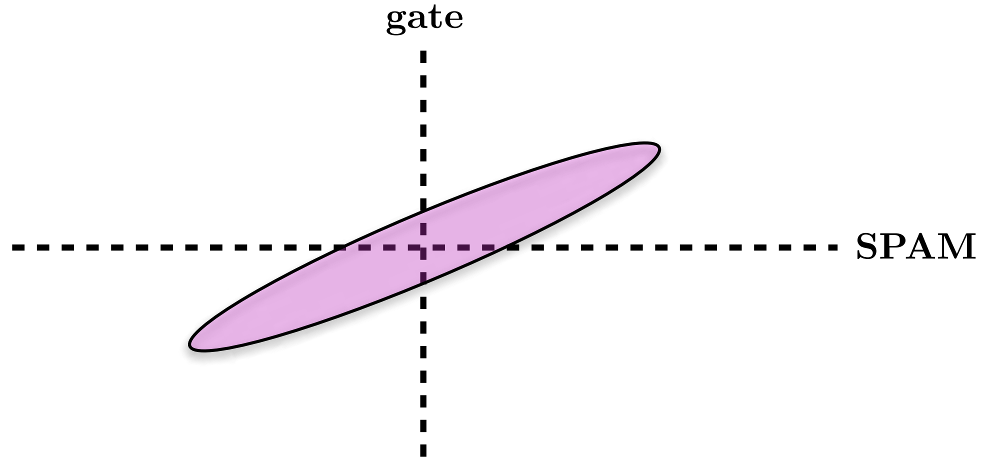
Is the gauge relevant to maximum likelihood estimation (MLE)?

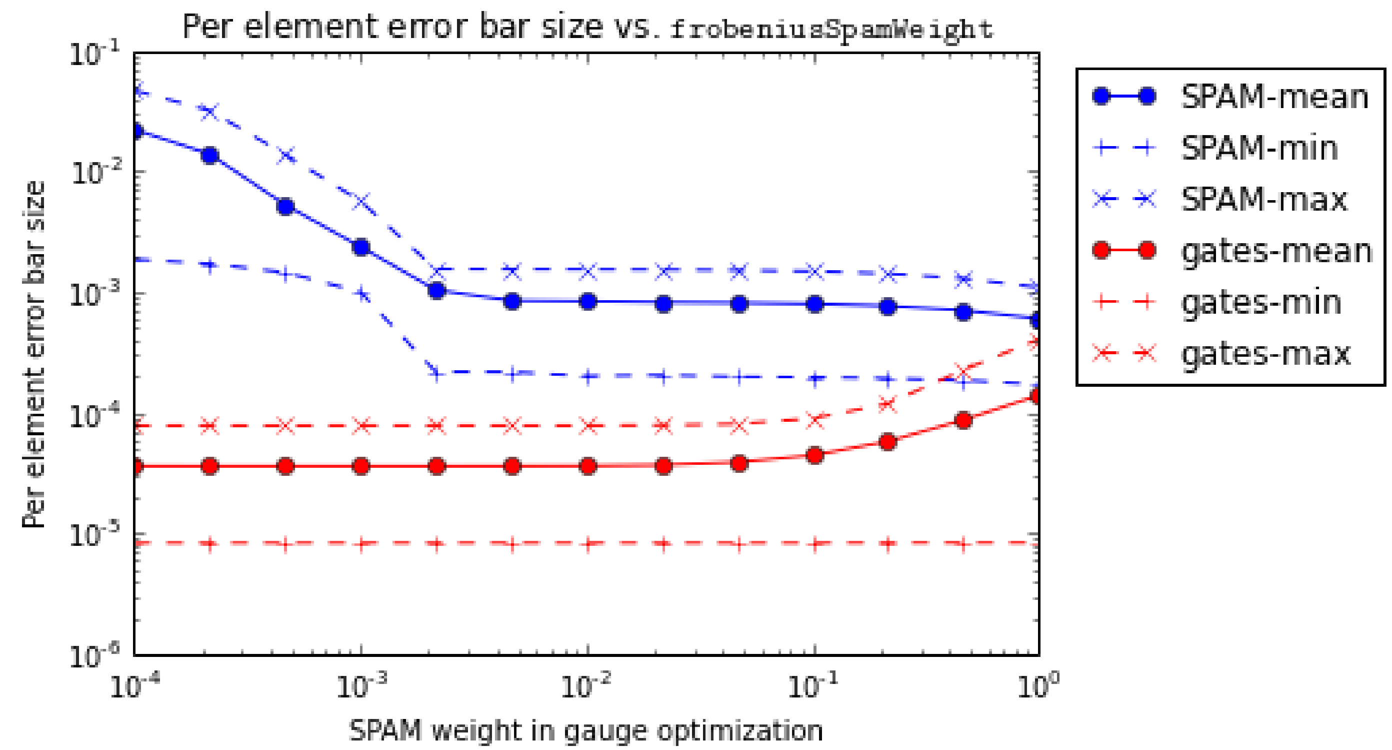
Yes. MLE over gatesets is possible, *but*:

1. **The likelihood is flat along gauge orbits — *and* not quasi convex. This presents problems for quite a few optimization algorithms.**

2. **Imposing CP is a complete nightmare, because the CP constraint *totally* doesn't play nice with gauge orbits.**

Does the gauge affect uncertainty quantification (error bars)?



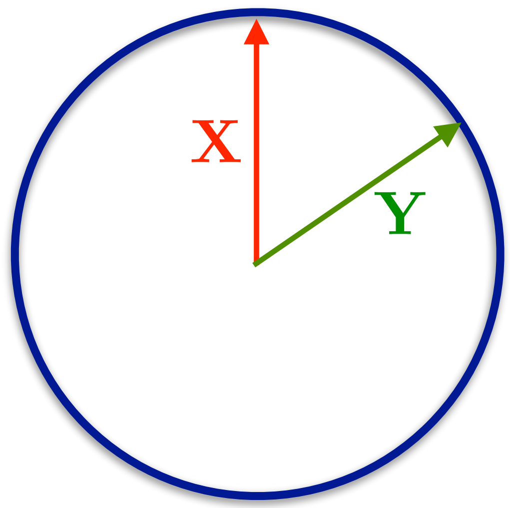


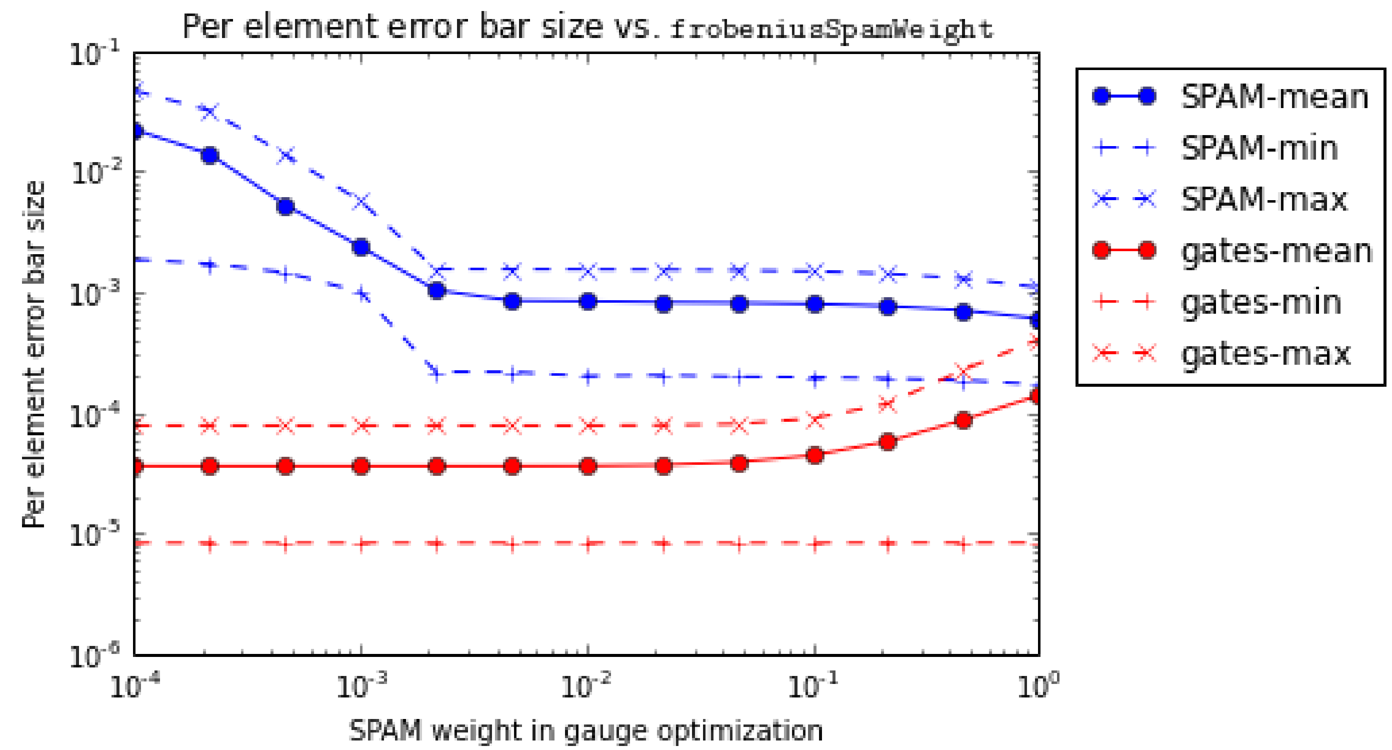
Does the gauge affect uncertainty quantification (error bars)?

Yes. It's a huge pain in the neck:

1. Technically, the fact that you can't "know" the gauge means your error bars are infinite on every gauge-variant quantity.
2. So, in practice, we fix the gauge => we "know" gauge parameters.
3. We have to fix the gauge very carefully to avoid "polluting" one parameter's error bars with another parameter's uncertainty.

You guys know how to fix the gauge when you do GST, right?





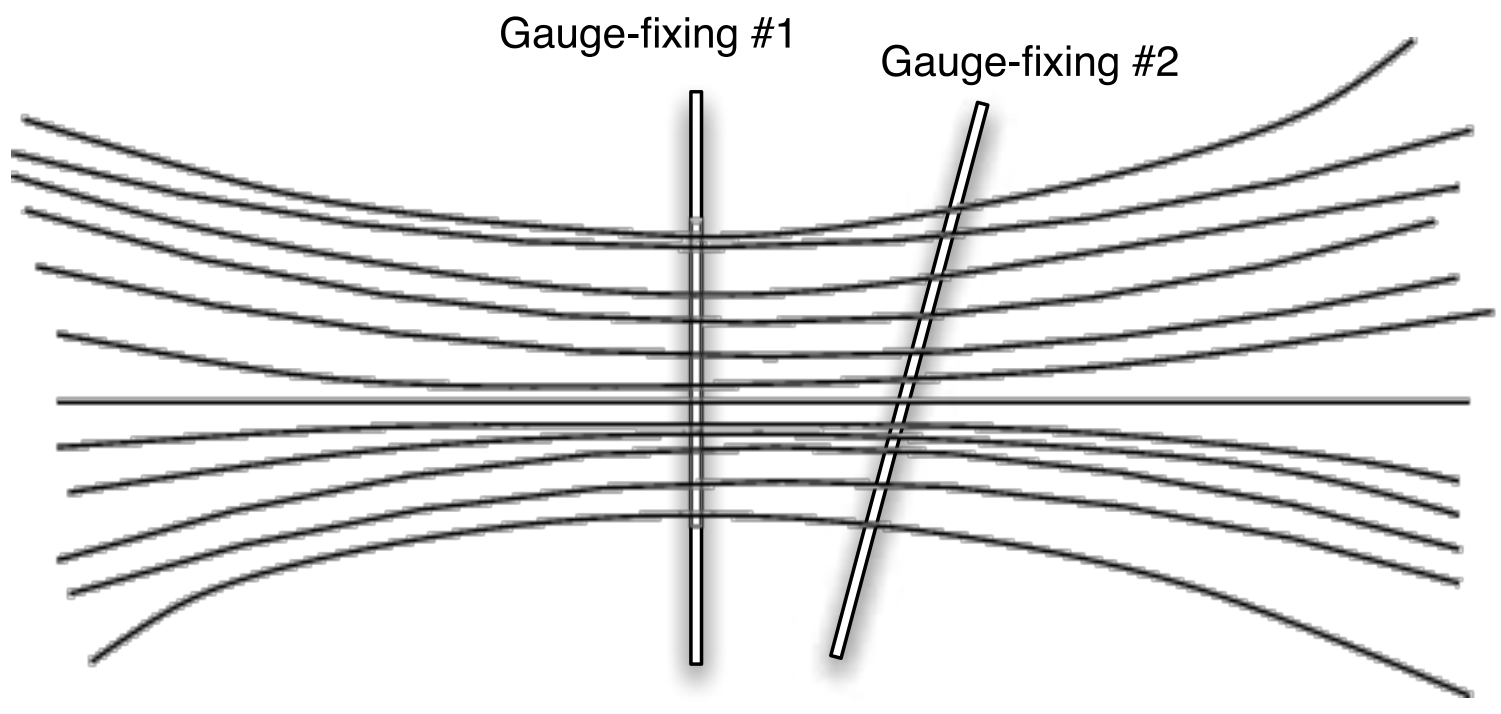
You guys know how to fix the gauge when you do GST, right?

No. We just do it anyway.

Any gauge-fixing assigns "error" to each individual gate.
But we know that sometimes the error is purely relational!

And "minimize error" usually conflicts with "be CP".

How does one *fix* the gauge?
What is a *gauge-fixing procedure*?

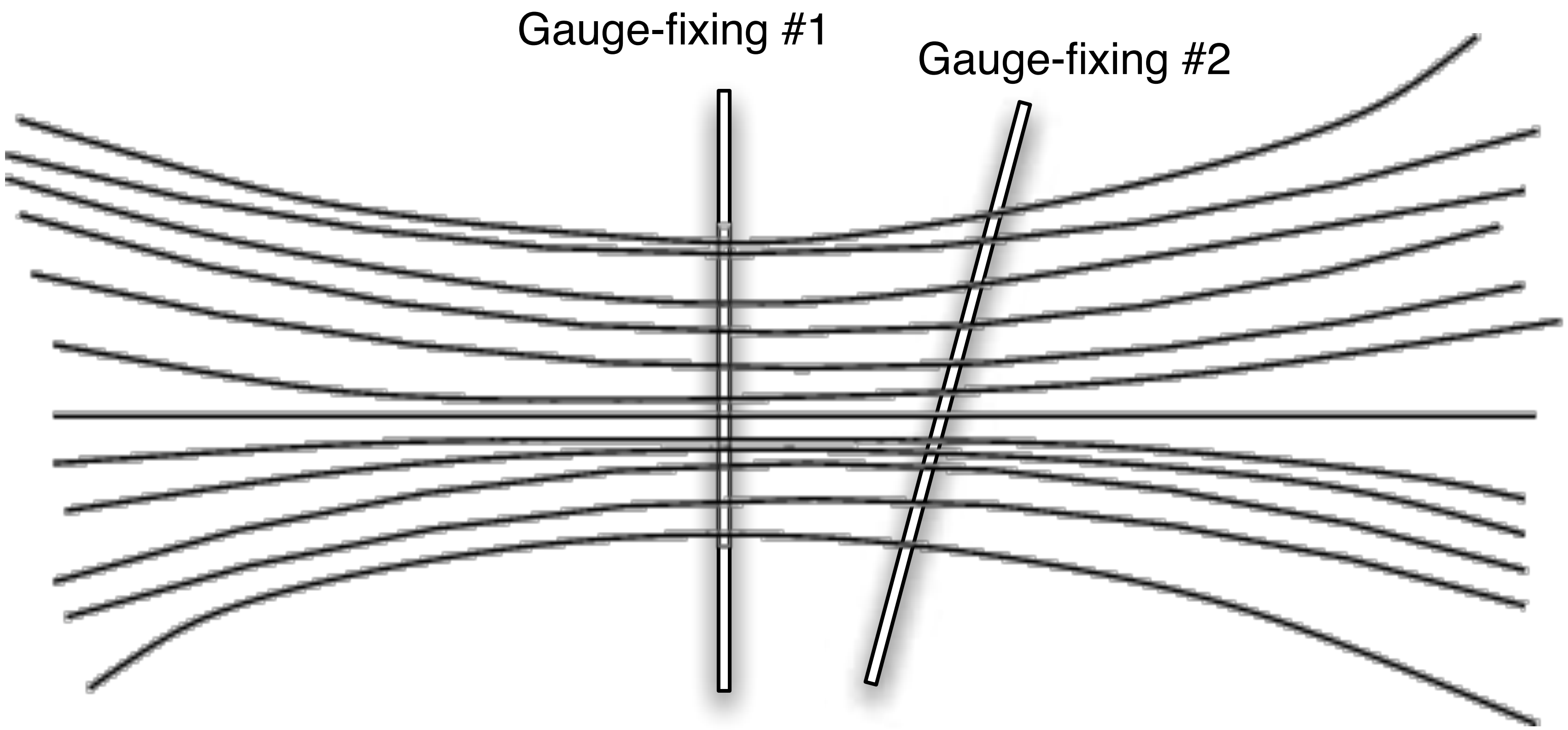


How does one *fix* the gauge?
What is a *gauge-fixing procedure*?

Gauge transformations partition gateset space into *gauge orbits*. A gauge-fixing procedure maps each orbit to a *single* "representative" point on that orbit.

Gauge-fixing procedures can be explicit ("enforce this condition"), implicit ("minimize this quantity") or algorithmic ("run this procedure").

Do you like fiber bundles?



Do you like fiber bundles?

No. Not really. But gauge freedom turns the space of possible gatesets into one.

At least I think so.

I don't actually know what a fiber bundle is.

I'm bored. I want something to do!

I'm bored. I want something to do!

Great! Here are some outstanding problems:

1. **Invent a useful, elegant *gauge-free* theory of QIPs.**

3. **Find gauge-invariant analogues of fidelity, diamond norm, etc.**
4. **Figure out whether a gateset is gauge-equivalent to a CP gateset.**
5. **Find a gauge-fixing procedure that ensures CP whenever possible.**
6. **Establish gauge-aware theories for your favorite QCVV protocols.**